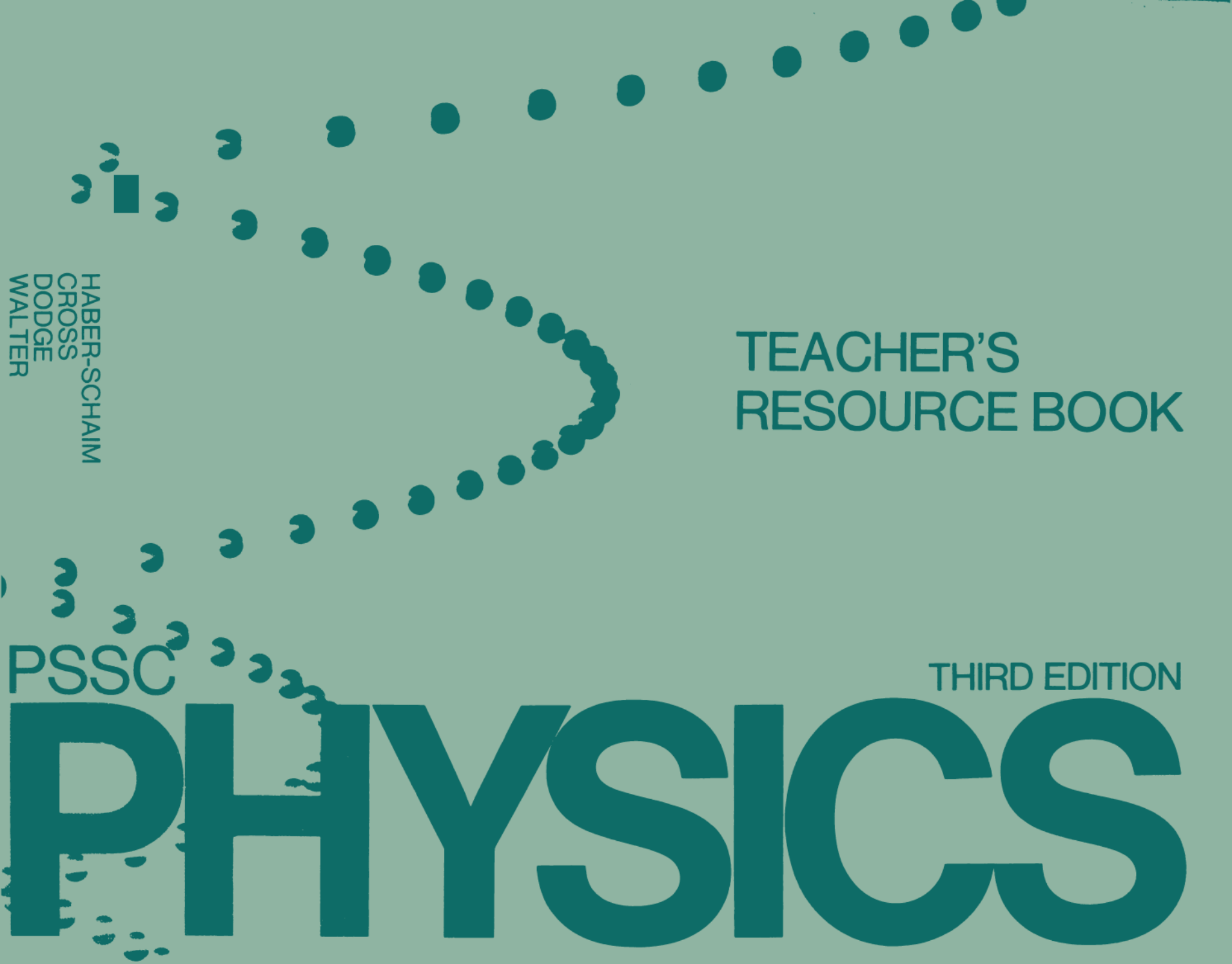

HABER-SCHAIM
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TEACHER'S RESOURCE BOOK

PSSC PHYSICS

THIRD EDITION

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Uri Haber-Schaim
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D.C. HEATH AND COMPANY
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Preface

If you are reading this preface, you are most likely in the process of planning your physics classes using the PSSC Physics Third Edition materials. In this case it is appropriate to raise the question, "What would you like your students to get out of the course?" The answers to this question, if taken seriously, will have a profound effect on what will go on in your classroom.

To help you formulate your own answers we shall list the goals we have in mind for the course. (They are reasonably close to those of the original author group.) You can then decide which are most suited to your class as a whole and to individual students within it, and emphasize those goals accordingly.

We feel that a course in physics must contribute its share to the general intellectual development of the student. That is, it has to give the student the opportunity—or, better yet, require him—to reason, to learn to express his thoughts clearly, and to be able to follow the development of ideas presented by others, whether orally or from the written page.

Physics was once defined as "the purposeful interrogation of nature." Thus the course should sharpen the student's skills as an observer and experimenter. There is a distinction between these two terms: to observe means only to note what is happening; to experiment includes also planning and making things happen.

Next comes the ability to analyze results and distinguish between essential and peripheral; without these skills there is no valid generalization. The limitations of a generalization must be recognized.

We would like the course to develop the student's aesthetic sense. This can be done in such diverse areas as learning to appreciate the beauty of a colored interference pattern or the abstract beauty of a concise mathematical formulation of a natural law.

All the materials for the course are geared to the goals which we have stated. It is the purpose of this Resource Book to help you and your students make the best use of all the components of the program.

The Laboratory

Almost throughout the year, the laboratory provides the backbone of the course. It is the place where new ideas are best introduced. This Resource Book indicates the optimal times for scheduling the experiments. It is, of course, most desirable that you have access to the laboratory at any physics period. This is an important point to remember when classroom schedules are arranged!

All too often an experiment degenerates into a "laboratory exercise" performed according to a rigid set of rules. The style of the students' Laboratory Guide is intended to prevent this from happening, it avoids "cookbook" instructions and keeps the experiments open ended. Whether you assign the reading of the experiment in the Laboratory Guide and discuss it in class before it is done, or simply discuss it first, is not very important. You may, in fact, start the year with discussion first and thereby help the students to read the Laboratory Guide intelligently. However, it is very important that your students have a clear understanding of the questions they are trying to answer by the experiment and the methods they are going to use to answer them.

The laboratory provides an ideal setting where you can adjust your role as instructor to the individual needs of your students. Some require considerable guidance, some are best left undisturbed, still others may enjoy an extra challenge. Learn to guide your students by raising questions rather than by providing answers.

Experiments can be performed by the students at their own stations independently of others, or by sharing the data with the rest of the class. For example, in the experiment on refraction of light, every student (or pair of students) can measure several angles of incidence and refraction and draw their own curve, or all data can be pooled and a much more reliable curve can be obtained. Not all the experiments can be treated in the latter way, at least without some manipulation of the data. However, we recommend that, whenever possible, you try this style of laboratory work. It has proved to be very effective. In any case, a summarizing discussion is necessary.

There seems to be a widespread belief that all experiments must give results accurate to three significant figures and that a result with an error of 50 percent is worthless. This is not true. For example, students have no *a priori* knowledge of the order of magnitude of the mass of the electron. The real achievement in Experiment 43 is to see that the mass is of the order 10^{-30} kg and not 10^{-25} or 10^{-32} kg. In general, the less we know about what we want to measure, the lower the accuracy we require to gain new information. On the other hand, if the student wants to determine whether the ratio of the angles of incidence and refraction or the ratio of their sines is constant, he must measure the angles accurately enough to tell the difference.

Concerning laboratory reports: More often than not, these are a chore for the student who has to write them and for the teacher who must read them. We recommend that each student keep a legible record of what he does at the time he does it. His notebook should contain answers to the questions raised in the Laboratory Guide.

Text

The reading of physics has to be learned, and the only place this can be done is in the physics class. If you want your students to be proficient in reading the text later in the school year, you must prepare to help them in the beginning.

It may be useful to introduce new material first by class discussion using photographs and drawings from the text and then assigning the appropriate section for reading. You may assign some sections for reading and have students summarize them in class. The most important point to remember is to adjust your assignments to the reading ability of your students.

Since one of the goals of the course is to show the development of ideas, the

text is written in a free-flowing style rather than in a style representing a collection of closed capsules containing factual information, a style in which it is impossible to convey the continuity and excitement of physics.

The importance of the text as a means of providing continuity varies. It is least important where the material is adequately carried by the laboratory and is more important where more reliance has to be placed on films.

"For Home, Desk, and Lab"

The problems at the end of each chapter vary from easy to quite hard. Some problems are quantitative; some are qualitative; some are more related to theory; some more to experiments. This means that you can tailor your homework assignments to the interest and abilities, not only of various classes, but to the individual student in each class. To do this effectively you must, of course, be familiar with the problems before you assign them. As with lab reports, we suggest that you do not require students to hand in written homework problems unless you plan to go through them thoroughly—not only checking the answers, but also commenting on the method of solution. As a general rule it is a good idea to encourage students to discuss their solutions at the blackboard.

The Third Edition of the textbook contains two kinds of problems for which answers are provided at the end of the book. Those marked by a "*" are generally simple one-step problems related directly to a given section of the text. Problems marked by a "†" are provided with an entire discussion of a method of solution.

Films

Many key experiments cannot be done in the classroom. Also, it is often very hard to perform a series of demonstrations in succession to introduce a new topic or provide an effective summary. Add to this the fact that physicists are not made from one mold and that your students should at least see and hear physicists other than yourself, and you will realize the tremendous value of the PSSC films. Their best use is indicated in this Resource Book. Just as with laboratory experiments, there is an optimum timing for films.

Screening a film in a physics class is not a show; do not hesitate to stop the projector to raise or answer questions on what was seen and to encourage predictions on what is likely to happen next. The films are an integral part of the course and when used properly save time in adequately studying a topic.

Judging Achievement

What a student learns in a physics class has so many components that no single tool can provide a just measurement of his achievement in the course. The course materials include a set of multiple-choice achievement tests with questions related to both laboratory work and theoretical material. Yet even these tests give only a partial indicator of student accomplishment. They tell you nothing about a student's ability to handle experiments, to analyze data, or to convey to others what he has done and learned. These things are hard to

measure in an objective way, and we suggest that you do not hesitate to give them considerable weight in determining a student's grade on a subjective basis. You may also have students who are very good at solving problems or getting results in the laboratory, but slow in working their way through a test. These students should not be penalized. In short, in a PSSC physics class there should be many ways of getting an "A."

If you feel that the general class work and the six achievement tests for the Third Edition are not enough, we suggest that you choose HDL questions to make up quizzes. If you wish to add some questions of your own, be sure they are in line with the objectives of the course.

In determining the final grade we suggest that you average grades only if a student did better at the beginning, but give more weight to grades in the latter part of the course if he improved his performance. In the PSSC course, as in the study of a language, the later parts build on the earlier ones. Some students develop more slowly than others. But if they achieve at a high level at the end, their slow start should not be held against them.

Acknowledgments

Large parts of this Guide were taken from the Second Edition. In checking the new parts and in rearranging some of the old material we were assisted by Mrs. Bette LeFeber. Benjamin T. Richards gave us valuable support as Production Editor.

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Judson B. Cross
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August 1970

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Curtain Raiser

To set the tone of the course and introduce your students to the kind of work they will be doing in it, it is useful to start off with Experiment 1 Analysis of an Experiment. This is an unusual experiment in that it does not involve the use of any apparatus or materials except graph paper, since the observations and measurements have already been made. The students' task is to take the data as presented in a table and analyze them graphically so that they can be used to arrive at a generalization and to predict the results of similar experiments. In future experiments your students will take their own data as well as analyze them in ways similar to those used here.

EXPERIMENT

1 Analysis of an Experiment

This experiment is designed to give your students the opportunity to devote a full laboratory period to the analysis of an experiment under your guidance. The experiment has been written to accommodate students with different mathematical backgrounds. All students should complete the graphs of t versus d and t versus $\frac{1}{d^2}$ and one graph of t versus h. The better students can be encouraged to proceed to the double interpolation and the investigation of the mathematical relation between t and h. Only students who have studied logarithms in mathematics should attempt to deal with the last two paragraphs.

Millimeter paper is recommended for the graphs. Students can make their own semi-log or log-log graph paper by using a plain sheet of paper and the C or B scale on a slide rule.

Answers to Questions

Figure (a) shows the t versus d plot for $h = 30.0$ cm on the largest scale. One can clearly see that the curve is not unique even if no unusual variations between measured points are assumed. The x in Fig. (b) shows the interpolated point at $d = 4.0$ cm. To extrapolate to $d = 8.0$ cm, the d scale has to be reduced as seen in Fig. (b). The upper curve in Fig. (c) shows that the conjecture $t \propto \frac{1}{d^2}$ is apparently correct for $h = 30.0$ cm. For that height, $t = 165 \frac{1}{d^2}$.

The other curves in Fig. (c) show that the conjecture appears to hold also for the other heights.

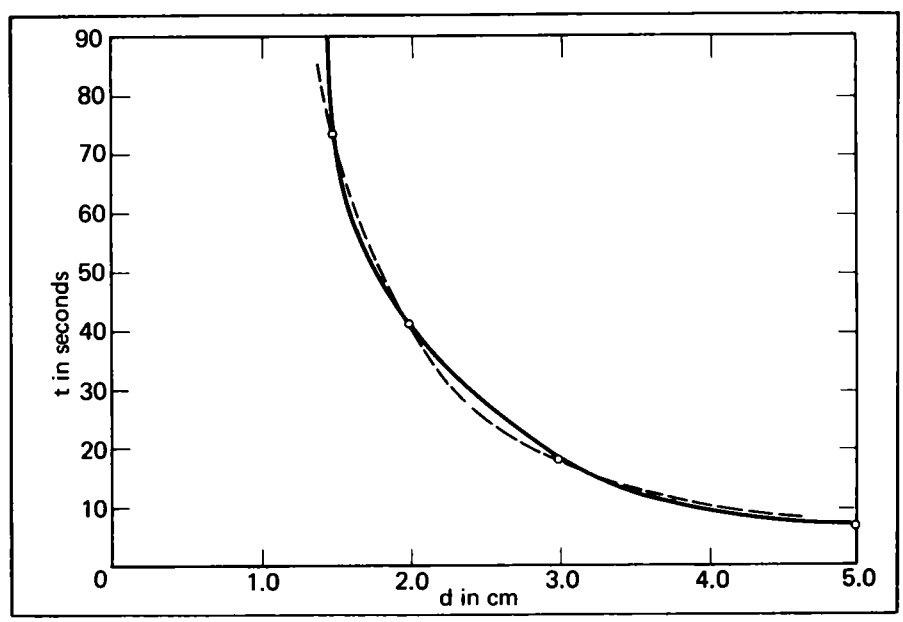
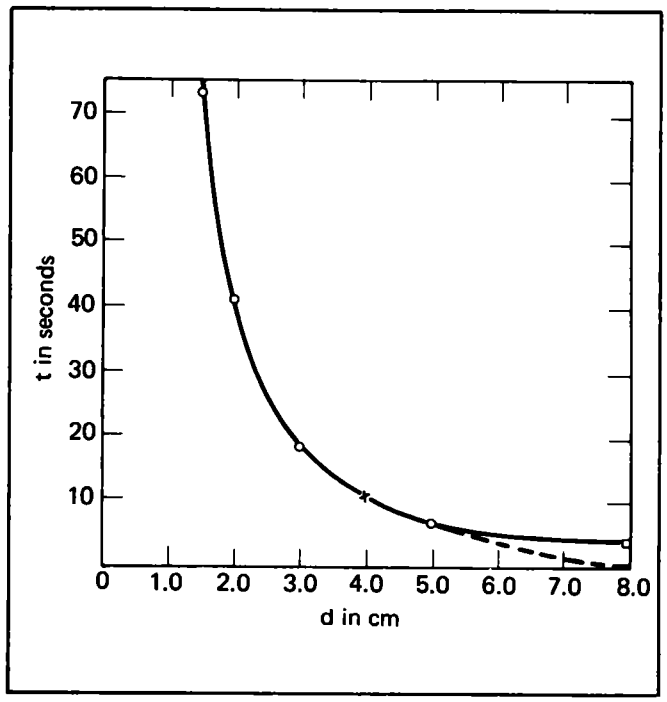


Figure (a)



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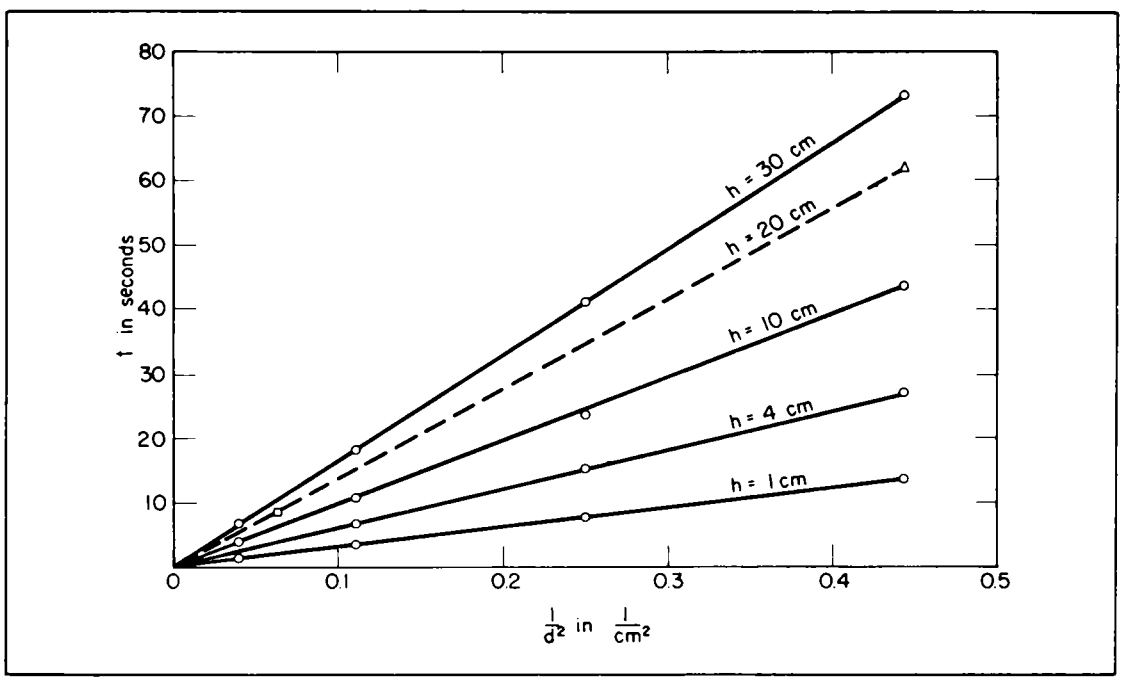


Figure (c)

On the enlarged t scale [Fig. (d)], one clearly recognizes that the experimental points are not on a straight line, but they are all within experimental error from such a line. We conclude that for $h = 1.0$ cm, the experiment is consistent with an inverse-square relation $t \propto \frac{1}{d^2}$, but it is not accurate enough to exclude sizable deviations from this relation. In practice, such a situation usually inspires more accurate measurements.

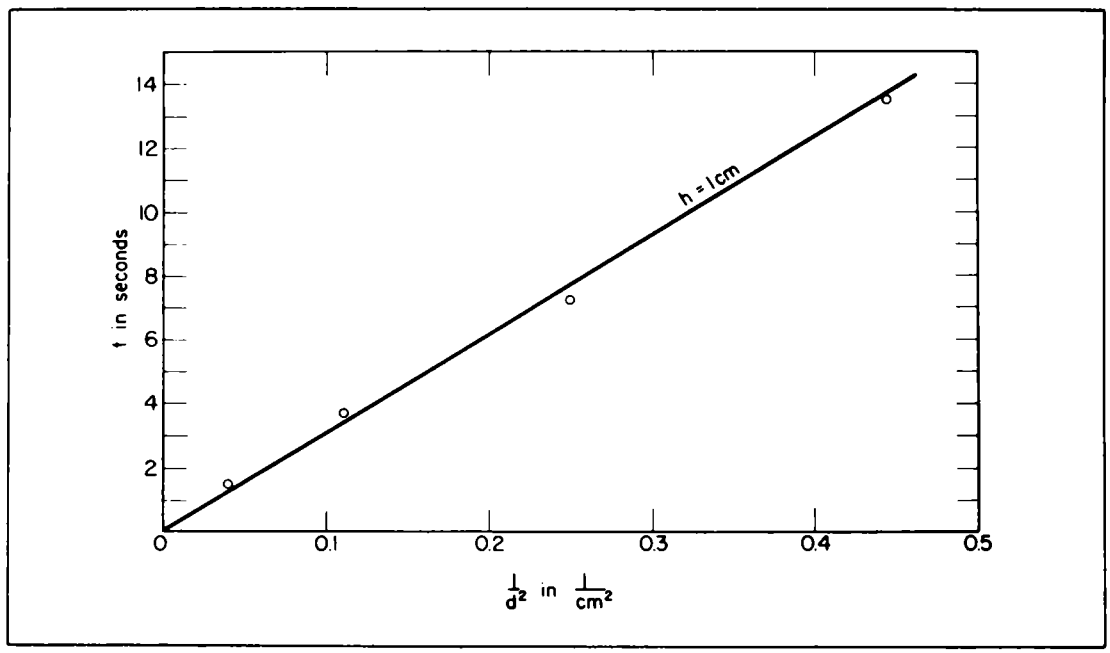


Figure (d)

The plot of t versus h [Fig. (e)] should, of course, pass through the origin since, if there is no water in the container, it takes no time to empty it.

To carry out the double interpolation for $h = 20.0$ cm and $d = 4.0$ cm, we first find t for $h = 20.0$ cm and $d = 1.5$ cm on Fig. (e). Since we have established that $t = \text{constant} \times \frac{1}{d^2}$, we can mark the point found in Fig. (e) on Fig. (c) and draw the straight line through the origin. On that line we read the value of t for $\frac{1}{d^2} = \frac{1}{4.0^2}$. A different method would be to read the values of t in Fig. (c) for all the four values of h and $\frac{1}{d^2} = \frac{1}{4.0^2}$ and then use these values to draw a new curve of t versus h on which we then interpolate for $h = 20$ cm. This method would also hold if t were not a linear function of $\frac{1}{d^2}$.

Students may find by trial and error that $t \propto \sqrt{h}$. This relation does not hold as accurately for $d = 5.0$ cm as it does for the longer times. Log t versus log h yields a straight line. The slope of the line of log t versus log h or the slope of t versus h on log-log paper is measured directly with a ruler and is equal to $\frac{1}{2}$ within the experimental error [Figs. (f) and (g)].

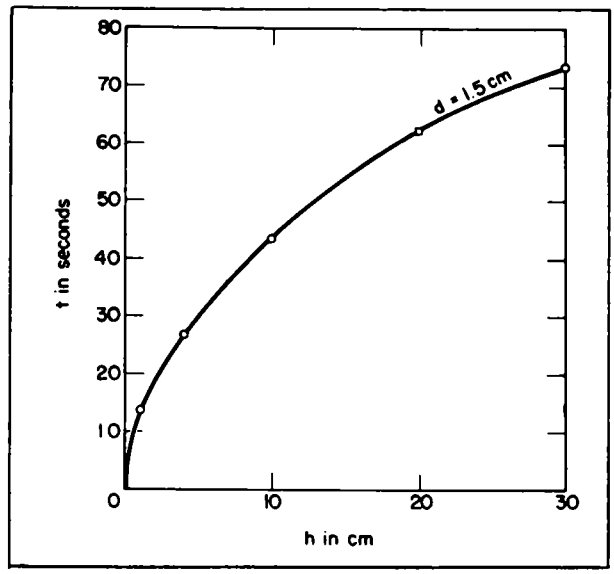


Figure (e)

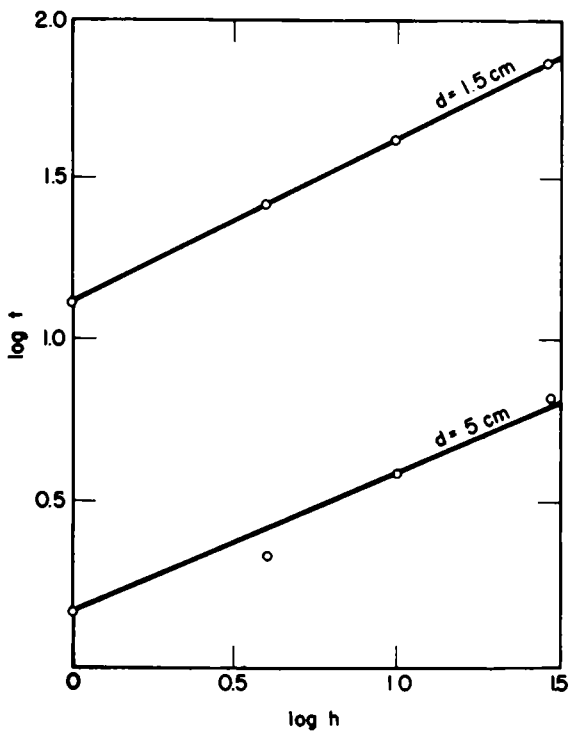


Figure (f)

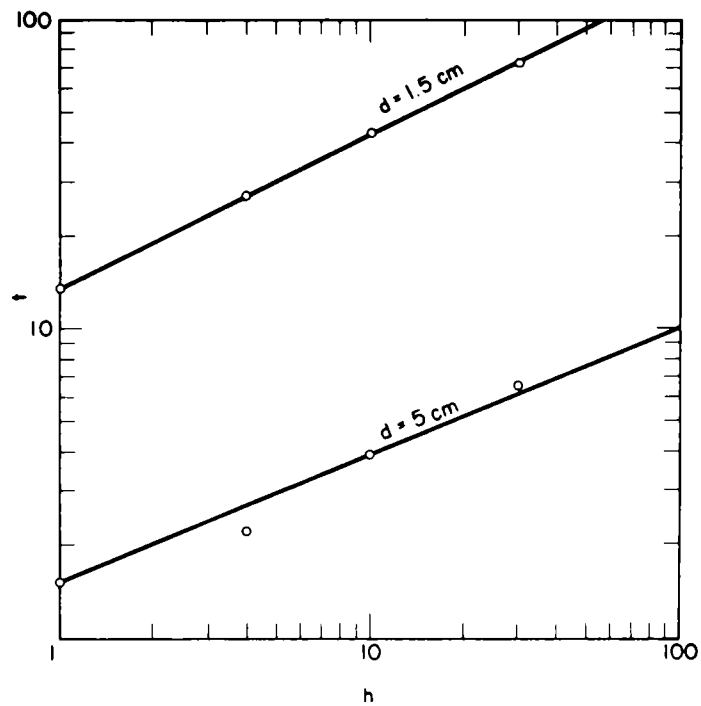


Figure (g)

Knowing that the dependence of t on d and h is of the form $t = \text{constant} \times \frac{\sqrt{h}}{d^2}$, the constant is found by multiplying the most accurately known time by $\frac{d^2}{\sqrt{h}}$, the result: constant = 30. The calculated value of t for $h = 20$ cm and $d = 4.0$ cm is more reliable than the value found by graphical interpolation.

Apparatus

4 Sheets rectilinear graph paper
1 Sheet log-log graph paper (2 × 2 min cycles)

CHAPTERS

The Study of Light 1-8

There is a coherent theme that draws these first eight chapters into a natural unit. They are devoted to the task of developing an adequate physical model to represent the properties of light. This book begins with an introduction that presents light in terms of its observable and familiar properties and establishes the geometrical optics of reflection and refraction at plane and certain curved surfaces. A particle model is proposed which gives the correct laws of reflection and refraction and predicts an inverse-square law for the variation of intensity with distance, but gives the wrong ratio for the speeds of light in air and in water.

Observations of one-dimensional waves in springs indicate that waves also exhibit some of the properties of light. The study of waves is extended to two dimensions by the use of water waves in a ripple tank and the laws of reflection and refraction are demonstrated for plane waves at plane boundaries. Interference and diffraction are studied with water waves and shown to exist for light, and the utility of a wave model for light is made clear.

In the course of this development, the students are drawn gradually into the geometrical and analytical arguments which are characteristic of physics. They are exposed to some basic theoretical physics in the contemplation of two models, one of which is "good" because it accounts for many observed phenomena, the other of which is "better" because it accounts for all those phenomena and for others in addition. In the process, the wave model acquires a rich content of meaning for them, so that when it is challenged later by the quantum theory of light, they have a real appreciation for the intellectual difficulties involved.

ADVANCE WARNING: In Chapter 6, your students begin to work with ripple tanks, one of the most interesting and instructive parts of the course. But, YOU NEED TO GET YOUR RIPPLE TANKS OPERATING NOW TO GAIN SOME EXPERIENCE BEFORE YOUR STUDENTS START TO USE THEM! No matter how busy you are, it is essential that you check your ripple tanks soon.

CHAPTER

1 How Light Behaves

This chapter begins the study of light by describing some of its properties, many but probably not all of which are already familiar to the students. It does not attempt to tell what light "is" but rather how it behaves. To answer the question, "What is X?" usually involves the naming of some more familiar entity from whose known qualities the behavior of X can be deduced. Here, however, our only reasonable course is to proceed in the opposite direction. The properties of light are already largely familiar, although some of them will need more precise description than can be had from common knowledge. When we know, or can discover, how light behaves, we are in a position to construct a model whose behavior corresponds. If it is a useful model, its properties may lead us to look for other properties of light we had not observed before, or to account for properties we had not understood. As the students follow the process of model building in the succeeding chapters, they may acquire some feeling for the way in which all physical theories are built and the extent to which they remain constructions of our minds.

Have your students read this chapter outside of class, and make it the basis of class discussion. The discussion should refer directly to the text, since the **Questions** do not cover all the points that are useful.

The content of this chapter might be thought of as a set of preliminary answers to the following questions. (The answers at this stage are necessarily crude.)

(a) How does light get started? From what kinds of material does it come?
(Section 1–1—Sources of Light)

(b) What happens when light hits an object? Does all the light enter the object? Does it go through? Does it bounce off the object?
(Section 1–2—Transparent, Colored, and Opaque Materials)
(Section 1–3—Reflection)
(Section 1–7—Shadows)

(c) Are there devices, other than eyes, which are sensitive to light?
(Section 1–4—Light-Sensitive Devices)

(d) Does light go in straight lines?
(Section 1–5—How Light Travels)

(e) Does light appear simultaneously at different places or does it travel with some definite speed?
(Section 1–6—The Speed of Light)

(f) How do we determine the location of an object that we see?
(Section 1–9—How We Locate Objects)

Emphasize the use of ray diagrams in the analysis of the formation of shadows from both point and extended sources of light; ray diagrams will be used repeatedly in the analysis of image formation by optical devices. Note that the way in which rays are used implies the properties observed in light pencils; for example, that light pencils can cross through each other without any change occurring in the pencils.

Since there are no student experiments for this chapter, it would be well to demonstrate as many of the phenomena discussed as you can with the equipment you have available.

CHAPTER

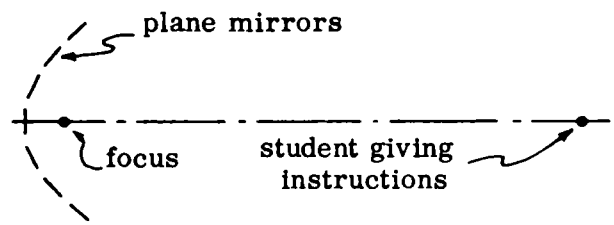
2 Reflection and Images

Chapter 1 introduced a physical definition of light rays and the means by which objects are located by sight. In this chapter, these ideas are used to treat reflection from plane and curved mirrors and the formation of images.

Experiment 2 should be performed before the laws of reflection are studied in the text. In the post-laboratory session, the meaning of such terms as "angle of incidence," "angle of reflection," and "normal to the surface" can be firmly established. Analysis of the experiment will show the equality of the angles of incidence and of reflection at the surface of the plane mirror. At the same time, the characteristics of the image formed by a plane mirror should be formulated.

Students should have little difficulty in following the development of a parabolic mirror from the description in Section 1–2 and 1–3. However, you may make it more meaningful, if you have available pieces of plane mirror of convenient size, by demonstrating the construction of an approximately parabolic mirror "by eye."

Arrange the small pieces in the form of a crude parabola about a nearby focus. Attaching the small mirrors with rubber bands to a soft aluminum strip might facilitate this procedure. Put a small, bright source (a flashlight bulb works well) at the intended focus. One student stays at the table with the mirrors to make adjustments and another student places himself, say, twenty feet in front of the mirror and gives instructions. The distant student looks at the small mirrors one at a time and calls instructions until he sees the source in the center of each section. A surprisingly accurate parabola can be constructed.



This section, while simple, is important to an understanding of what a curved mirror does. Again, make sure that your students have firmly in mind the meanings of the special terms that are introduced: principal focus, focal length, and axis of revolution.

Your students should understand and be able to reproduce accurately the constructions used in the analysis of image formation by parabolic mirrors. By means of a quick sketch, the students will always have the means of keeping straight the relationships involving object and image heights and distances. The same sort of procedure will be found advantageous in work with lenses later on. Such constructions are also a key in the understanding of the differences between real and virtual images.

Some students with good mathematics backgrounds may be familiar with the algebraic and geometric properties of the parabola. However, it would be an irrelevant diversion to devote any class time to a discussion of such matters.

CAUTION The analytical formula, $S_iS_o = f^2$, is derived partially as an illustration of how the geometry can be used to locate an image. The equivalent formula, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, could also be derived simply, but the reciprocals confuse some students and the correct assignment of signs to p, q, and f is more complicated than in the equation $S_iS_o = f^2$ (p is used in many physics texts as the distance from the object to the mirror while q is the distance from the image to the mirror). Each of these formulas has some advantages and some disadvantages. You can easily design problems which make either one seem superior. Every physicist has a slight preference for the form that he knows best; he naturally thinks of problems and situations in which the form he knows is most useful.

If you are most familiar with $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, a little practice will give you all the facility you need with $S_iS_o = f^2$. The inconvenience involved will be slight and it will not last long. Of course, students who do not have prior experience with either formula find $S_iS_o = f^2$ somewhat easier to learn and to work with.

Physicists who are adept with optics problems sometimes use both formulas interchangeably. However, when both formulas have been introduced in an early physics course, some students have become confused. They measure distances to the mirror when they should measure them to the focal point, and vice versa.

Inasmuch as you will be encouraging students to draw ray diagrams and to rederive the formula when they need it, this warning against introducing p and q as distances to the mirror may seem unnecessary. However, there will be times when you will want the students to use ray diagrams in which it is convenient for them to label the distance from the image or object to the mirror. In such cases let them call this distance anything they please. Labeling it with the pair of letters corresponding to the terminal points may be best. As soon as you reserve a particular letter for a particular distance, some students will be tempted to memorize and reuse any formulas they may derive.

The S_iS_o formulation will also be used to work with lenses in Chapter 3.

Some teachers may wonder why the text uses the so-called Newtonian form of the relation between image distance, object distance and focal length ($S_iS_o = f^2$), when the Gaussian form $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ is more commonly used.

For thin lenses, it is true, the Gaussian form has an apparent advantage of a single reference point (the lens position) from which one measures object and image distances and focal length. However, for thick lenses, not only does this apparent advantage disappear, but the question is raised as to just what is meant by the focal length of a thick lens. The Newtonian form still

uses the same pair of reference points (the principal foci) for thin lenses and for thick lenses (or combinations of thin lenses treated as a single thick lens).

Consider, for example, Experiment 4. It can be performed in exactly the same way, using a single thin lens or two converging lenses mounted at the ends of a cardboard tube 10 to 15 cm long. In this experiment one determines the locations of the principal foci of the lens system and establishes the relation S_iS_o = constant. The positive square root of this constant is then defined as the focal length of the lens system.

The Gaussian form is also valid for such a lens system provided one measures the object distance and the image distance from two different points, called the principal points. These are located at distances equal to the focal length toward the lens from each principal focus. More often than not, they lie inside the thick lens or lens system, and thus are difficult to locate experimentally.

EXPERIMENT

2 Reflection from a Plane Mirror

Only nails and pins are used as objects in this experiment; unlike triangles and letters, they are symmetrical, and finding their images in a plane mirror will not raise the matter of reversal.

Nails long enough to project above the mirror should be used in finding the image by parallax. Finding the image is then just a matter of "putting the top" on the image nail. (See Fig. 2–8 of the text.) Another way to hold the mirror is to fit it into a saw cut in a block of wood.

Because the angles between the rays and the reflecting surface are seen directly on the paper, these angles are the ones we compare. You may delay the introduction of the angles between the rays and the normal until you take up the subject in class or, if you prefer, call the student's attention to it during this experiment.

It is worth emphasizing that the more carefully the lines of sight are established, the more accurate the results will be. Sharp, hard-lead pencils should be used.

The lines of sight drawn to locate an image should form a large angle with each other (30° or more). If the angles are small, the lines of sight are nearly parallel and it is difficult to determine their point of intersection. The rays, angles, reflecting surface, object, image, and distances from the mirror should be carefully labeled. A sample diagram is not included in the Laboratory Guide. This enables the student to discover the paths of the reflected rays for himself.

With this apparatus, it is difficult for the student to see that the reflected ray, the normal, and the incident ray lie in the same plane; the proof is therefore left to Section 2–1 and Fig. 2–3 in the text.

Answers to Questions

When the head is moved to the left, the nearer of the two pencils will appear to move to the right. This exercise will refresh the student's memory of parallax and make clear to him at the outset that there is nothing mysterious about locating objects or images by parallax. In addition, noting that the nearer of two objects appears to move in the opposite direction to that of the

eye will be useful to the student in locating images by helping him to decide which way to move his parallax "indicator."

Many students will think that the image of the nail is in the plane of the mirror. If they come to this conclusion before making measurements, let them have the fun of discovering where the image really is.

Accurate measurements using a mirror with its rear surface silvered will show that both object and image are nearly the same distance from the back reflecting surface. Refraction, however, makes the image distance slightly smaller than the object distance (about 1 mm for a mirror 2 mm thick) as shown in Fig. (a). It is not necessary to discuss this small discrepancy at this time.

The image and object are the same size, although the image appears smaller because it is farther away. When a nail identical to the object is used as a parallax indicator, the indicator and the image are clearly seen to be of the same size.

The three images formed by two mirrors at right angles can be found as shown in Fig. (b).

Apparatus

2 Plane mirrors (about $1\frac{1}{2}'' \times 1\frac{1}{2}''$) (Front surface, if available)
2 Nails—flat head (about $1\frac{1}{2}''$ to $2\frac{1}{2}''$ long)
3 Pins (1" long)
3 Sheets of paper ($8\frac{1}{2}'' \times 11''$)
1 Sheet of soft cardboard or similar material ($8\frac{1}{2}'' \times 11''$)
1 Sharp, hard-lead pencil
1 Protractor
1 Wood block ($2'' \times 1'' \times 1''$)
1 Rubber band
1 Metric ruler

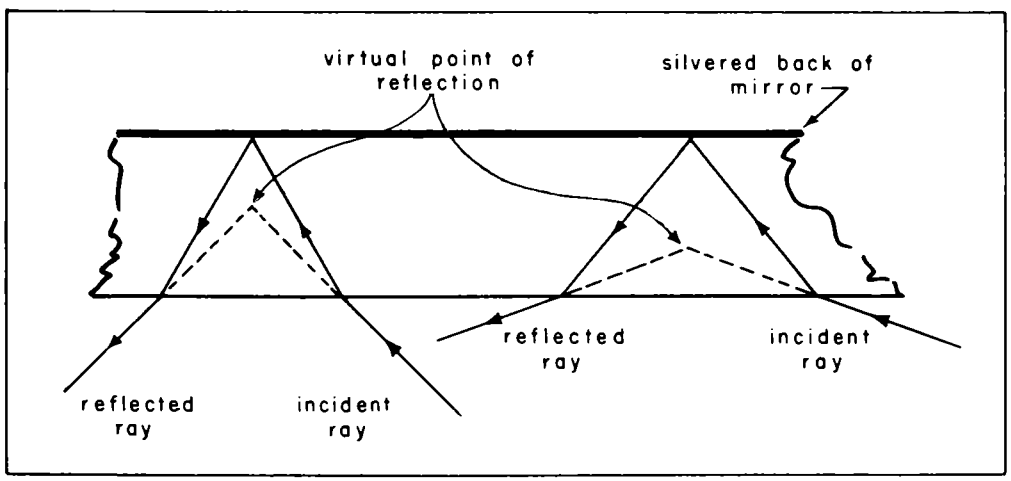


Figure (a)

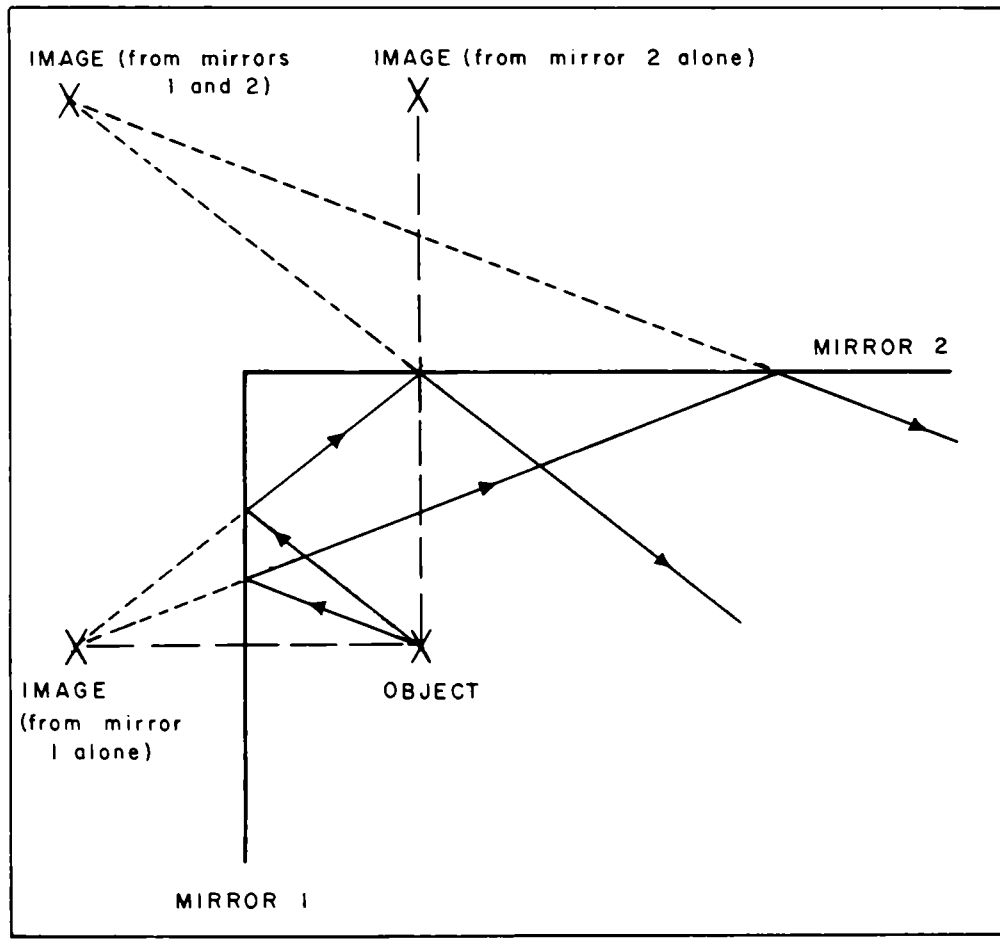


IMAGE (from mirrors 1 and 2)
IMAGE (from mirror 2 alone)
MIRROR 2
IMAGE (from mirror 1 alone)
OBJECT
MIRROR 1

Figure (b)

CHAPTER

3

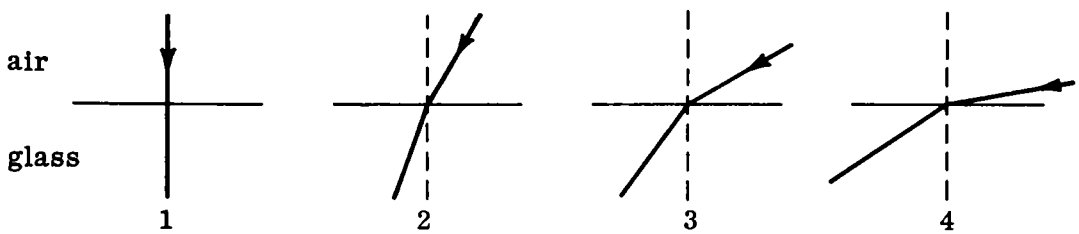
Refraction

The first sections are an experimental search to determine whether the behavior of a light ray as it bends in passing from air into another medium can be reduced to simple laws. Before getting into the first law of refraction, make it clear to your students that they are trying to summarize refraction phenomena with a few laws, just as they summarized all reflection phenomena with the two laws of reflection. If they can find laws which describe how a single light ray refracts, they will have tools that will enable them to reduce to simple terms the action of complicated refracting devices, such as lenses and prisms, in which the refraction of many rays must be considered.

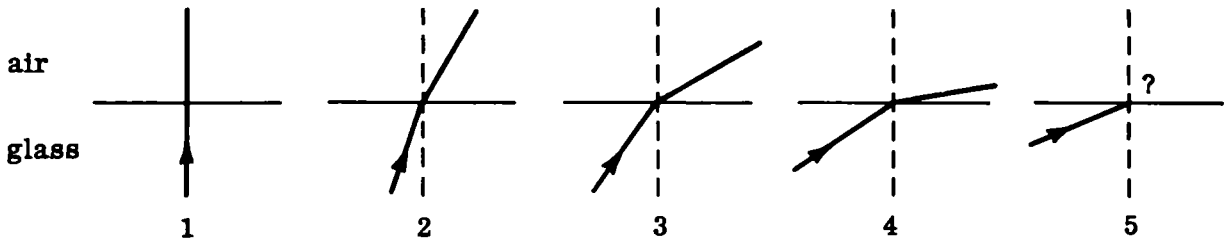
The purpose of Experiment 3 Refraction is to discover Snell's law. Therefore, the experiment is most appropriately done before the discussion of Sections 3–2 and 3–3. The text will then serve to organize their thinking and to point out relations which they may have overlooked in doing the experiment. If your students are not familiar with simple trigonometry, you may wish to defer completion of the post-laboratory discussion until after they have read the discussion of sine functions in Section 3–3 of the text. This material can also be used to show students how one looks for regularity to summarize experimental data.

It may be worthwhile to call to students' attention the fact that not every equation printed in a physics textbook becomes a "formula." Some are simply intermediate steps in an argument, and those which should be treasured for use are the most general and most easily understood. Of all the equations from Section 3–3 through Section 3–7, the form of Snell's law that is most worth remembering is

$$n_1 \sin \theta_1 = n_2 \sin {}_2.$$



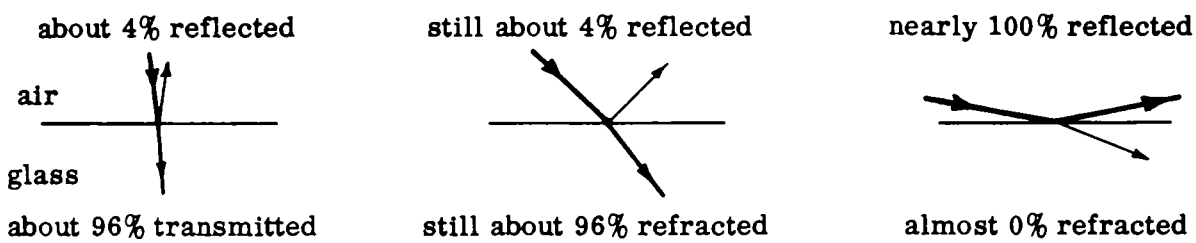
As you look at larger and larger angles of incidence for, say, air to glass, it is natural to apply reversibility and ask about a case not pictured above:



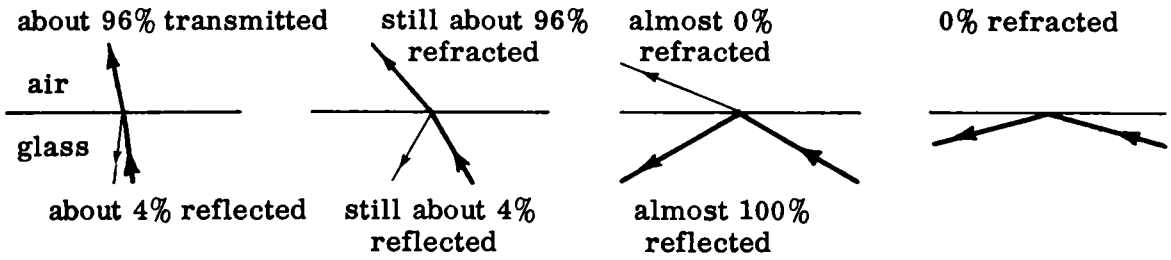
When presented with diagrams of this kind, students usually infer most of the facts about total internal reflection. In particular they should see that if a ray did refract into the air in case 5 above, Snell's law would be violated. There is nowhere in the air for it to go! Students should see from this development that the possibility of total reflection occurs only when light "attempts to leave" a material of higher refractive index to go into a material of lower refractive index.

You can use the idea of reversibility to describe the critical angle. From the previous development you will have shown (or be able to show) that, no matter what the angle of incidence for a ray traveling from air to glass ($n = 1.5$), the ray cannot be refracted more than 41.8° from the normal. Thus, for any given point on a glass surface, any incident light will be refracted within a cone (in the glass) whose vertex angle is 83.6°. Using reversibility, the student can see that, to escape from glass to air through the given point of incidence, a ray would have to travel within the same cone.

COMMENT Students should not get the idea that all of the incident light is refracted until the critical angle is reached, then suddenly all of it is reflected. If a ray is incident on glass from air, the percentage of reflection depends on the angle in air, θ_a. At $\theta_a = 0$, the percentage of light reflected is $\left(\frac{n-1}{n+1}\right)^2$.



As θ_a increases, this percentage remains constant through about 50° and then increases rather rapidly to 100%. In considering rays which start in the glass and make an angle of θ_g with the normal to the air surface, you find that the percentage reflection for θ_g is the same as the reflection for the corresponding θ_a. As sin θ_g approaches $1/n$ (and sin θ_a approaches 1), the reflection approaches 100%.



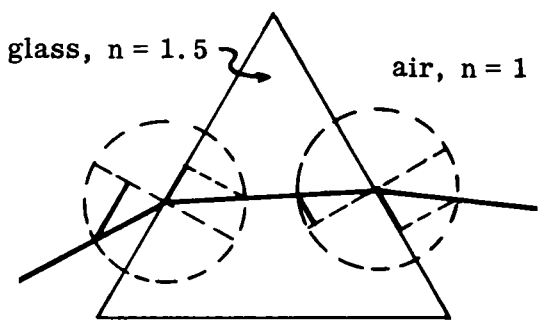
If you have the apparatus of Fig. 3–2, you can demonstrate the gradual change of reflected and transmitted intensities. The student should not be left with the impression that, at $\theta = \theta_c$, an intense light beam skims along the surface.

After students have a qualitative familiarity with total internal reflection it will take very little class time to establish the equation for the critical angle:

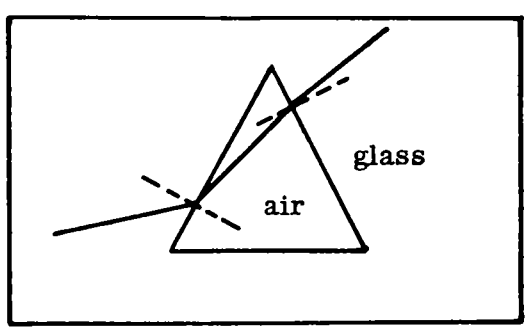
$$\theta_c \text{ is the angle where } \sin\theta_c = \frac{n_2}{n_1}.$$

Students should now realize that Snell's law (with constant n) is only an approximation for white light. But remind them that if only a single color is present, Snell's law is exact.

You will want to discuss at least briefly the path followed by a light ray of one color through a prism. Students should see such ray tracing as an extension of the work on parallel plates to nonparallel faces. At each surface they have a refraction problem to solve. A careful drawing is a good exercise that guarantees student understanding.



Also, students may enjoy the notion of an "air prism" embedded in water or glass.



Just as mirrors can be shaped to converge light by reflection, so most transparent substances can be made to converge light by refraction. Beginning with a set of tapered prisms, as a means to this end, the idea is extended first to a smoothly curved cylindrical lens and then to a spherical lens. The focal length is used as the basis for drawing ray diagrams to find images and to determine magnifications.

In Experiment 4 Images Formed by a Converging Lens, the student becomes familiar with the images formed by a converging lens and finds the mathematical relation between S_o and S_i. The best time to do it is before the discussion of Section 3–11. Work through Section 3–11 thoroughly until students can handle problems easily. Emphasize the ray diagrams. Discourage routine, unthinking use of the formula $S_iS_o = f^2$.

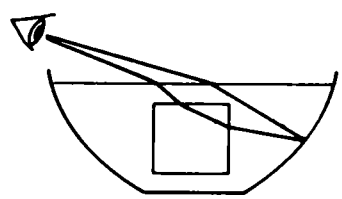
Experiment 5 A Diverging Lens is interesting but not essential to the development of the course. It may be done after the converging lens has been thoroughly discussed. It is not necessary to dwell on the sign conventions for dealing with diverging lenses.

At the end of this chapter, it would be appropriate to summarize the properties of light which have to be accounted for in developing a model for light.

Questions

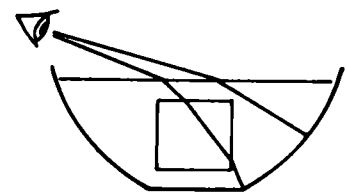
1. Here is a challenge for your class: Without looking it up, find whether the refractive index of ice is more than, less than, or the same as that of water. If you are lucky, someone will point out that since you can *see* ice submerged in water it must not have the same refractive index. A student who knows that water expands a little as it freezes may guess correctly (without logical foundation) that the refractive index for ice is less than for water.

   The nice problem is to show experimentally that ice has the smaller refractive index. Tell students that a coffee cup, water, and an ice cube (a clear one) are all they need. Hold the ice cube beneath the water and look just over an edge of the cube at a spot on the cup. If $n_i < n_w$, it is possible to see two spots simultaneously; otherwise not.



$n_i < n_w$

two spots visible

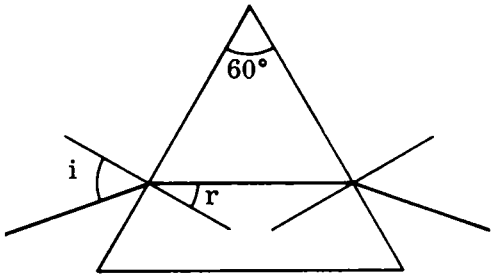


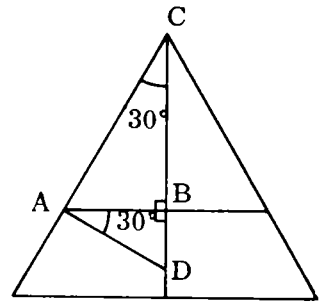
$n_i > n_w$

only one spot visible

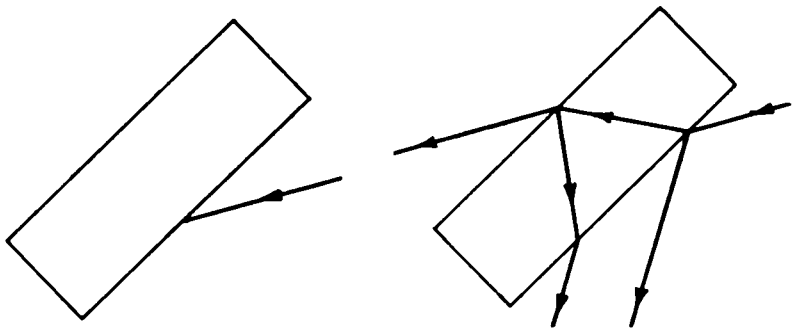
2. An equilateral glass prism has an index of refraction $n = 1.5$. For the ray of light that travels through the prism parallel to the base, find the incident angle.

   If we can determine the angle r of refraction, Snell's law will give the angle of incidence i immediately. Since the ray in the prism is parallel to the base, it is perpendicular to the bisector of the apex angle. The triangles ABC and ABD are similar, and $r = 30°$. Thus, sin i = 1.5; sin 30° = 0.75 from Snell's law, and $i = 49°$.



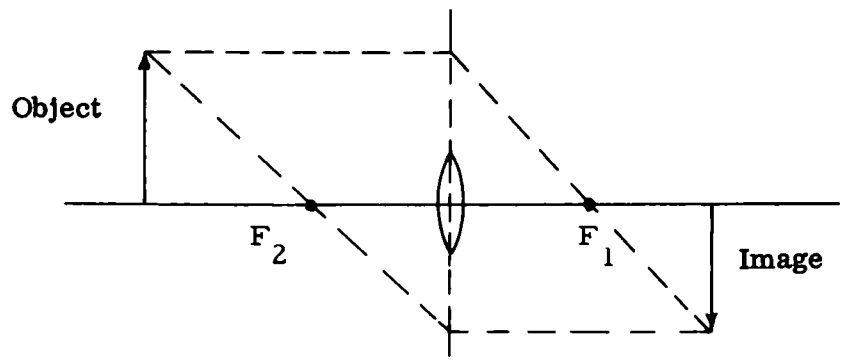


3. A good way to make reversibility clear is to draw a diagram as at the left below. Ask students to sketch in the resulting rays. They should know that there will be four rays. In addition to the refracted ray and the transmitted ray, there are the two reflected rays, one from each surface. (Notice that the diagram at the right looks like an upsidedown view of Fig. 3–7.) You should then make sure that students realize that what reversibility has to say about this new situation is simply that the refracted and transmitted rays in the new diagram coincide with the refracted and incident rays in Fig. 3–7. The limited form of reversibility discussed here says nothing about either the relative intensities or the reflected rays.

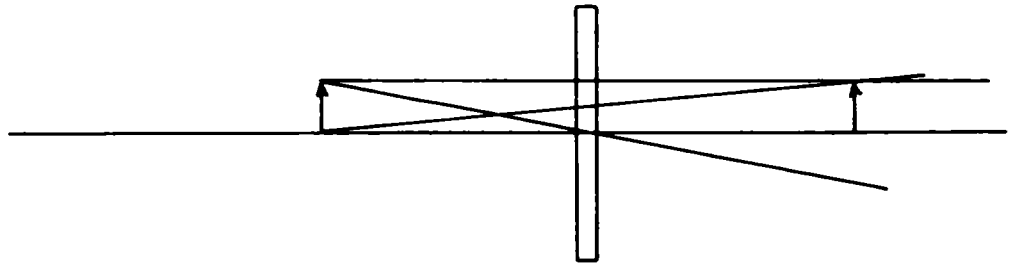


4. How would you arrange a selection of glass pieces such as this to concentrate a beam of light most intensely near a line?

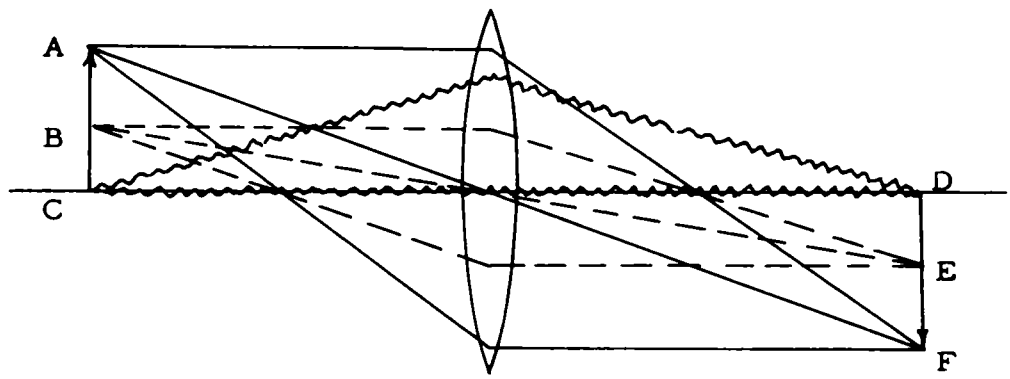
5. How can you find the image for the object shown below which is larger than the lens?



6. What is wrong with the following ray diagram and corresponding location of the "image"?



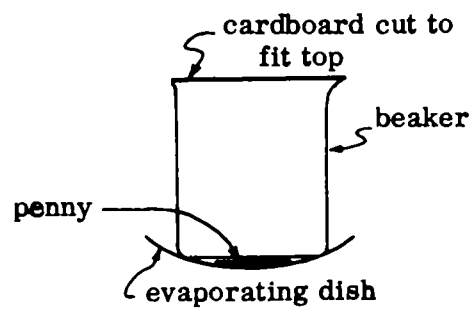
7. If your students dispose of Question 6 in short order, fine! If they do not, remind them that only the intersection of rays which *leave the same point* is useful in locating an image point. You can emphasize this by drawing a figure on the board like the one below. Draw (*in colored chalk*) all of the rays leaving point *A* in red, all leaving point *B* in yellow, and all leaving point *C* in white. Image points such as *D*, *E*, and *F* are the ones to be found when an image is to be located. Although this drawing looks like a mess in black and white, it will be very clear in a drawing using colored chalk. You will find that some students remember for a long time that the image occurs where "rays" of the same color intersect.



8. Have students locate the image of a tilted object.

Demonstrations

1. With a coin underneath a small, empty beaker in an evaporating dish, the coin is visible through the sides of the beaker. But when the beaker is filled with water and the top of the beaker covered with cardboard, the coin disappears.



2. A penny in the bottom of an evaporating dish on the lecture table cannot be seen by students at their seats. But if water is added to the dish the coin becomes visible. See below.

EXPERIMENT

Refraction 3

The vertical line on the straight side of the plastic box is more easily seen through the liquid if it is darkened with red pencil. With polar coordinate graph paper, it is possible to read the angles directly from the pin holes to within 0.2°. If other graph paper is used, the incident ray and refracted ray should be constructed with a sharp pencil to avoid unnecessary errors in angle measurement. Corrugated or soft cardboard placed under the graph paper makes pin placement easier.

The object pin should be about 4 cm from the vertical line on the plastic box. Otherwise, at greater distances and large angles of incidence, the image becomes dim and distorted. It may be necessary at large angles to place the pin closer to the vertical line than 4 cm.

You may wish to have different students do the experiment with different liquids. Some liquids that will not discolor or dissolve the plastic box are: glycerine, mineral oil, motor oil ($n \sim 1.5$); salt solution; sugar solution. Do not use carbon tetrachloride, carbon disulfide, turpentine, acids or bases.

If the experiment is performed with care, sin r vs. sin i is more nearly constant than r vs.i. [Figs. (a) and (b)]. But if a student fails to obtain accurate

Figure (a)

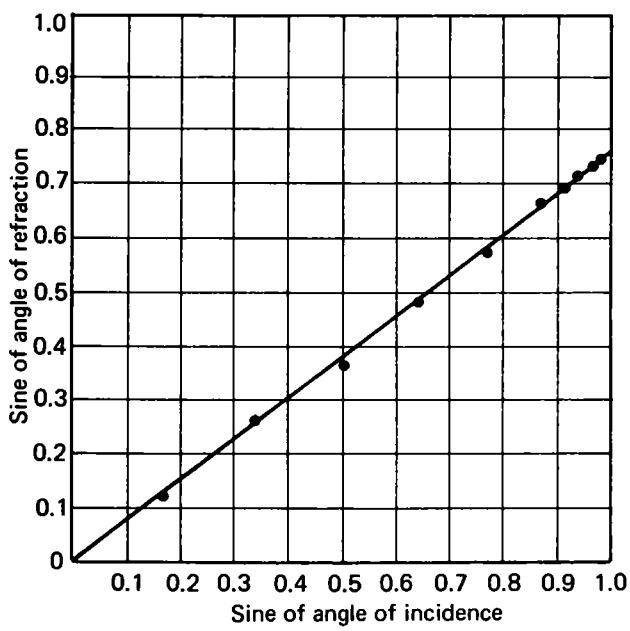
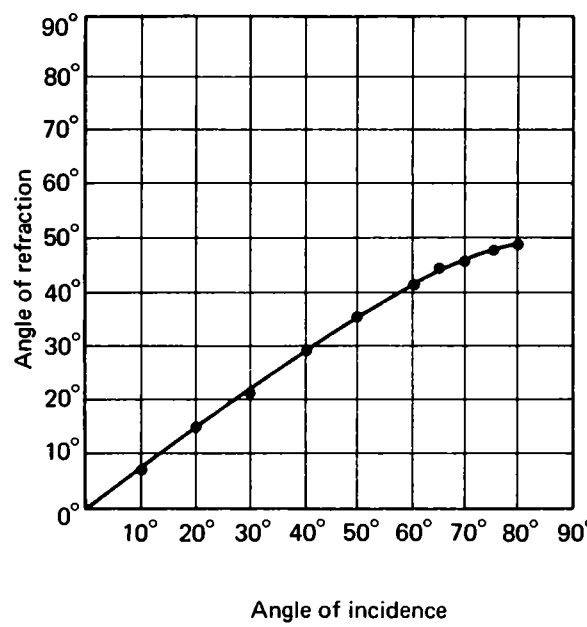


Figure (b)



data at large angles of incidence, he will not be able to decide which is the better relation. In borderline cases, suggest to the student that he repeat some of his measurements at large angles.

Answers to Questions

When the angle of incidence is 0°, light is not refracted.

Neither the difference between the angles of incidence and refraction nor their ratio is constant.

From the graph it can be see that sin r is proportional to sin i; therefore, the best mathematical relationship to describe refraction is $\frac{\sin i}{\sin r} = \text{constant}$.

The path of light is reversible.

The rays which have been refracted by the block of glass will be parallel to those entering the glass.

Another liquid will yield a different ratio for $\frac{\sin i}{\sin r}$.

Apparatus

1 Semicircular clear plastic box (6 cm radius, 3 cm deep)
4 Sheets rectangular coordinate graph paper, or 2 sheets rectangular and 2 sheets polar coordinate paper.
1 Sheet of soft cardboard or similar material about 8″ × 11″
2 Pins ~ 1″
Mineral oil (about 75 cm^3) or other suitable liquids
1 Sharp, hard-lead pencil
1 Protractor or a circular scribe (if polar paper is not used)
1 Rectangular block of glass or plastic, 2 sides clear polished (optional)
1 Ruler

EXPERIMENT

4 Images Formed by a Converging Lens

Because students will measure the object and image distances from the principal foci, a small error in measuring the focal length f will introduce a large error in the plotted results. A lens with a focal length of about 20 cm is best. However, if the focal length is more than 15 cm, a longer working space will be needed to determine it accurately. An accurate determination of the focal length by parallax can be made by locating the image of an object that is 5 meters or more from the lens.

Students will need a work area of about 2 meters. A 2-meter strip of paper like that to be used with the timer in Experiment 19, or adding-machine paper, fastened to the table with tape can be used as an "optical bench."

Two meter sticks, placed parallel to the line on the paper with one end of each corresponding to a principal focus, enable the student to read S_o and S_i directly. Note that S_o is always measured from the principal focus on the object side of the lens, while S_i is always measured from the principal focus on the other side of the lens. To avoid a sign convention, both S_o and S_i are considered distances and are, therefore, always positive. Make sure that the

student always looks at the object through the lens. The object should never be between the eye and the lens. If it is, the student will see images formed by reflection from the convex lens surface.

A flashlight bulb is used as the object because it is bright and stands out from the background. A small porcelain socket is best for holding the bulb in place. A dial light socket mounted on a small strip of wood also makes a good mounting. If neither of the above is available, the bulb can be wired by soldering two connecting wires to the base of the bulb, and then molding a lump of modeling clay around the base to hold the bulb upright. The filament of the bulb and the center of the lens should be the same height above the table.

Finding the image by parallax may be difficult for some students. You can quickly check on proper manipulation by comparing the $S_o S_i$ product with f^2. If the two values are off more than 5%, it may be necessary to give some individual help in locating the image. Since one of the purposes of the experiment is to have the student discover the relation between S_o and S_i, do not suggest that the student check each pair of values when he measures them.

A convenient parallax indicator is made of a piece of wire or a pin mounted in a cork stopper. The mounting is not essential, provided one is careful to hold the indicator vertically. Make sure that your students see that the real image stands out in front of the lens and that the virtual image is back of the lens.

Results with an error of up to 5% are not unusual. Care and patience will improve results, although this can be overdone and the purpose of the experiment lost. Measuring S_o and S_i to the nearest half centimeter will give satisfactory results for most object positions.

A half lens makes it much easier to locate the virtual image because it eliminates distortion due to spherical aberration and provides a straight surface to sight over. It is possible to cut a thin lens by scratching it across the diameter with a glass cutter and then breaking it. To avoid injury, smooth the sharp edges with sandpaper.

Answers to Questions

The image is smaller than the object when the object is beyond the principal focus by a distance greater than f, and larger than the object when the object is between the lens and the principal focus and a distance less than f beyond the principal focus.

The image is inverted when the object is beyond the focus, and right side up when the object is between the lens and the focus.

To convince yourself that the lens has two principal foci, simply turn the lens around.

The plot of S_1 vs. $1/S_o$ will give a straight line with the slope equal to f^2.

The image will not be visible when the object is at the principal focus or near it.

Apparatus

1 Converging lens (about 20 cm focal length)
1 Long strip of paper (about 1.5 m × 10 cm)
1 2.5-volt flashlight bulb (No. 41) and socket

1 No. 6 dry cell; 1½-volt
2 Connecting wires
1 Lump modeling clay (about 10 gm)
1 Sharp, hard-lead pencil
1 Meter stick
1 Piece of straight wire (about 3″ of No. 22) or a florist pin
1 Cork (No. 2 or No. 4)
1 Sheet graph paper

EXPERIMENT

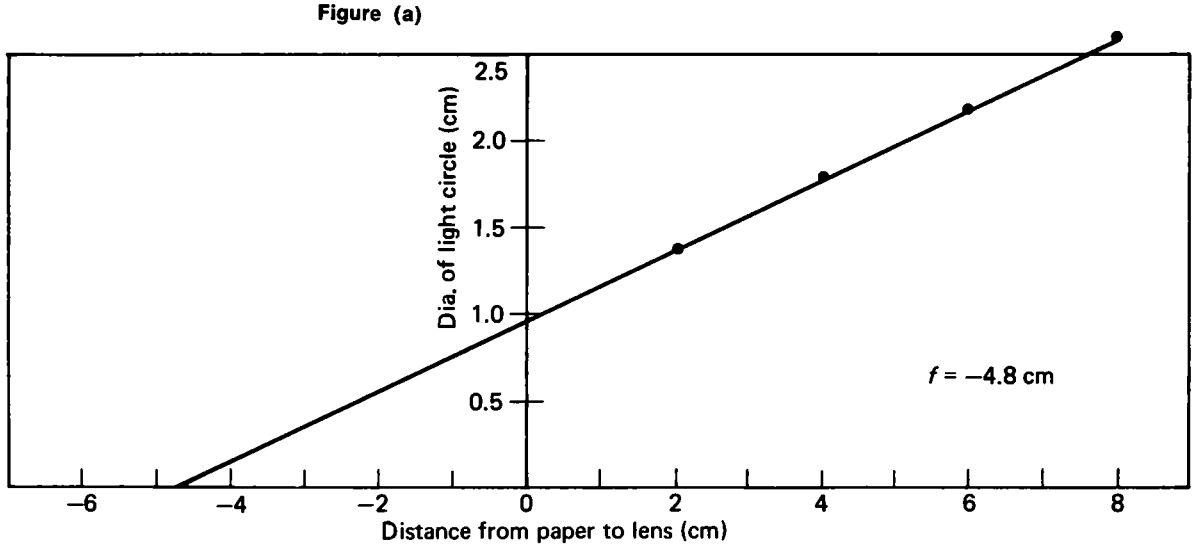
5 A Diverging Lens

This experiment employs an interesting technique to measure the focal length of a diverging lens. It will be easier to measure the diameter of the light circle if a piece of white cardboard is used on which the outside edges of the circle may be marked; the cardboard may also be mounted on plasticine. Students may have to be reminded not to block out the object while locating the image in the final part of the experiment.

If the diverging lens has a focal length between −5.0 cm and −10.0 cm, the change in the size of the circle's diameter per cm is reasonable (it is between 0.1 cm and 0.2 cm) and the image may be more easily located in the last part of the experiment.

Sample data are given in Figure (a). The focal length of the converging lens was −14.5 cm.

Figure (a)



Answers to Questions

The principal foci can be found from the graph. It is the value on the distance axis where the diameter is 0 cm. See Figure (b).

No. With a diverging lens the image will always be smaller than the object.

No. The image will always be virtual with a diverging lens.

When the light bulb is placed at one of the principal foci, the image will be located at one-half the distance between the principal focus and the lens; it will be one-half the size of the object. See Figure (c).

Figure (b)

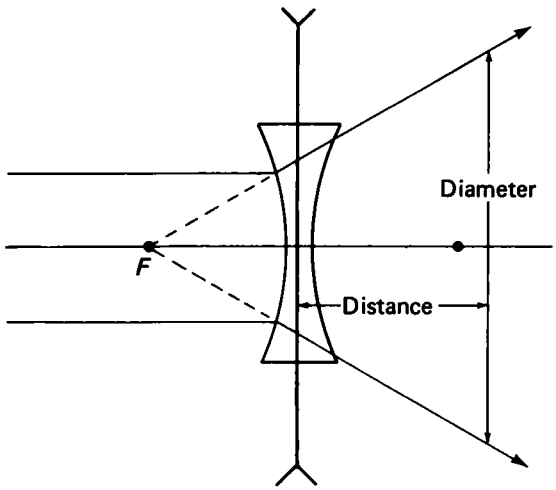
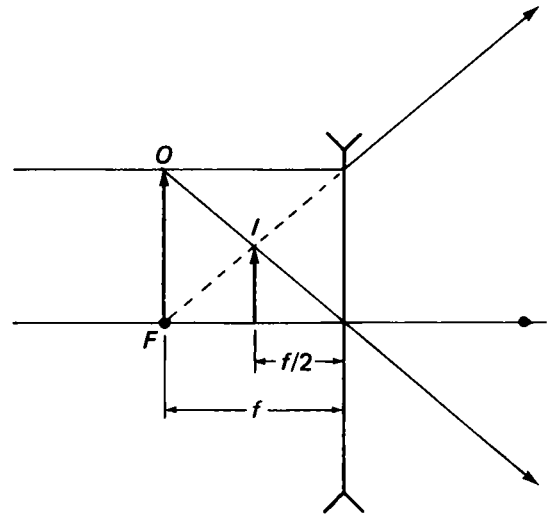


Figure (c)



Apparatus

1 Converging lens
1 Diverging lens (f between -5.0 cm and -10.0 cm)
1 Barrier with circular hole
1 2.5-volt flashlight bulb (No. 41) and socket
1 No. 6 dry cell; $1\frac{1}{2}$ volt
1 Meter stick
1 Sheet graph paper
Cardboard
Modeling clay or plasticine

CHAPTER

4 The Particle Model of Light

This chapter takes rather careful handling. The students will come to it with the conviction that all scientific theories are either true beyond question or false and to be completely rejected. The ideas which they should consider here are that scientific theories are inventions of the human imagination, that a theory may be partially successful in applying to some phenomena but not to others, and that a theory's usefulness is judged by the range of phenomena to which it applies and the valid predictions that can be made from it.

The particle model in its original form will be discarded eventually, but first help your students to realize that it is a successful model in many respects. The parts of the chapter which deal with reflection and refraction, with the inverse-square law of illumination, and with the pressure of light should be taken at their face value. Do not skip the experiment on the refraction of particles. It is most profitably done before discussing Section 4–5.

Only after the successes of the particle model have been appreciated is it time to consider its difficulties. Some of these can be overcome by assigning rather complex properties to the light particles and transparent materials. However, there is no dynamical model, in the Newtonian sense, that will predict a lower speed of light in refractive media than in a vacuum. Do not dwell on the incorrectness of the particle model. (You probably should point out explicitly that this is a very simple model.) Instead, just lead into Chapter 5 by asking what else, except a particle, moves.

EXPERIMENT

6 The "Refraction" of Particles

The aim of most of our experiments is to have the student discover new physical relations. For example, in Experiment 3 he discovered Snell's law. Here, we are not concerned per se with the change in direction of steel balls rolling down a slope; rather, we want to find specifically whether the change in direction can be described by Snell's law. In other words, the students are asked to test a hypothesis about light.

The incline and the upper surface of the platform should be of a smooth, rigid material such as Masonite. The two pieces, about $12'' \times 12''$ and $3'' \times 12''$, can be jointed together with a masking-tape hinge. The larger section can be placed about $1\frac{1}{2}''$ above the table. It may be necessary to tape a sheet of paper to the incline to prevent the ball from sliding.

Both the upper surface and the table should be smooth and level. If the table is rough, another piece of Masonite can be placed at the base of the incline. A quick visual check on whether further leveling is necessary is provided by rolling the steel ball slowly across the upper and lower surfaces. Paper wedges can be used to level the two surfaces.

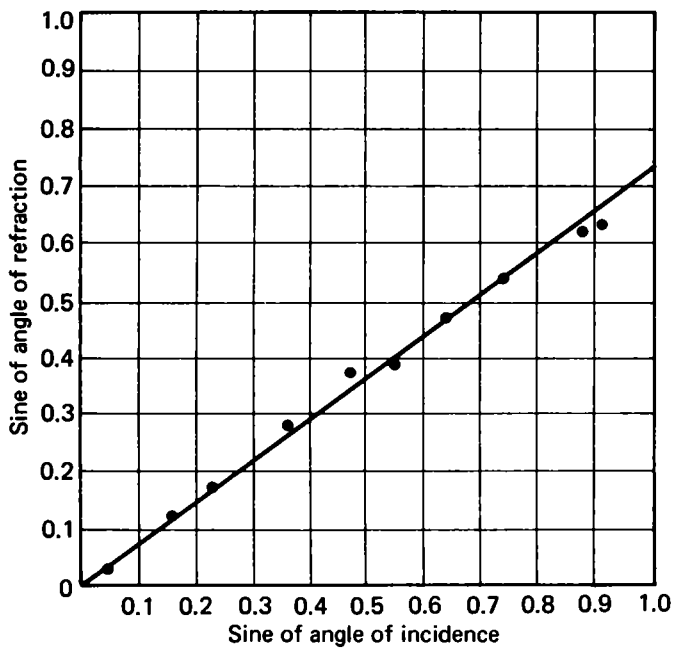
A ramp, to give the ball its initial velocity, can be made from a plastic ruler with a groove down the center. A 3″ section and a 1″ section of the ruler can be fastened together with glue. It is necessary to hold the ramp while the ball is accelerating so that the ball gains approximately the same speed on each trial.

The papers taped to the upper and lower surfaces must be carefully placed so that their edges are parallel to each other and remain so for each succeeding trial.

With the short starting ramp, it is possible to get large incident angles; however, it may be necessary to shift the paper on the lower surface to the right or left to pick up the trace.

Different students may find different indices of refraction because the initial speed of the ball or the height of the upper surface is different. The results of a typical run are shown in Fig. (a).

Figure (a)



Answers to Questions

The particle model of light predicts that the speed of light in water is greater than the speed of light in air.

It is possible to make a "lens" that will focus rolling balls if the upper and lower edges of the slope are parabolic in shape.

The ball must be started from the same height each time so that it will always have the same speed on the upper level. This corresponds to light having a constant speed in air.

An Extension of the Experiment

Total internal reflection can be illustrated by rolling a ball up the incline. A piece of Masonite must be placed on the table next to the incline to provide a continuous surface. The paper and soft carbon should be arranged on the two surfaces as described earlier. A starting ramp made from a ruler and a block of wood, about one inch higher than the upper surface, will give the ball sufficient speed to get over the top of the incline. The angle of incidence can be increased until the critical angle is reached. When the angle of incidence exceeds the critical angle, the ball will not reach the upper surface, but will curve and come back down.

Apparatus

1 Steel ball, 1″ diameter
2 Pieces ¼″ Masonite about 12″ × 15″ and 3″ × 15″
2 to 4 Sheets white paper, 8½″ × 11″
2 to 4 Sheets soft carbon paper (Carter No. 1432 for noiseless typewriter)
Grooved ruler (ramp) (Block of wood, 2″ × 1″ × 1″, to support ruler for launching ball, or other launching device)
Protractor
Heavy masking tape

Films

"The Pressure of Light," by Professor Jerrold B. Zacharias of M.I.T., is intended for use with Section 4. The film gives remarkable visual evidence that light exerts a pressure. It also shows how some experiments, designed to prove one theory, will introduce new problems and avenues of investigation. Running time: 21 minutes.

"The Speed of Light," by Professor William Siebert of M.I.T., is appropriate in connection with Section 7. The film shows an experimental measurement of the speed of light in air and an experimental comparison of the speeds of light in water and air. It should be scheduled after the class is ready to consider this discrepancy in the particle model. Running time: 23 minutes.

CHAPTER

5

Introduction to Waves

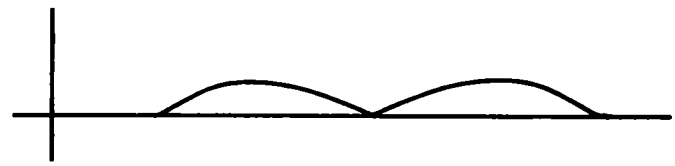
At this point some of the major characteristics of the behavior of light have been described and a first step has been taken to develop a model for the behavior of light. A simple particle model has been seen to be inadequate. The course now turns to another model—waves. The model must describe the travel of light through air, vacuum, glass, and water; it must encompass the laws of reflection and refraction; it must cover the noninteraction of intersecting light beams; it must predict the observed speed of light in various media; and it must explain diffraction effects. The next four chapters explore the characteristics of waves and their applicability in a model for light.

If some students show a little irritation at "developing a model, then throwing it away," this is a good sign of their involvement in the subject; the facts of life are that models are hard to give up. The Ptolemaic model of the solar system was clung to for centuries after its inadequacies were shown clearly. In any case, students should not get the idea that, with waves, we are now getting the "right" model. With waves we are taking the next step in the development of a model for light. Later it will be shown that, currently, particle and wave concepts must be combined.

If your students are to consider a wave model of light, they must first become familiar with the properties of waves on the laboratory scale. The essentials of this chapter are to be found in Experiment 7, which it is imperative that the students perform. This experiment covers essentially the whole chapter and, therefore, should not be done in one laboratory period in spite of the fact that it is primarily qualitative. If your schedule does not permit two half periods, the experiment is best done about the middle of the chapter. If you have a fixed amount of laboratory time per week, you may need to augment it with classroom demonstrations.

In order to understand superposition, it is important for students to understand the difference between a *picture* (or sketch) of a wave and a *graph* of a property of the wave. Students may have some difficulty with this because in Fig. 5–3 and 5–4 the graph closely resembles a picture. Both displacement-distance and displacement-time graphs are somewhat deceptive because they have the same general shape as the wave itself. If students still have difficulty with this distinction, you can mention that if the wave looked like this,

a graph of the absolute value of the displacement against a horizontal distance would look something like this:



In discussing the constructions involving superposition such as those given in Fig. 5–6, 5–8, and 5–9 it may be well to check to make sure that students understand that the dashed curves represent the displacement which would exist if each pulse were present alone.

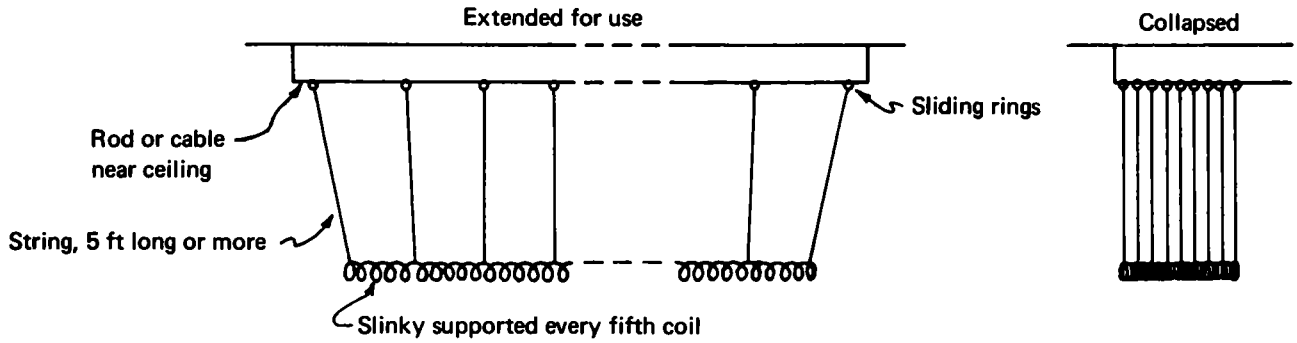
It is important for students to realize that partial transmission and partial reflection occur at boundaries. They should also know that the polarity of the reflected wave depends on the exact nature of the boundary. They should not be expected to have a good mechanical idea of why the pulses reflect as they do. Notice that the waves with which the students deal here are pulses rather than periodic waves, and you should take pains not to use the language of periodic waves in referring to them. In particular, reflected pulses should be described as "erect" or "inverted" rather than by any use of the word "phase." Phase is a concept that pertains to periodic waves and should be reserved until they are introduced.

The point of discussing "idealization" and "approximation" is not to produce definitions of them. Instead, the discussion should be directed to the idea that, in studying or working with complex phenomena, it is highly important to separate the essential from the inessential. Far from being a superficial explanation, an appropriate idealization or approximation is often a powerful intellectual tool for dealing with a complicated matter. Making the right idealization is often the most important part of the problem.

Having seen that waves can cross without interfering with each other, and that waves can be partially reflected and partially transmitted at a boundary, you next wish to inquire whether waves can be refracted. To do this, you must consider waves which are not confined to a single straight line.

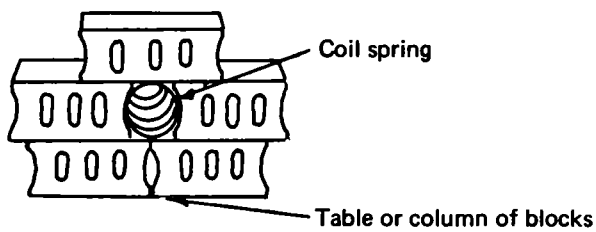
Demonstrations

1. A long piece of rubber tubing or flexible clothesline can be hung outdoors for demonstration. It has the advantage of small attenuation.
2. SETTING UP A STRING-SUPPORTED COIL SPRING The triple-length coil spring "Slinky" (supplied as a PSSC laboratory item) is the most satisfactory coil spring for this purpose. About every fifth or sixth coil should be supported by a string. Pick a spacing, say, every fifth coil, and then stick to this spacing. In order to produce waves of high amplitude which can be seen easily by an entire class, the supporting strings should be five or more feet long. When it is being used, the spring will be extended to a length of about 20 feet. In most cases it will need to be on a collapsible mounting, which can be made by running the individual supporting strings up to curtain rings which can slide along a rod or taut cable near the ceiling.



For good viewing, the coil spring should be about waist high. It can be a little higher if it is necessary to clear tables or benches. *Caution:* If you use an overhead cable for the rings to slide on, it needs to be a rather heavy one and stretched tightly. Tight clothesline is a borderline case. This is because waves in the coil spring set up waves in the supporting cable which, in turn, influence the wave on the coil spring. If you support the cable at several points, clothesline will do, but then you cannot collapse the coil spring for storage because the supports will prevent sliding the rings together. Another warning: once you have a coil spring extended 20 feet or so, do not let it snap together. It can become so inextricably tangled that destructive surgery is the only recourse.

In order to keep the coil spring extended to about twenty feet, some horizontal force must be applied. The figure above shows the end supporting strings somewhat diagonal. Such slanting supports will not keep the coil spring extended. The ends must be held. A fixed end is easy. Use a table or a pillar of concrete blocks to firmly "pinch" the end. Concrete blocks, either starting from the floor or starting on a table, are particularly convenient because they can be made to grip the coil spring rigidly without damaging it.



The person who is generating the pulses can stand at the other end. Shake a pulse into the spring, and it will travel to the fixed end and reflect (coming back on the opposite side) back toward you. If you want the pulse to reflect again from your end, you must grip the end of the spring as rigidly as possible(rest your elbow on a table, etc.). Even so you will not do as well as another pillar of concrete blocks. But holding one end with your hand, you will be able to trace a single pulse up and back between six and a dozen times.

Note: To observe these waves, students need to gather at the ends of the spring. A student near the middle of the spring does not see the wave nearly as well as does a student near an end.

Producing a free end is more difficult than producing a fixed end unless you have an extra long room. Ideally you produce an open end by holding that end with a long string (half the length of the extended spring or more).

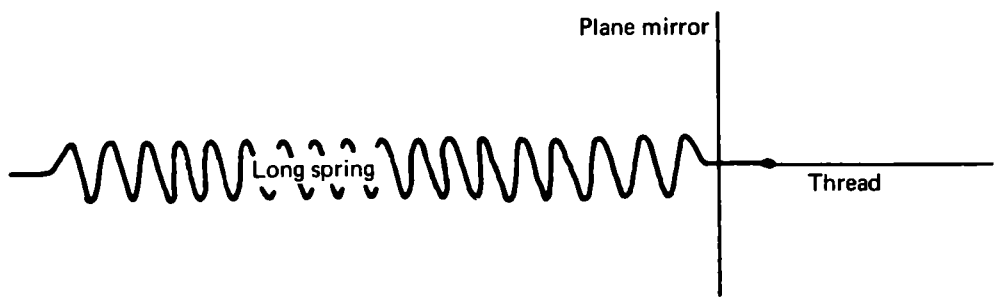
If you do not have a room long enough for both the extended spring and an open end, you will need to experiment with compromises involving shortening the length of the spring in order to increase the length of string at the open end.

Since there is very little natural damping in a suspended spring, one of the first problems you will meet is how to make a "disturbed" spring return quickly to rest. Most attempts to force the spring into a rest position with "bare hands" only result in more total agitation. (If you learn to "trap" the wave near one end, you can cut down its amplitude fairly quickly.) What you need is a way of extracting energy from the spring. Hold a large piece of cardboard or similar material loosely against the spring near the middle. Each time the wave crosses the cardboard the spring should scrape back and forth. The motion will die out quickly. If the cardboard is held so tightly against the spring that it cannot move, the motion will not die out rapidly. Experiments with different materials, and different forces of application, should result in finding a satisfactory method of rapid absorption.

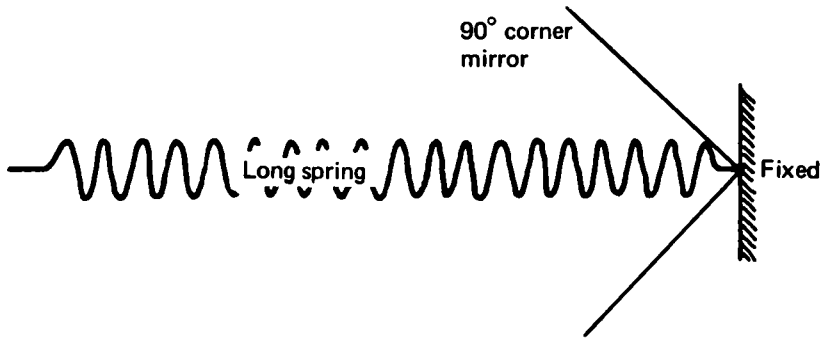
If you suspend a coil spring, you may want to keep it available for the introduction of Chapter 7.

3. PRESSURE OF WAVES The particle model predicts that light will exert pressure. The fact that waves also exert pressure can be demonstrated with a Slinky. Place a section of cardboard tubing about 2 inches in diameter and six inches long inside a Slinky and shake some pulses. Each time a pulse goes by, the tube will move a little farther away from the source. As it approaches a fixed end, it will slow down, since the reflected pulses push it in the opposite direction. The tube will not move if standing waves are produced, because the pushes balance out when waves are moving in both directions on the spring.
4. WAVE REFLECTIONS (Perform upon completion of Experiment 6.) The following demonstrations have been found helpful in giving the student a better understanding of reflections from free and fixed ends of a spring. The mirrors to be used in these demonstrations should be at least a foot square.

(1) Hold a plane mirror above the end of the spring, as in the diagram, so that the student shaking the spring can see its image in the mirror. When he shakes a pulse, it will seem to go into the mirror and the reflection of the pulse will seem to come out of the mirror and down the spring. The action of the Slinky at the junction will be the same as if two identical pulses from opposite directions met at the mirror. A long thread approximates a free end.

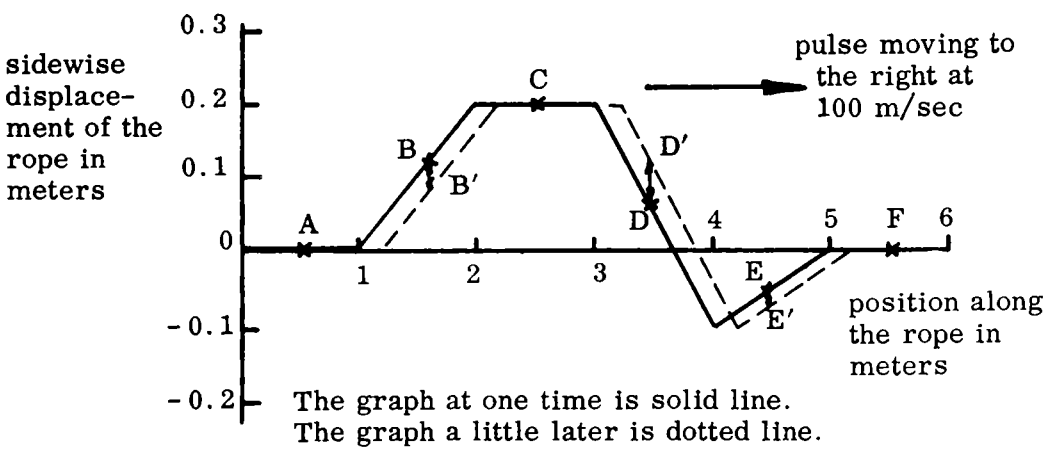


(2) A 90° corner mirror held above the fixed end of a spring, as shown in the diagram, will show the action in the case of a fixed end.



Questions

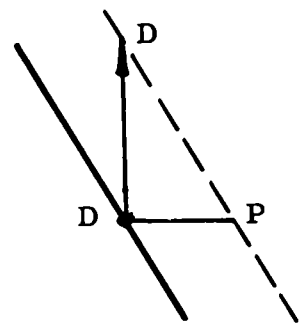
1. Ask students whether the graphs in Fig. 5–3 and 5–4 really look like one of the instantaneous frames from Fig. 5–1 or 5–2. (You can return to the blurring later.) They should realize that an actual pulse would not have a sharp corner. (It is impossible to produce an actual pulses whose slope changes so abruptly.) The sharp corners are used merely as an approximation convenient for preliminary study.
2. In Fig. 5–3 and 5–4, what indicates the average velocity of each point? The arrows. Their direction gives the direction of the piece of rope; their length gives the speed.
3. What is the velocity of the wave? Be sure that the students realize that:
   (a) The wave itself must have a single velocity; otherwise its shape would change;
   (b) The direction of the *wave's velocity* is different from the direction of motion of an individual point on the rope;
   (c) The magnitude of the wave velocity is not directly dependent upon the magnitude of the particle velocities. If the displacement of each point were doubled, the particle velocities would double but the wave velocity would be unchanged (or almost unchanged).
4. Ask about points *A, B, C, D,* and *E* in the drawing. Which ones are moving up, down, or standing still? Many students will know immediately. For other students be sure to draw in the dotted-line curve showing where the



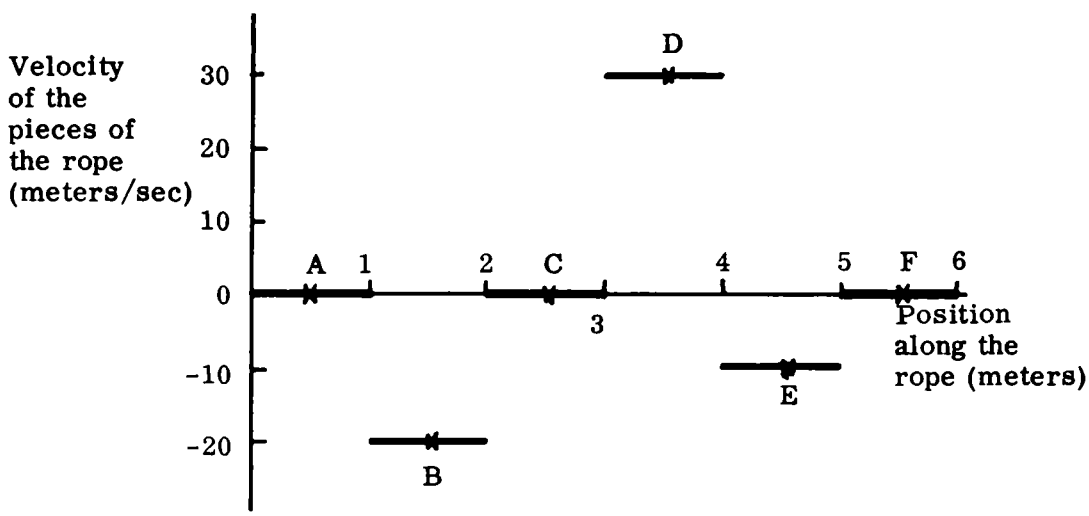
pulse will be 0.002 second later. (At this qualitative stage you don't need to specify the 0.002.) It should be clear immediately that points *A*, *C*, and *F* have not moved and, therefore, that they have zero velocity. Point *B*, during this 0.002 second, moved down and therefore has a velocity down. *D* moved up to *D'* and therefore has a velocity up. *E* to *E'* shows that *E* is moving down.

Next you may inquire about which points are moving fastest. It should be clear, just from the drawing, that *D* moved farthest in the given 0.002 second, *B* next, and *E* the smallest distance—except, of course, for those points that remained at rest.

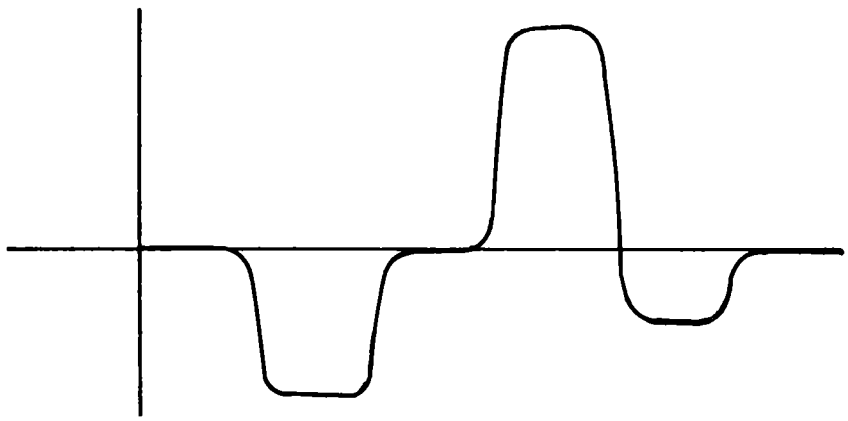
The next question is a quantitative one. What is the velocity of point *D*? To answer this, refer to a larger graph.



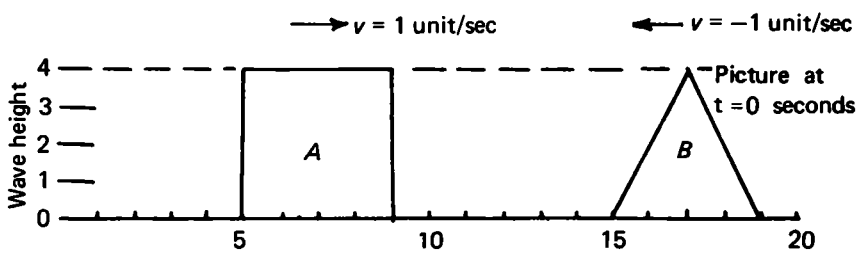
During the given time interval, *D* moved a distance *DD'* while the *wave* moved a distance *DP*. In our particular case, the distance *DP* is 0.002 sec × 100 m/sec = 0.2 meter. Referring to the original graph it is easy to see that $\frac{DD'}{DP}$ $\frac{0.3 \text{ meter}}{1.0 \text{ meter}}$. Then $\frac{DD'}{DP} = 0.3$. $DD' = 0.3 \times 0.2 = 0.06$ meter. Since this motion occurred in 0.002 second, point *D* is moving with a speed 0.06 ÷ 0.002 = 30 meters/sec. We can do the same for points *B* and *E*, obtaining 20 and 10 meters/sec, respectively.



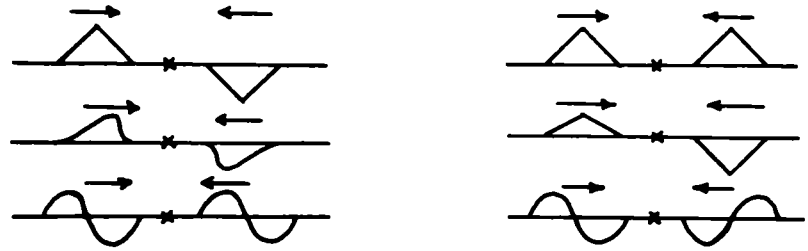
It is now time to point out, if some student hasn't by this time, that the *whole piece of rope* from 1 to 2 meters is moving with the velocity of point *B*. The whole piece from 2 to 3 meters is standing still, etc. Now a graph of velocity vs. position along the rope can be made. The graph above is discontinuous. This is because our wave had sharp corners. Actual waves do not, so that this velocity graph in a practical case might look like the one below:



5. Ask a student to explain Figure 5–8B in order to see how well he appreciates the superposition principle and the distinction between a graph of a pulse and a picture of the pulse. Be sure the class realizes that the dashed curves are graphs of the displacements that would be caused by the separate pulses. A picture of the rope at the instant of part B of Figure 5–8 would be merely a horizontal line.



6. If we had only pulse *A* in the drawing, what would be the shape of the rope after 1 second? After 2 seconds? After 10 seconds? If we had only pulse *B* what would be the shape of the rope after 1 second? After 2 seconds? After 5 seconds? After 10 seconds? If only pulse *A* were present, where would the point at horizontal position 12 be after 2 seconds? If only *B* were present? If both were present? Etc.
7. What similarity must exist between two pulses if the point at which they meet is to remain undisturbed? Since this same question recurs in the next section and at several places in Chapter 7, it is worthwhile to discuss it in class. You may wish to draw several examples on the board and make sure that the students understand which ones leave their midpoint on the rope undisturbed. Those on the left do and those on the right do not:



EXPERIMENT

7 Waves on a Coil Spring

Chapter 5 presents many new ideas and concepts, which will be better understood if the students have the opportunity to study them in the laboratory. This experiment covers essentially the whole chapter and, therefore, should not be done in one laboratory period in spite of the fact that it is primarily qualitative. If your schedule does not permit two half periods, the experiment is best done about the middle of the chapter.

The various paragraphs are almost independent of one another, so that certain parts of the experiment can be done as class demonstrations. For example, you may wish to demonstrate the part that requires two different coils hooked together.

We use a coil spring in this experiment because pulses propagate slowly enough along it to be easily observed. A smooth floor is essential. The combination of the small round-wire coil and the large flat coil (Fig. 1) gives an optimum splitting of a pulse into a reflected and a transmitted part.

The partial reflection and transmission at the junction of the two coils is best seen when the two coils are stretched to equal length. To achieve this, only about one third of the large coil should be stretched.

The laboratory guide does not contain instructions on how to shake pulses. We believe that this is best learned by seeing it done. To obtain short pulses a rapid snapping motion of the arm is required. Care should be taken not to overshoot the starting position during the backswing of the arm; this will avoid pulses of complicated shapes. A student can avoid overshooting by using his foot as a stop. While the large coil is stretched, a twist is introduced which causes a different attenuation of the head and the tail of the pulse, thereby distorting it. To avoid this, have your students attach a thread to one end of the coil as shown in Fig. 2 and hold the thread while stretching the spring. The thread permits the spring to "unwind," thus preventing the build-up of the twist.

To study the reflection of a pulse at the junction of the large coil and the thread, the thread must be several meters long. If a short thread is used, the head of the pulse will reflect from the fixed end of the thread and superpose with the tail of the pulse before the latter reaches the junction.

Answers to Questions

The pulse definitely attenuates as it travels along the spring. Only if the floor is smooth enough, the spring tight enough, and the twist removed, can

the student be expected to conclude that the shape of the pulse remains basically unchanged.

The speed of the pulse does not change as it moves along the spring; it is independent of the size of the pulse within the experimental accuracy obtainable with a stop watch.

The assumption that the speed of the pulse does not decrease upon each reflection can be checked by comparing the time it takes the pulse to make two or more trips down and back with the time required for one round trip.

The speed of the pulse increases as the tension in the spring increases. Two springs of the same material stretched to different lengths are therefore different media.

When pulses collide, they seem to pass through each other without change.

The maximum displacement of the spring when the pulses meet is approximately the sum of the two separate displacements. The students can determine this to an accuracy of about 10 to 20 percent.

When a pulse reaches the junction between the two springs, it is partly reflected and partly transmitted. Pulses traveling on the small spring are reflected right side up, while pulses on the large spring are reflected upside down.

Pulses on the large spring reflect right side up when they reach the thread; pulses are reflected upside down from a fixed end. The speed of the pulse in the thread is much higher than that in a spring.

Hints for Demonstrations

A long piece of rubber tubing or flexible clothesline can be hung outdoors for demonstration. It has the advantage of small attenuation. Indoors a coil spring can be suspended horizontally from the ceiling as shown in the Teacher's Resource Book. The spring must nevertheless be under considerable tension to carry the kind of waves we are studying.

Apparatus

1 Flat-wire coiled spring, 4″ long, 3″ diameter
1 Meter stick
1 Long thread
1 Round-wire coiled spring, 1.75 m long, 2 cm diameter

CHAPTER

6 Waves and Light

The purpose of this chapter is to familiarize students with the phenomena associated with wave propagation in a plane. The ripple tank is introduced in the text and in the laboratory as an instrument for observing and experimenting with these phenomena. Observation of the ripple tank gives evidence that waves undergo reflection, refraction, dispersion, and diffraction. The analogy between the behavior of water waves and the behavior of light is so strong that it suggests that light may be propagated as a wave disturbance of some kind.

Work on this chapter should be built around the laboratory experiments, using the text to give continuity and to review what has been learned in the laboratory. It is not sufficient that students merely observe "demonstration experiments" with a ripple tank. Students must work in small groups and study each of the phenomena closely in order to get an adequate understanding of wave motion.

The work in this chapter has proved to be among the most exciting in the course. Very few students will begin it with any real familiarity with the ideas presented, despite years of casual observations of water waves. Here is a real opportunity to drive home some of the aspects of "the scientific method." Experience with quantitative and careful observation of wave motion—isolating events, correlating phenomena, generalizing results—can teach students more science than hundreds of pages in books. You will need to be generous with allotments of laboratory time.

The laboratory work should be prefaced by a brief explanation of the light-focusing process by which the ripple tank waves are seen.

Experiment 8 Pulses in a Ripple Tank acquaints the student with the ripple tank and is best done as an introduction to the first three sections of Chapter 6. The use of periodic waves and the wave generator have been delayed purposely until Experiment 10 to allow a gradual transition from the pulses on a line to periodic waves in a plane.

Experiment 9 Parabolic Mirrors shows that water waves and light will focus at the same point. It may be done at the time Section 6–3 is being discussed.

Experiment 10 Periodic Waves is most appropriately done after studying Section 6–4. Its main purpose is to drive home the use of the relation, $v = f\lambda$. The experiment also demonstrates that the speed of propagation of waves in a ripple tank varies with the depth of the water, thus preparing the ground for Experiment 11 on refraction.

Since this is the first time students encounter the stroboscope, allow time to familiarize them with its characteristics and use. A good way to do this is to

secure a fan that can be operated at more than one speed, if necessary connecting it in series with a resistance. Mark one of the blades with a strip of white tape; darken the room somewhat and illuminate the fan strongly. Have the students practice "stopping" the motion of the fan. They should see that this can be done at different speeds of rotation of the stroboscope disk, and with different numbers of slits in the stroboscope disk taped over. They should see that the motion of the fan can be "stopped" in such a way that more than one strip of white tape can be observed, and should realize that the appearance of such a pattern means that they are observing at a rate that is faster or slower than the fan is actually turning. Hence, to measure the speed of rotation of the fan they must use the highest rate at which the stroboscope can be rotated while stopping the motion of the fan with a single strip of white tape visible.

A few practice problems would be in order: A six-slotted stroboscope disk rotates 20 times in 10 seconds in stopping the motion of the fan; how fast is the fan turning? Since each rotation of the disk gives six glimpses of the fan, the fan was seen 120 times in the 10 seconds; hence, the viewing rate is 12 times per second. If this was the highest rate at which the stroboscope could be turned so as to stop the fan with one white strip visible, then the frequency of the fan is 12 sec^{-1}; if not, the frequency of the fan was some integral multiple of this.

Now when the stroboscope is applied to the study of the waves in a ripple tank, they know that the maximum stroboscope frequency, f_s, which gives a "stopped" pattern of the right wave length, gives the frequency of the wave disturbance that is being observed:

$$f_w = (f_s)_{\text{stopping maximum}}.$$

Experiment 11 Refraction of Waves is basically qualitative and should precede the study of Section 6–5.

It probably will be worthwhile to review in class the proof of Snell's law for water waves as it is presented in the text. It is important for all students to understand this reasoning.

The purpose of Experiment 12 Waves and Obstacles is to show that waves bend when they pass by an obstacle and that this bending decreases with the wavelength. This does not exclude a wave model for light, but suggests that the wavelength of light is very small. The experiment is best done before studying Section 6–7. When we infer properties of light from observation of ripples in a tank, we are making a large extrapolation, *which must be tested by an experiment with light.* Thus, the observation of the bending of water waves, by itself, does not *prove* that light bends any more than the fact that we can hold water in a bucket proves that we can hold light in a bucket. However, in this case, our model gives us the key to the proper experiment with light by pointing up the importance of the quantity λ/d in determining the amount of bending.

EXPERIMENT

Pulses in a Ripple Tank 8

The depth of water in the tank should be about 5–7 mm for all experiments unless otherwise stated. (Operation is much better if water and damper covers are changed after a day's use.) Generating a straight pulse requires

some practice. Smaller pulses are always present preceding and following the main pulse. The pulses may be curved if the dowel is moved too rapidly; if it is moved too slowly, the pulses will be too weak to be visible. A ruler can be used instead of a dowel.

The opposite end of the tank may be used as a straight reflecting barrier provided the edge of the tank is vertical. By changing the direction of the dowel, the incident angle is varied.

Answers to Questions

The pulse starts almost from a point and expands in a circular pattern; hence the speed is the same in all directions.

Straight pulses remain straight as they move along the tank, provided they are wide enough to cover the whole tank. When a pulse does not cover the whole tank, it curves at the ends. This is particularly noticeable after the pulse has been reflected.

The angle of reflection appears to be equal to the angle of incidence.

The virtual source of a circular pulse reflected from a straight barrier is as far behind the barrier as the source is in front of the barrier. This can be explained only with the assumption that the angle of incidence equals the angle of reflection. A straight barrier reflects a straight pulse similar to the way that a plane mirror reflects a parallel beam of light; it reflects a circular pulse similar to the way that a plane mirror reflects a divergent beam of light.

Apparatus

1 Ripple tank and dampers
1 150-watt clear-glass light bulb (straight filament), shield and socket
Large white paper screen about 2′ square

EXPERIMENT

9 Parabolic Mirrors

Answers to Questions

Straight pulses reflected by a parabola are brought to a focus.

The direction of motion of a small segment of a pulse may be indicated by drawing a directed normal to the segment. Connecting the successive normals to a segment results in a picture similar to the ray diagrams studied in Chapter 2 (Fig. 2–11).

When circular pulses are generated at the focus of the parabola, the reflected pulses are straight.

Light from a distant source will converge at the same point the water waves did.

Apparatus

1 Ripple Tank and dampers
1 Rectangular piece of metal (app. 3 cm × 30 cm)
1 150-watt clear glass light bulb (straight filament), shield and socket

To obtain clear waves, connect the battery in such a way that the motor will turn in the direction away from the tank. A low and steady frequency can be obtained by setting the rheostat slightly above the stalling point of the motor. This will result in a frequency of about 5 to 7 cycles per second. For low-frequency waves, adjust the nut on the shaft to give higher amplitude. Before taking quantitative measurements some practice may be necessary, to make sure that the generator will operate at low frequency long enough to take several measurements of frequency and wavelength. It is important to work with a low frequency. For frequencies of 10 c.p.s. or more, a change in the depth of the water from the standard $\frac{1}{2}$-$\frac{3}{4}$ cm to 2 cm will not produce an observable change in the speed of the waves. Figure (a) shows the dependence of the speed of water waves on frequency and depth.

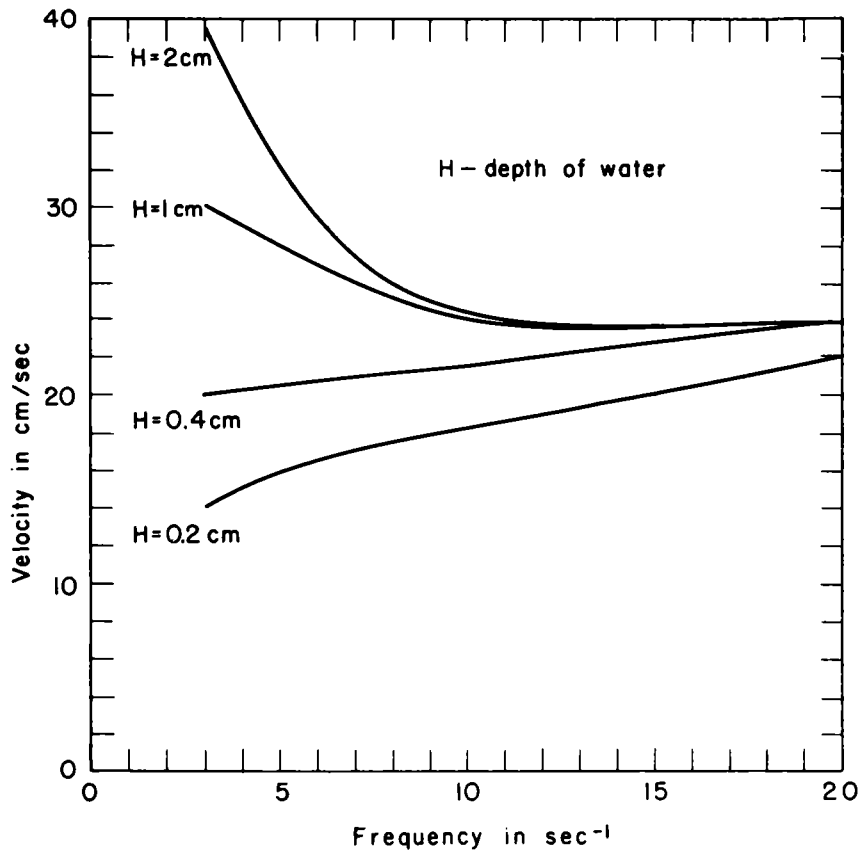


Figure (a)

In the process of generating waves on a coil spring and in later ripple-tank experiments using barriers, standing waves will be observed by most students. You may wish to treat them briefly as a special case of superposition and as another way of measuring wavelength. Standing waves are discussed in more detail in Chapter 27.

If a small and a large coiled steel spring are tied together (see Experiment 7), it is best to allow only about one third of the large spring to extend while the rest of it is held in the hand.

Answers to Questions

When the waves are stopped by the stroboscope with the two slits open, their frequency is twice the frequency of rotation of the stroboscope; with four slits open, the frequency of the waves is four times that of the stroboscope—provided no higher frequency of rotation stops the waves.

The accuracy of the speed determination can be found by first calculating the mean of all measurements, then calculating the deviations from the mean value, and finally, the mean of the absolute value of the deviations. (This procedure is simpler than finding the root mean square.) The mean deviation divided by the overall mean yields the fractional error, which can be expressed in percent.

Because the images of the waves are magnified on the screen, it is necessary to scale down the measurements of distance accordingly. The students can try to figure out how to determine the scaling factor. There are several ways to do this. One way is to place an opaque block of known length in the tank and compare its length with that of its shadow on the screen. To avoid the indistinct shadows cast by the water meniscus at the edges of the block, the block can be held just below the glass in the ripple tank. Another way is to measure the distance from the light source to the tank and from the light source to the screen and take the ratio.

The distance between two bright bars in the standing wave pattern is one half the wavelength of the traveling wave. The wavelength can be determined from the standing wave pattern by finding the distance across, say, ten bright bars and dividing by five.

Both ends of the spring can be considered to be nodes as long as the hand maintaining the standing waves moves with a small amplitude. This will happen only if half the wavelength fits an integral number of times into the length of the spring. At this point, it is not important for the students to find the quantitative relation between possible wavelengths of standing waves and the length of the spring; but if they do, this will be very useful in their study of energy levels of atoms. You can also mention standing waves in sound-producing instruments.

If you use the two coil springs of Experiment 7 tied together, the frequencies in both will be the same; the wavelength will be less in the small spring than in the large spring; hence the velocity in the small spring is less than that in the large spring.

Apparatus

1 Complete ripple-tank apparatus; this consists of:
- 1 Ripple tank and dampers
- 1 Wave generator and support
- 1 Support for lamp
- 1 $1\frac{1}{2}$-volt battery, No. 6 dry cell
- 1 6-ohm rheostat
- 1 Connecting wire—rheostat to battery
- 1 Paper screen, about 2′ square

2 Barriers, paraffin blocks
1 150-watt clear glass straight-filament bulb with socket, shield and cord
2 Hand stroboscopes
1 Stop watch or clock with sweep second hand
1 Ruler or meter stick

EXPERIMENT

Refraction of Waves 11

By taking careful measurements, the quantitative relation between the angles of incidence and refraction can also be established, but with the present equipment only a few students should be encouraged to do this. The main problem is to keep the frequency constant.

Waves generated at the rate of 5 to 7 cycles per second will be noticeably refracted when passing from the deeper section into the shallower section at an angle. A point source of light should be used to guarantee wave images that are equally sharp regardless of their direction.

During this experiment, it is very important to keep the motor running so that the frequency will remain constant. This will prevent untimely discovery of dispersion. At the end of the experiment, you may wish to call the students' attention to dispersion by suggesting that they observe the change in the angle of reflection when the frequency is increased.

Answers to Questions

The refracted waves are straight. Without taking quantitative measurements, one sees that the angle of refraction is less than the angle of incidence. Both the wavelength and speed of waves in the shallower section are less than in the deeper section.

Because the speed of the waves in the shallow water is less, the wave model agrees with the refraction of light better than does the particle model.

The angles of incidence should be chosen from a range of 20° to about 70°. The ratio of sin i/sin r will be nearly constant. However, the accuracy is not sufficient to conclude that the above ratio is closer to a constant than i/r.

Supplement

Several problems in the HDL section of Chapter 6 relate to situations which can easily be realized in the ripple tank. The value of these problems will be greatly enhanced if your students work them out in the laboratory.

PROBLEM 7.† A satisfactory ellipse can be made in the following way: first bend a piece of rubber tubing into a circle, connecting the two ends by a short dowel or glass tube. Place the tube in the ripple tank and squeeze it into an elliptical shape by pressing it with two blocks opposite each other.

PROBLEM 9. To obtain satisfactory results be sure that the reflectors are straight and at right angles to each other. Any deviation is amplified by the reflection.

PROBLEM 23. You can use a glass plate resting on the tank on one side and a few washers on the other. It is important to keep the slope of the plate small to obtain gradual bending. A depth of about $\frac{3}{4}$ cm is suitable.

PROBLEMS 25 AND 26. A lens can be sawed from a piece of lucite. It should be long enough to cover about half the width of the tank or more, but should be rather narrow so that the waves will not attenuate much over it. The exact shape of the lens is not important; two intersecting arcs or an ellipse will be satisfactory. In the latter case, you can use the outside piece of lucite as an elliptical reflector for Problem 7. The depth of the water in the deep part of the tank should be at least 1.5 cm and no more than 2 mm over the lens.

Apparatus

1 Complete ripple-tank apparatus (see Exp. 10)
1 Trapezoidal glass plate
Supports for glass plate (coins, washers, etc.)
1 Hand stroboscope
1 Stop watch or clock with sweep second hand
1 Meter stick or ruler

EXPERIMENT

12 Waves and Obstacles

To obtain clean waves at high frequency, the edges of the generator must be smooth. Have the students follow the instructions supplied with the kit. Air bubbles can be removed by rubbing a finger along the edge of the generator after it is placed in the water. Before high-frequency waves are used with obstacles, have your students check the straight waves without them. If strange lines and patterns appear, the generator is not smooth enough. This check is essential.

The ends of the paraffin blocks used as barriers should be cut smoothly to form a 45° trapezoid as shown in Fig. (a). The longer side should face the generator. A paraffin block about 5 cm long makes a good obstacle. Under normal conditions its apparent length on the screen will be about 10 cm.

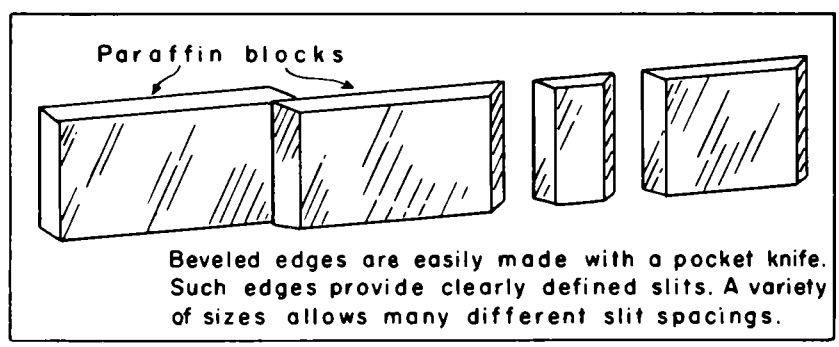

Paraffin blocks
Beveled edges are easily made with a pocket knife. Such edges provide clearly defined slits. A variety of sizes allows many different slit spacings.

Figure (a)

Answers to Questions

When the wavelength is about half the size of the obstacle or larger (both measured on the screen), the waves bend strongly around the obstacle.

Far behind the obstacle the waves fuse together and the presence of the

obstacle cannot be sensed; in effect, the block does not cast a sharp shadow. As the wavelength decreases, the shadow becomes sharper.

The diffraction by a slit shows a similar dependence on wavelength. Some patterns are shown in Fig. (b). We have drawn only the central maximum of the pattern. In some cases, students will observe secondary maxima. (This is not the time to elaborate on the detailed structure of a single-slit diffraction pattern. This is done in Chapter 8). At this time it suffices to mention that the secondary maxima are very weak and that they too appear at smaller angles to the original direction of propagation as the wavelength decreases.

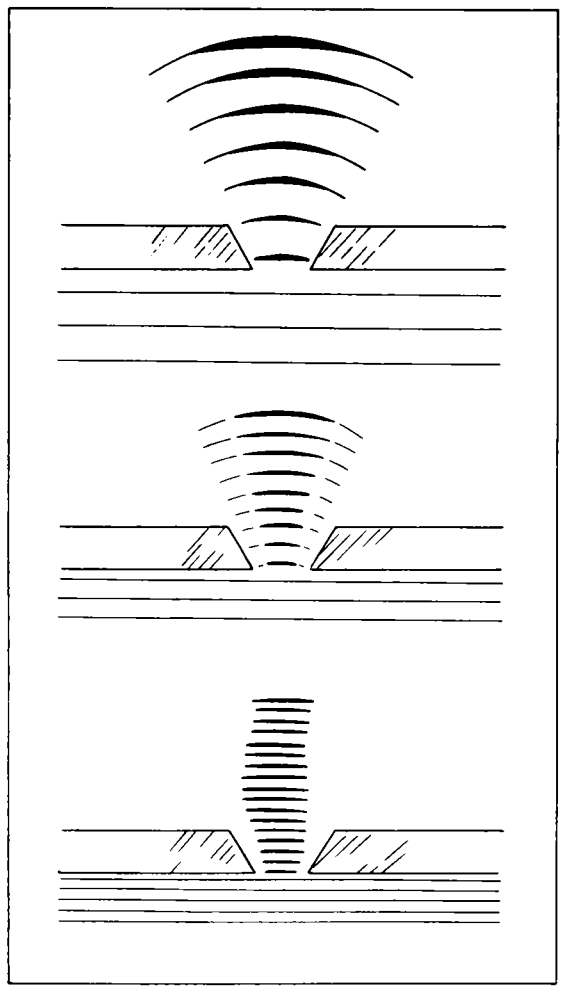


Figure (b)

For a constant wavelength, progressively smaller slits produce more bending. The change in pattern which results from the decrease in the width of the slit can be compensated for by decreasing the wavelength. A quantitative study would show that the pattern is determined by the ratio of wavelength to width of the slit. It is not wise to attempt to do this in the ripple tank because the quality of the wave images seen on the screen changes with frequency. Details seen at one frequency may not be seen at another even if λ/w is the same.

Apparatus

1 Complete ripple-tank apparatus (see Exp. 10)
1 Small barrier, paraffin
4 Barriers, paraffin blocks
1 Hand stroboscope
1 Meter stick or ruler

Questions

1. You might wish to let some interested students try to produce total reflection of water waves in the laboratory. It is not easy. You had better first be sure that your equipment is good enough so that it can be done. You will need to make your "slow" region very shallow, and you will have to run your generator at a very low frequency. (It is essential to be able to run at 8 cycles per second, and 5 cycles per second will be even better.)

CHAPTER

7

Interference

In this chapter the experimental study of water-wave interference is approached both qualitatively and quantitatively. A student who has seen the interference pattern of a two-point source and studied visually the effect of varying the separation and frequency of the sources has a rich background for the understanding of similar phenomena in light, sound, etc. The ideas in this chapter are not inherently difficult, but for students to master them requires close attention to detail and all the resources of text, laboratory, classroom, and outside study.

The concept of nodes on a coil spring should be introduced by demonstrations, preferably with the spring suspended as described in the Resource Book for Chapter 5. Such demonstrations will reinforce the idea of periodic waves, and make clear the conditions for interference. The material must be understood, but do not spend too much time here.

Experiment 13 Waves from Two Point Sources shows the wave pattern for two point sources in phase. It is divided into two parts: the first part is qualitative and serves as an introduction to Chapter 7; the second part is quantitative and should come after the discussion of Section 7–4. The text discusses means of locating the nodal lines in the pattern. The graphical construction in Questions 6, 11, and 14 bring together the theory set out in the text and the qualitative observations of the laboratory. The classroom is needed to pin down the geometry of the text and to indicate the techniques by which the graphical questions are to be handled.

There are two degrees of mathematical difficulty to be met here. The first involves the location of the curved (hyperbolic) nodal lines near the two sources with the condition that the points on a line must be farther from one source than from the other by an odd number of half wavelengths. This is a straightforward geometry, which has only to be followed closely to be understood. The second involves the location of the straight nodal lines (asymptotes) far from the sources in terms of the angles they make with the central axis. Here the students are asked to consider an isosceles triangle (in Fig. 7–12) both of whose base angles are approximately 90°. This is the first of a number of approximations they will meet in this course, and it is worth a little time to dispel the suspicion that physicists cheat in their mathematics. A graphical construction which can be measured may help to justify the approximation.

You should stress the crucial role of the path difference. Students can get the qualitative ideas necessary to understand interference in light if they realize

that a path difference of $\frac{1}{2}\lambda$, (or $\frac{3}{2}\lambda$, or $\frac{5}{2}\lambda$, etc.) implies destructive interference. It is the concept of path difference, more than the shape of the nodal lines, which should be stressed.

A qualitative feeling for the effects of phase delay in one source is all that is necessary at this point. The ideas can be made more precise when they are needed again. Notice that phase is defined here in fractions of a period, and that this definition is perfectly adequate for the course.

Experiment 14 Interference and Phase is best done at the beginning of the discussion of Section 7–5.

EXPERIMENT

13 Waves from Two Point Sources

It takes only a few seconds to look at an interference pattern, but it requires a much longer time to observe the details and to be able to describe the pattern. Learning to describe what one sees is an important part of science. This experiment should not be replaced by a demonstration!

A wavelength of about 2 cm on the screen is a good starting point.

We are primarily interested in the pattern far from the sources. Therefore a high amplitude is desirable. In order to observe the waves close to the sources, the amplitude must be reduced by adjusting the nut and by arranging the wave generator so that the beads barely touch the water.

In the second part of the experiment the wavelength of a periodic wave is calculated from measurements on the interference pattern and the separation of the sources. The result is then checked by direct measurement on the screen to give students confidence in the method they will use in Experiment 15 to measure the wavelength of red light.

Make sure your students measure x, L, and d on the screen, which gives them the wavelength as measured on the screen.

You may suggest that your students cover one slit and see what happens. They will repeat this with light in Experiment 16.

Answers to Questions

The main features of the pattern are: waves travel outward from the sources; the waves are separated by paths of no disturbance (nodal lines); far from the sources these lines are straight and appear to radiate from the point midway between the sources; closer to the sources they curve. "Stopping" the waves with a stroboscope will show that the waves separated by lines of no disturbance are out of step: each crest is flanked by two troughs. With small amplitude, a standing wave may be observed between the sources.

When the wavelength is decreased, the number of lines of no disturbance increases. The patterns are always symmetrical about the perpendicular bisector of the line between the two sources. We always see moving waves along the bisector. When the source separation is increased, the number of nodal lines increases.

Students should be encouraged to draw sketches on the screen as they answer these questions.

The results of the two measurements agree within 5 to 10 percent. The main sources of error are variations in the frequency of the motor and failure to "stop" the waves long enough to measure their length accurately.

Students need not repeat all the measurements with slits; they need only to obtain the pattern to convince themselves that they can measure the wavelength from the pattern.

To obtain wide enough diffraction, the width of the slits has to be less than one wavelength. If, on the other hand, the slits are too narrow, not enough intensity will get through. The 5-cm block used in Experiment 12 can serve as the barrier between the two slits. An interference pattern can be expected to occur only where the waves from the two slits overlap, which is most clearly visible in the central region.

Apparatus

1 Complete ripple-tank apparatus (see Exp. 10)
1 Hand stroboscope
1 Meter stick or ruler
1 Small barrier, paraffin block
4 Barriers, paraffin blocks

EXPERIMENT

Interference and Phase 14

The dial on the glass disc variable-phase wave generator controls the phase of the right-hand rippler as one faces the dial. Whether it advances or retards the phase of this rippler depends upon the direction of rotation of the motor, which depends in turn upon the battery connections. Students who do this experiment should have time to discover for themselves which way the motor rotates and which rippler starts down first when the dial is set at 90°. Then they can proceed with some assurance to relate the shift in the interference pattern to the change in phase of the ripplers.

Figure (a) shows another way of generating waves with adjustable phase delay. The arrangement of the paraffin blocks is the same as in Experiment 13. The width of the slits must be less than one wavelength so that they will ap-

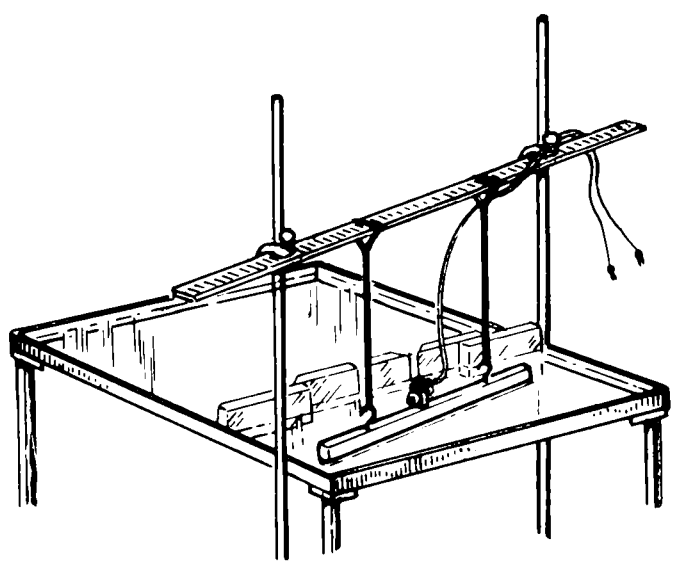


Figure (a)

proximate point sources. Rotating the generator changes the phase of the two sources (slits). If the generator support is rotated very slowly without disturbing the generator, the effect of varying the phase can be observed.

Answers to Questions

When the right-hand source is delayed with respect to the left-hand source, the pattern shifts to the right. When, for example, this phase delay is $\frac{1}{2}$, the first nodal line to the left has taken the position of the central maximum in the in-phase pattern; the first nodal line to the right has moved to a position about halfway between the first and second nodal lines in the in-phase pattern. When the phase delay is increased to 1, the sources are again in phase; the first nodal line has moved to the position of the second line and so on.

Changing the phase while the generators operate would result in a sweeping motion of the nodal lines and their curving away from the direction of the motion.

Apparatus

1 Complete ripple-tank apparatus (see Exp. 10)
1 Hand stroboscope
1 Variable-phase wave generator
1 $1\frac{1}{2}$-volt dry cell
(For alternate method)
2 Meter sticks
1 Small barrier, paraffin block
4 Barriers, paraffin blocks

Films

"Sound Waves in Air," by Richard H. Bolt, Massachusetts Institute of Technology. The wave characteristics of sound are investigated with large-scale experiments on reflection, refraction, interference, and diffraction. Running time: 35 minutes. This film can be shown any time during this chapter, or even during the next chapter.

Helpful film loops for reviewing ripple-tank phenomena are "Interference of Waves" and "The Effect of Phase Differences Between Sources."

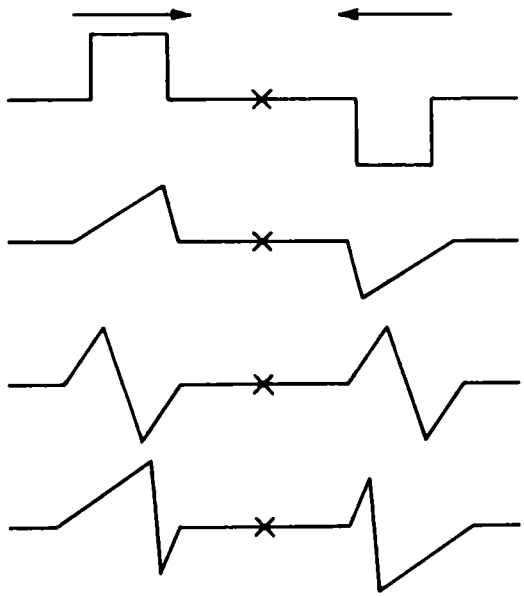
Demonstration

While you have a coil spring set up, it is a good idea to prepare the class for the ideas of path difference and phase difference in two dimensions by using the following exercise. Send a student to each end of the spring and ask each to start a pulse (on the same side of the spring) when a third student counts three. Then give the class the problem of deciding whether the students really start their pulses at the same time. Even if students do this without supervision in a laboratory, they will rapidly agree that watching the pulses as they add at the midpoint, where the maximum displacement should occur, is the best way to decide. If one student sends a pulse to one side of the spring and the other a pulse to the opposite side of the spring, a node should occur at the midpoint.

Ask what could be done to make the pulses cross at a different point. Ask them to decide which student should move (and which way) in order to keep the pulse crossing at the center even though one student starts his pulse later than the other. Be sure that students realize the relation between the point of crossing, the difference in starting time of the two pulses, and the two path lengths.

Question

Students must understand that both the shape and size of crossing pulses must be right to produce a node. Draw one of the pulses as shown in the left-hand side of the drawings, and ask someone to draw the pulse which should be introduced from the other end of the spring to produce a node. The student should reply by drawing the pulse shown at the right side of the corresponding diagram.



Students get a clearer idea of cancellation when you use some pulses that are not symmetric; otherwise some students may think that you get a node only if each pulse is symmetrical with respect to its own center.

CHAPTER

8 Light Waves

Beginning with Chapter 5, the students have seen an increasing degree of correspondence between the behavior of waves and the properties of light. The wave model appears promising; this chapter completes the discussion by showing that light does exhibit the phenomena of interference which the wave model predicts. We have taken the degree of correspondence between the properties of light and the behavior of the model as the criterion for the model's usefulness. Hence the first three sections of the chapter, which discuss the problem of the interference of light from separate sources, are worth some degree of attention. The later sections are well supported by Experiments 16, 17, and 18. Since this chapter is the culmination of this part of the course, it should be covered thoroughly.

Experiment 15 Young's Experiment is in a sense the climax of the study of waves. It is best done before the study of Section 8–4

Before doing this experiment, it should be worthwhile to take a good second look at the two-source patterns for water waves in Figures 7–6 and 7–9, using methods and ideas suitable for application with light sources; for instance, we must imagine the back side of the ripple tank (the side farthest from the sources) replaced by a screen and the two sources of water waves replaced by two light sources which are somehow made to emit light waves in phase. If this is done, we can only see the places on the screen where the nodal lines intersect, and the places where the double crests arrive at the screen. To calculate the wavelength (actually the ratio λ/d) we must now apply the geometry outlined in Figures 7–12 and 7–13. Unless you have already done so, by all means you should evaluate λ/d for Figures 7–6 and 7–9 by this method. The simple geometry used should be thoroughly understood since it is used over and over again in the chapter, sometimes with minor changes. Make sure also that everyone understands qualitatively and quantitatively what happens when the ratio λ/d changes.

Students should also re-examine Figures 7–16 and 7–17 which show, better than words, the effect of changing the phase delay between the sources.

The first part of Experiment 16 Diffraction of Light by a Single Slit serves as an introduction to the study of single-slit interference. It emphasizes the similarities and differences in the appearance of single and double-slit interference patterns. It is best done immediately after Experiment 15, perhaps even in the same laboratory period. The second part can be done after the discussion of Section 8–6 and is of less importance.

In discussing the theory of single-slit diffraction, the method of pairing slits to show cancellation at $(Nd)\sin\theta = (\text{integer})x\lambda$ only works if N is a power of 2. Cancellation does occur for other values of N, but it is more difficult to prove.

Experiment 17 Resolution shows how diffraction by a small aperture affects the resolution of two closely spaced light sources. The experiment does not require a full laboratory period and is best done while studying Section 8–8.

A crude estimate of the resolving power of a lens used in viewing distant objects can be obtained from an extension of the discussion of HDL 20 (see the Problem Section). In an optical system used to look at near objects (microscope), the situation is somewhat different but the result is quite similar. (See HDL 19.)

The diameter of the pupil of the eye is of the order of 2 mm $= 2 \times 10^{-3}$ meter. Taking $\lambda = 5 \times 10^{-7}$ meter we find:

$$\theta \approx \sin\theta = \frac{\lambda}{w} = \frac{5 \times 10^{-7}\ \text{meter}}{2 \times 10^{-3}\ \text{meter}}$$
$$= 2.5 \times 10^{-4}\ \text{radian.}$$

An angular resolution of 2.5×10^{-4} radian implies that a good eye should be able to resolve two lines a distance 2.5×10^{-4} meter apart, i.e., 0.25 mm, at 1 meter. This is not very far from the actual resolution.

Experiment 18 Measurement of Short Distances by Interference, like the second part of Experiment 16 uses the wavelength of light to measure very short distances. It is best done after studying Section 8–9.

Sections 10 and 11 should require very little class discussion. Aside from whatever preferences you may have for or against "reviews" in class, it will be worthwhile to spend a little time discussing in terms of Chapters 1–8 the idea of a model and how the validity of a model is established.

EXPERIMENT

Young's Experiment 15

In preparing the slits, the best results are obtained by using microscope slides coated with a colloidal suspension of graphite in water. A thin, opaque layer of the colloid can be applied with a soft brush. The slits should be made as suggested in Fig. 1 within an hour after the coating has dried.

As an alternative, the slides can be coated with carbon black. This is done by holding the slide above a candle flame until a thin layer of carbon is deposited. Attempts to prepare good slits with slides coated with other kinds of material have not been successful; the coating chips when scratched.

Two clean, double-edged razor blades tilted slightly forward as shown in Fig. 1 will produce slits that are separated by a distance equal to the thickness of one blade.

A 40-watt clear-glass showcase bulb is a good light source because of the long, straight filament. If this is not available, a ripple-tank bulb or a weaker clear-glass bulb with a straight filament can be used. In either case, the slits should be parallel to the filament.

It might be worthwhile to have one ripple tank set up during this experiment. The similarities and differences between measurements of the directions of nodal lines in the ripple tank can then be related to the measurements with

light. Note that in this experiment the pupil of the eye subtends the entire interference pattern of the light waves. The analogous situation in the ripple tank would require an "eye" as large as the tank.

Answers to Questions

Light from the bulb passing through the narrow slits is diffracted. The light from the two slits overlaps, producing an interference pattern. Dark bars are seen along the nodal lines and bright bars along the direction of maximum disturbance.

The bars near the end of the pattern will be colored if the colors making up white light are waves of different wavelength. Such waves will have maxima and nodal lines in different directions.

When the bulb is covered with red cellophane the other colors are cut out, leaving a sharper pattern of red and black bars. More bars are seen clearly.

From Fig. 2 it is seen that θ_n equals half the distance between $2n$ nodal lines divided by the distance between the ruler and the slits. (Since θ_n is small, the distance from the slits to the ruler along any of the nodal lines is the same, within experimental error.)

The wavelength of red light can be determined to an accuracy of about 10 to 20 percent. The main sources of error are the measurements of x and the separation of the slits. It should be remembered that even the most accurate measurements give a value dependent on the range of wavelengths passed by the filter.

Blue light will show alternating light and dark bars that are closer together than bars from red light; thus, blue light has a shorter wavelength.

The interference pattern is spread out when the slide is rotated to form a horizontal angle of about 30° with the line of sight. This is caused by an effective decrease in the slit separation [Fig. (a)].

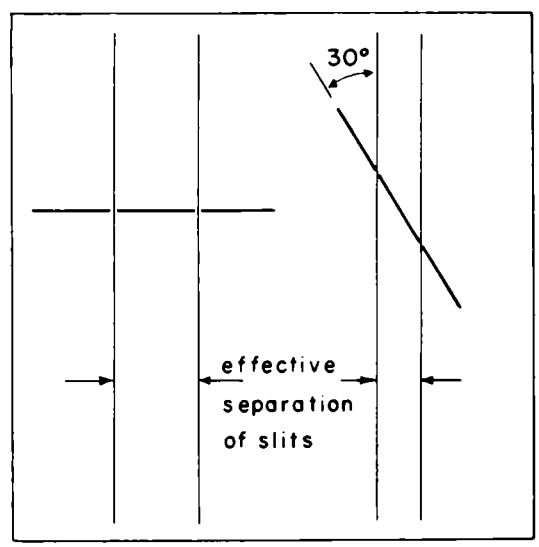


Figure (a)

Apparatus

1 Showcase lamp, clear glass, 40-watt
1 Socket and cord
2 Microscope slides, 1″ × 3″
2 Double-edged razor blades
1 Piece of red cellophane, 4″ × 4″
1 Piece of blue cellophane, 4″ × 4″
2 Rubber bands
1 Ring stand
1 Burette clamp
1 Ruler
2 Markers for ruler
Colloidal suspension of graphite or candle for lampblacking
Masking or cellophane tape

EXPERIMENT

Diffraction of Light by a Single Slit 16

The slides used in making single slits should be prepared as described in Experiment 15.

One of a pair of slits can be blocked off by moving the razor blade slowly along the slide until the edge of the blade covers one of the slits.

Answers to Questions

A very narrow slit made with a razor blade shows a broad bright band. A slit made with a needle shows a pattern of bright and dark bars. The central bar is twice as wide as those on both sides and much brighter. (With a narrow slit only the central maximum of the diffraction pattern is visible.)

The dark bars seen with the double slit disappear, leaving the broad diffraction band of a single slit. The places which were dark are now light! A particle model of light could not account for this.

The width of the slit, w, can be calculated from the relation $\frac{\lambda}{w} = \frac{x}{L}$ where x is half the width of the central maximum as read on the scale and L is the distance between the slit and the scale. With wider slits the accuracy of the determination of x can be increased by measuring the distance separating the nodal lines on both sides of the second maxima and dividing the width by four.

The error in the determination of the width of the slit is about 50 percent.

For comparison, a direct way to measure the width of a single slit is to project the slit on a screen with a slide projector. The magnification can be found by placing a flat, transparent scale in the projector in place of the slide.

Apparatus

Same as for Experiment 15
1 Sewing needle or straight pin

The best way to make smaller apertures is to place the foil on a smooth, hard surface and push the point of a pin or needle through the foil to different depths.

It is necessary for the filament of the clear-glass bulb to be directly behind the two holes to allow enough light to pass through the hole to make the diffraction rings readily visible. As many as four or five concentric rings around the two holes will be visible through some of the apertures. Speckled patterns or irregular rings which rotate when the aperture is rotated are caused by irregularities in the apertures. It is difficult to prevent this, especially with the smaller apertures, but it will not interfere with the experiment. Students with poor eyesight and no correction may have difficulty resolving the source at a moderate distance without the apertures.

In this experiment there is no interference pattern even when the two sources overlap; the light coming through the two holes originates from different parts of the filament and is, therefore, not locked in phase.

Answers to Questions

The two separate points of light can be resolved with the eye at distances of several meters. The size of the aperture is the size of the pupil of the eye, about 0.5 cm.

The two sources will probably be resolved with one of the middle-sized apertures at a distance of 1 to 2 meters, depending on the size of the aperture. As the distance is increased, the edges of the two sources gradually come together until they begin to overlap.

With increasing distance from the sources, there is a decrease in the angle between the two directions in which we see the centers of the sources. The central maxima of the diffraction pattern of the sources are cones diverging from the aperture. These cones finally overlap when the angle between the directions to the sources becomes smaller than the apex angle of the cone.

As the aperture decreases in size, the angular width of the diffraction cones of the central maxima increase until the cones overlap.

Red light diffracts more than blue light, because it has a longer wavelength. Thus, the diffraction cones of the red light are wider and the sources overlap sooner.

The resolution of the two sources will not be affected by changing their size if their minimum separation is kept constant, because it is the diffraction of the light from the inner edges which determines the resolution.

Supplement

If a monochromatic light source such as a sodium lamp is available, the largest resolving distance from the two sources can be studied quantitatively as a function of the separation of the sources. Plotting the results on a graph will show a linear relationship.

Apparatus

1 Incandescent clear glass lamp (150-watt ripple-tank lamp)
1 Socket and cord for lamp

1 Ringstand and burette clamp (or other stand for stock and lamp)
1 Thin cardboard rectangle, about 3″ × 6″ (to support aluminum foil)
1 Pin or needle
Red cellophane (or other red filter)
Blue cellophane (or other blue filter)
1 Meter stick or ruler
Aluminum foil
Cellulose tape

EXPERIMENT

Measurement of Short Distances by Interference 18

Very flat and thoroughly cleaned glass plates about 3 cm wide and 20 cm long are best.

Place one clean plate on top of another under a monochromatic light source. Pairs that show only a few dark and light bands will be flat enough to use.

A monochromatic light source such as a sodium lamp is best. The light from a clear glass fluorescent tube will also give good results. If neither is available, a yellow or green filter placed over an ordinary fluorescent tube will produce countable bands.

Various thin materials such as the thin nylon fiber from the Coulomb's law experiment or Saran wrap can be measured with this device. In the range of 10^{-3} cm this method is as accurate as a precision micrometer caliper.

Answers to Questions

The irregular light and dark bands are caused by interference between light reflected from the lower surface of the top plate and the top surface of the lower plate. This distance will vary from one place to another over the surface.

When one band replaces another, the top plate has been pushed $\lambda/2$ closer to the lower plate.

The separation of the plates differs by one-half wavelength between two adjacent bright bands.

The range of thickness that can be measured with these plates is limited by our ability to resolve and count the bands. The lower limit is determined by the flatness and cleanness of the plates.

Apparatus

2 Glass plates, 20 cm × 5 cm × 1 cm thick, flat plate glass
2 Rubber bands
1 Fluorescent lamp (at least 10 in. long) with fixtures and cord or other monochromatic light source
Thin materials; small pieces, 5 cm long, about 1 cm wide, 0.002 to 0.006 cm thick
1 Ruler
Micrometer caliper

Demonstration

Figure (a) shows a simple arrangement which you can use to demonstrate that many point sources close together produce the same diffraction pattern as a straight source behind a slit.

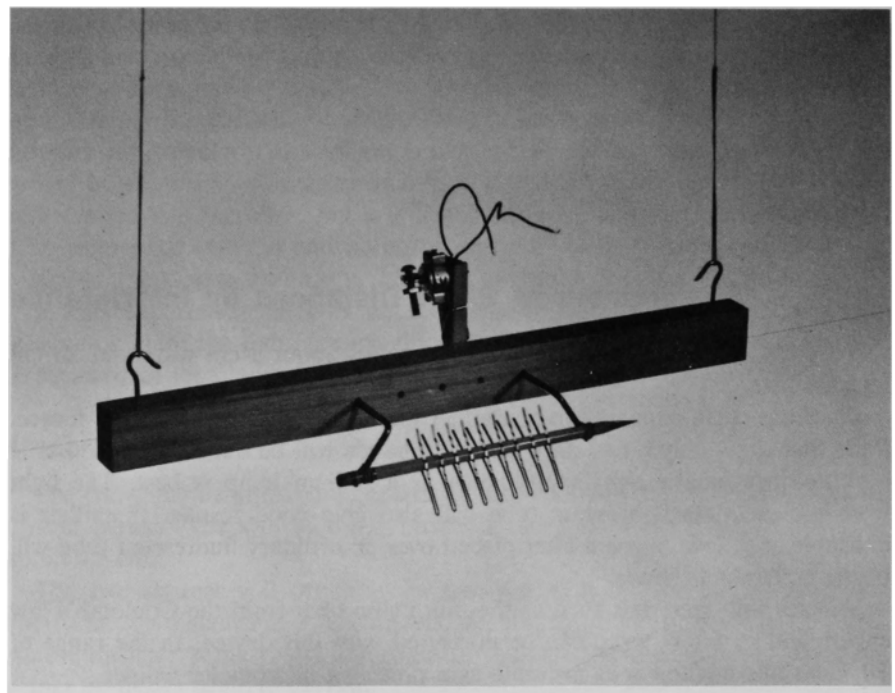


Figure (a)

Arrange the paraffin blocks to form a slit equal in width to the "multi-point source." Show the diffraction pattern with the straight generator, marking the directions of the nodal lines on the screen. Then place the row of point sources in the slit and adjust the wavelength to be the same as before. Compare the directions of the nodal lines with those marked on the screen.

To be effective the demonstration should use as short a wavelength as possible so that the ratio of slit width to wavelength is as large as possible; a ratio of at least 3 : 1 and preferably higher should be used. If the patterns are sufficiently distinct, it will be worthwhile to establish the relationship sin $\theta = \lambda/w$ for the first nodal line (the second nodal line will probably be hard to see).

Note that if you succeed in establishing the relationship between λ/w and sin θ from the ripple-tank experiment, a reasonable interpretation of the diffraction of light as the basis of the wave model is available independent of the theoretical predictions developed in Section 6. Since some students may find the arguments of the text rather difficult, this alternative approach may be helpful to them.

Films

Helpful film loops for reviewing ripple-tank phenomena are "Single-Slit Diffraction of Waves" and "Multiple-Slit Diffraction of Waves."

Force and Motion

CHAPTERS
9-13

These five chapters form a logical unit devoted to Newton's law of motion. They take up in succession the kinematics of straight-line motion; the kinematics of motion in two dimensions; Newton's law and the relation of inertial mass to gravitational mass; applications of Newton's law to free fall, projectile motion, uniform circular motion, and simple harmonic motion; finally, the use of Newton's laws to give the motions of the solar system.

These chapters contain a good deal of solid material, much of which is essential to a fundamental understanding of physics; you may find it necessary to take a somewhat slower pace than you used with light. Bear in mind, however, that your students are now well seasoned and somewhat accustomed to thinking in physical terms. You should notice also that since these chapters form a unit, not every topic is completed at the point where it is first introduced. In those cases, you can save your sessions on drill-for-mastery until the topic reappears on its second round.

CHAPTER

9

Motion Along a Straight-line Path

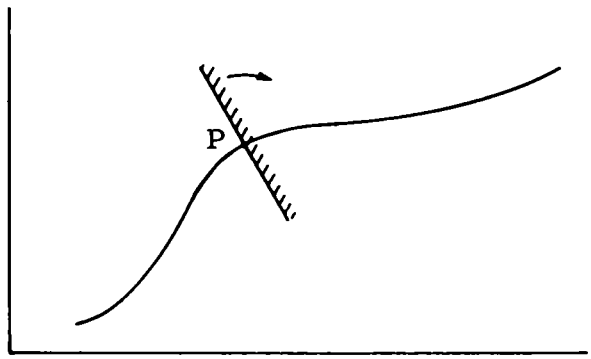
In developing the fundamental concepts of kinematics in Chapters 9 and 10, rely heavily on graphical construction and analysis. This is not to deny the students practice in applying the more formal operations of mathematics (trigonometry and calculus, for example), but to provide them with a method of calculating the various aspects of motion and at the same time with a "picture" of what actually is going on in kinematics problems. Students can learn to handle graphically problems that would be difficult with a formal mathematical treatment.

Graphical analysis should be made *quantitative*. Students should be urged to draw accurate graphs and make reasonably precise measurements of the slopes of lines and the areas under curves. Their ability to relate algebraic and graphical approaches should grow as they proceed through these chapters, but not all students can be expected to become familiar with purely algebraic methods the first time through.

Students should have no difficulty with the representation of the position of an object on a position-time graph. Be sure the students understand the difference between *position* and *displacement,* and that displacements can be positive or negative independent of whether the position coordinates are positive, negative, or mixed.

It is a natural step to consider the changes of position with respect to time, first on the straight-line sections of a position-time graph and then on curved sections, associating the slope of the straight-line sections or the slope of the tangent to the graph at any point as the velocity. Some students may already be familiar with the slope of a line from mathematics classes; there the slope can frequently be expressed as the tangent of the angle between the line and the horizontal axis. It should be emphasized that in applications in physics the slope of a line must be obtained by dividing the difference in the vertical coordinates of two points on the line by the difference in the horizontal coordinates of the same points. Besides a magnitude the slope also has appropriate physical units. With the concept of slope firmly in mind, students can easily understand how to draw a velocity-time graph from a position-time graph.

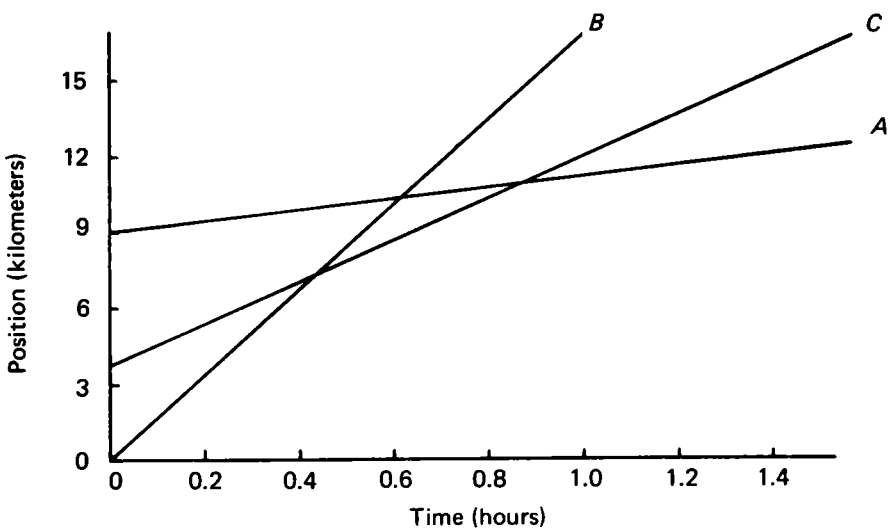
One problem which has bothered some classes is how to draw a tangent to a curve. It should be understood that we do not require an exact geometric construction.



A method of drawing the tangent which may interest some students is the following. (This method defines the perpendicular to the curve at a given point. Then the tangent is drawn at a right angle to the perpendicular.) Place the reflecting surface of a plane mirror over the graph approximately perpendicular to the curve at the point *P* where the tangent is desired. Rotate the mirror about *P* until the graph and its image form a smooth, continuous curve. Draw a line along the edge of the mirror. The tangent is then perpendicular to this line.

If it is the rear surface of the mirror that reflects, remember to redraw the line so as to cross the curve at the point where the reflecting surface intersected the curve.

Interpretation of position-time graphs is probably best done by asking exactly what is happening to the object at each position represented on the graph. You can use a blackboard graph like that shown below as a basis for asking questions about the physical situation described by the graph.



Which man is traveling fastest? (*B*)
Are all three ever at the same point on the road? (No.)

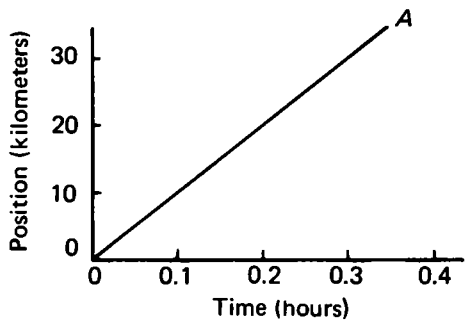
When B passes A, where is C? (About 1.5 km back.)

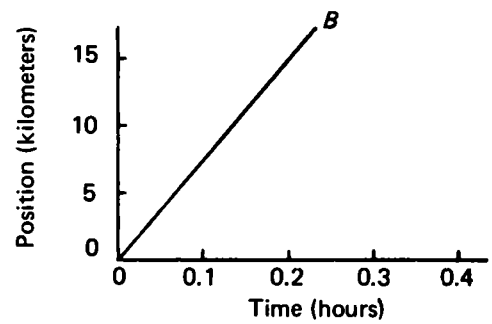
How far did B travel between the times he passed C and A? (About 3 km.)

Which men are on bicycles? (B must be riding in order to cover 15 km in 0.9 hour. Since C's speed is a little over 5 miles per hour, he at least needs to trot. Since A's speed is about 2.4 km per hour, he should be able to walk.)

After the class has worked with position-time graphs, you might put on the board a graph using numbers but no physical units indicated on the vertical scale. Ask for the speed of the car at a certain time. Some students will be prepared to answer following the previous pattern, while the more cautious ones will demand that you complete the graph by telling them what is represented by the vertical scale.

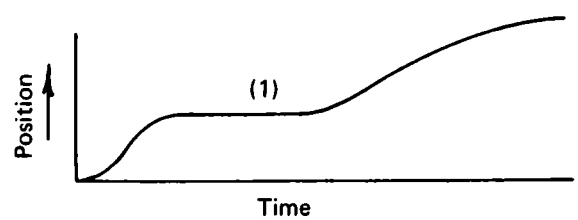
Some classes may have trouble with graphs plotted with different scales. To clarify this point, plot two separate graphs, such as those shown below, and ask which car goes faster:





Car B has the "steeper" graph, but the car travels only 15 kilometers in 0.2 hour; hence it has a velocity of 75 km/hr, whereas car A goes 30 kilometers in 0.3 hour with a velocity of 100 km/hr.

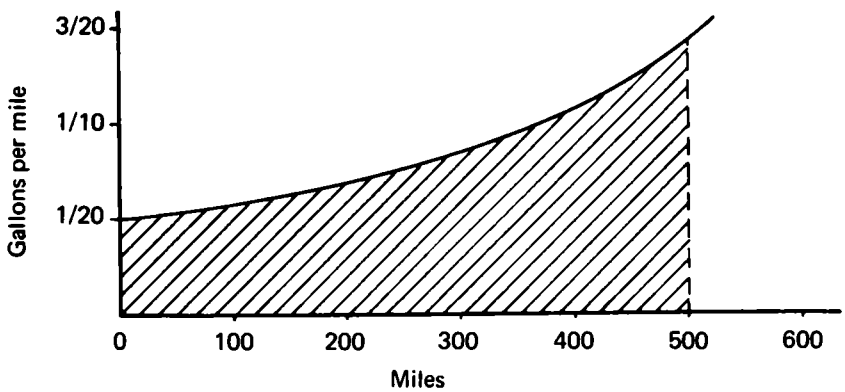
The text carefully distinguishes between the cases of constant velocity and variable velocity by discussing the former in Section 9–2 and introducing the latter in Section 9–3. With an able class, some teachers may wish to depart from this sequence and present from the beginning a graph like that below, possibly containing a region (1) where the car is stopped. Students may be able to answer such questions as: "When is the car going fastest? When slowest?", etc.



The reverse process—that of finding a displacement-time graph from a velocity-time graph—will probably be unfamiliar to most students. The idea that the displacement can be thought of as an "area" on a velocity-time graph will probably be new. For the case of constant velocity, the result is almost trivial, but it is, nevertheless, worth considerable attention, since the idea that an "area" can have such units as kilometers may be puzzling. Emphasize that the "area" which represents the displacement is the area between the velocity curve and the axis of *zero* velocity.

The extension to motion with changing velocity is made readily, by noting that the "area" under any section of a velocity-time graph can be approximated by a sequence of narrow rectangles, an approximation which becomes better and better as the rectangles are made narrower and narrower. In complex variations of velocity with time, displacements can always be found with sufficient precision by plotting the velocity-time curve on graph paper with a grid of appropriate size, and counting the squares below the graph. The area below the graph can often be found by dividing it into rectangles and triangles, and the square-counting process shortened by finding their areas. Emphasize the units involved; miles per hour must be plotted against hours if the displacement is to be in miles, for example. There never is any need to resort to the formalities of calculus, though on occasion students who know how may use it.

The method of measuring areas is of much more general use than just for calculation of displacement from a velocity vs. time graph. As a simple illustration, you might ask the following question of the class: A car starts a trip using $\frac{1}{20}$ gallon of gasoline per mile. As the sparkplugs foul up, the car uses more and more gasoline per mile as shown in the accompanying graph: How many gallons of gasoline were used to travel 500 miles? The shaded area gives the answer (gallons per mile) $\times$ (miles) = (gallons).



EXPERIMENT

Motion: Velocity and Acceleration 19

This experiment is essential to the study of kinematics. It may be done in parts corresponding to the progression of your coverage of the chapter.

The degree to which the velocities found by using slopes agree with those

on the approximate velocity vs. time graph depends on the slope of the position vs. time graph at the points which are checked. Where it is a straight line, the velocities will be the same, otherwise, there will be some difference.

Although much of the student's time will be spent in analyzing the information from the tape and in constructing the graphs, as much of this analysis as possible should be done in the laboratory where you can help the student learn how to make the graphical analysis.

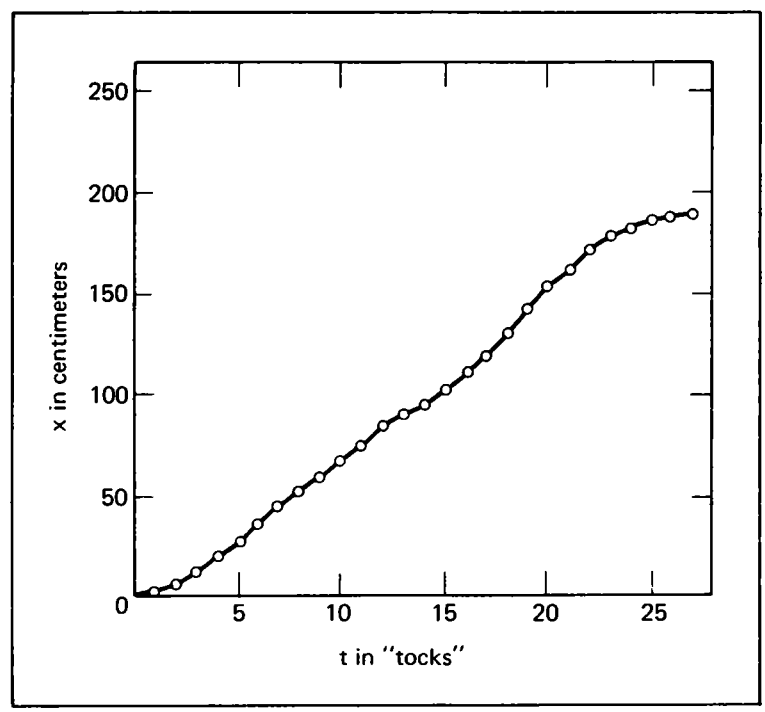
The experiment demonstrates that complicated forms of motion occur and that they can be described and analyzed. By performing this experiment, the student will gain practice and confidence in graphical analysis as a tool in studying motion.

One or two timers, set up as shown in Fig. 1, can be used to run off tapes for the whole class. Let each student pull a tape through the timer while walking at a normal rate and then construct his own set of graphs.

About 1.5 meter of tape will be sufficient to get 20 to 25 points on the graph with 4 "ticks" as the unit of time (4 "ticks" = 1 "tock"). Make certain that your students begin work on the graphs in class. They can then continue this work at home. A typical analysis is shown in Figs. (a), (b), and (c).

It is not necessary to know the calibration of the timer for this experiment; any unit of time is good.

Figure (a)



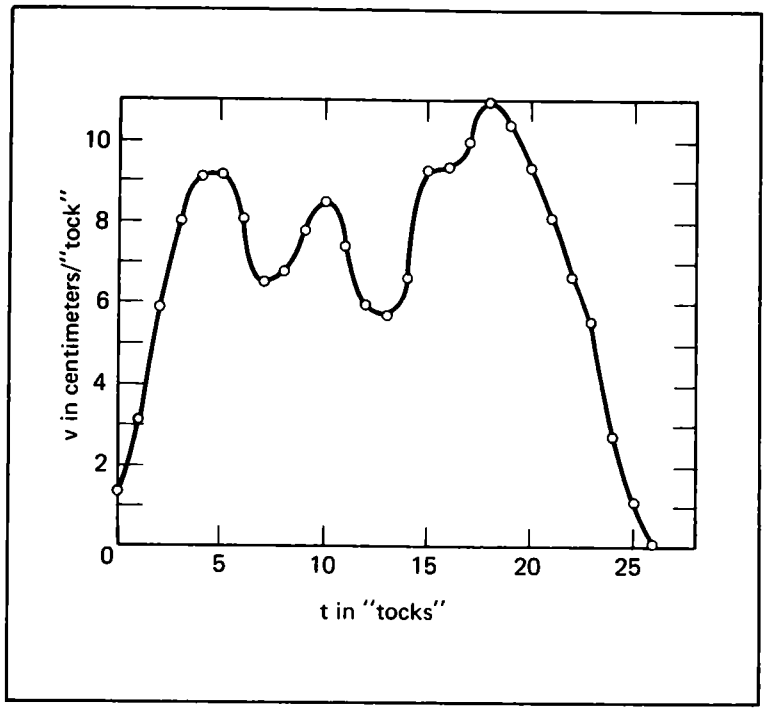


Figure (b)

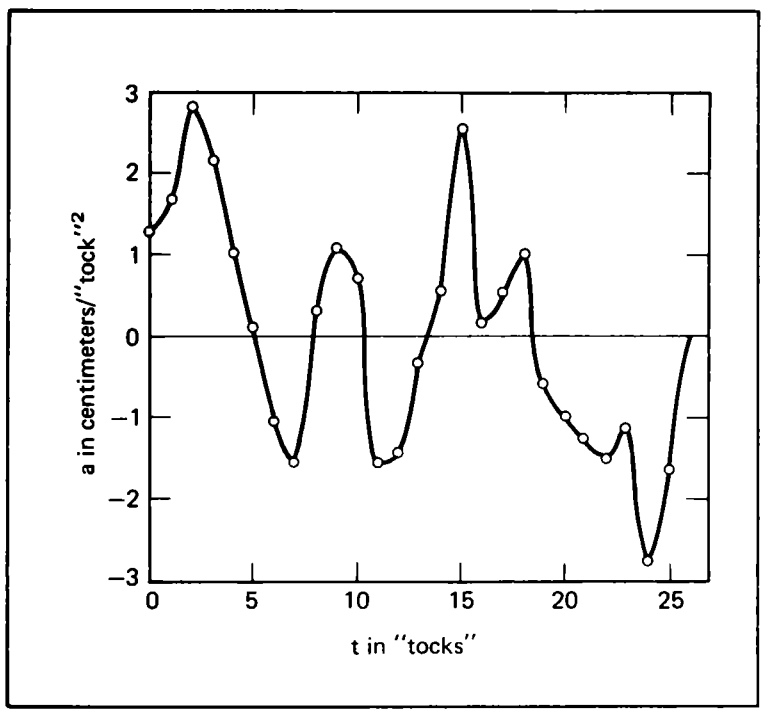


Figure (c)

Answers to Questions

The displacement (in cm) between any two adjacent marks on the tape represents the velocity of the hand in centimeters per "tick."

It will be possible to see, by inspecting the tape, where the velocity was greatest, and where it was smallest. However, it is more difficult to see where the acceleration was greatest and smallest.

The position as a function of time, obtained by counting squares under the velocity curve, will be the same as the position measured directly from the tape except for small errors in graphing and in finding the areas.

Apparatus

1 Ruler
4 Sheets rectilinear graph paper
Timer
1½-volt dry cell
Ticker tape
Carbon-paper disc
1 C clamp (3″ or 4″)

CHAPTER

10

Vectors

The kinematics of one-dimensional motion (Chapter 9) is extended in this chapter to motion in a plane. For this purpose, the chapter introduces vector quantities. The chapter has three natural parts: (1) the nature of vector quantities and vector algebra; (2) vector acceleration; and (3) a discussion of frames of reference and the distinction between kinematics and dynamics.

This chapter includes centrally important ideas and should be covered thoroughly through class discussion and problem solving. Like the preceding one, it is critically important. While Sections 10–1 through 10–7 should be covered thoroughly, most of the important concepts will be encountered again in later chapters.

Some teachers will feel that considering the kinematics aspects of circular motion here, and the dynamic aspects in Chapter 12, is an artificial division. This choice was made (a) because it is logical to combine space and time measures in kinematics, and (b) because many teachers have found that the natural review provided in dynamics makes such a split effective. Considering the kinematics of circular motion at this point emphasizes that constant speed does not always imply zero acceleration.

While you will want to seek a good understanding of vectors, you should remember that later chapters will make their contribution. Avoid getting bogged down in excessive detail at this point.

While there are no experiments specifically intended for Chapter 10, laboratory time can be used to excellent advantage for supervised work in ruler-protractor solution of vector problems. In addition to learning to describe position and displacement with directed line segments, the student must become thoroughly familiar with graphical procedures for handling vectors. The displacement vector is the prototype for all further vector work in the course.

Don't spend much time on the position vector at the beginning; its use will become clear in later work. The more important and more natural vector is the displacement, or "change of position." As a beginning, you can call these "trips." Describe some trips between local points familiar to the students as examples of quantities having magnitude and direction. Then describe a two- or three-part trip and calculate the resultant displacement. (*Caution:* If you invent "map" trips between major cities in the United States and use flat maps, keep the cities within 100 miles or so of one another or the distortions of the projection of the spherical earth on a plane will cause trouble.)

Trip vs. displacement: A trip from London to Paris is not generally considered to be the same as a trip from Buffalo to Baltimore, even though they

are approximately the same direction and distance. If two pencils are at different spots on a desk and each is moved 1 foot northeast, the two pencils have not made exactly the *same trips,* but *something* was certainly the same. The pencils have made the *same displacement.*

Vectors, as usually used by physicists, are like *displacements;* they are considered equal if they have the same magnitude and direction. It therefore makes no difference where a vector is moved if it is kept parallel to itself.

It is important for the students to draw vectors and to do vector additions graphically. Precision pays here. With sharp pencils and good rulers and protractors, two vectors can be added graphically to about 0.3% accuracy—better than can be done trigonometrically with a slide rule.

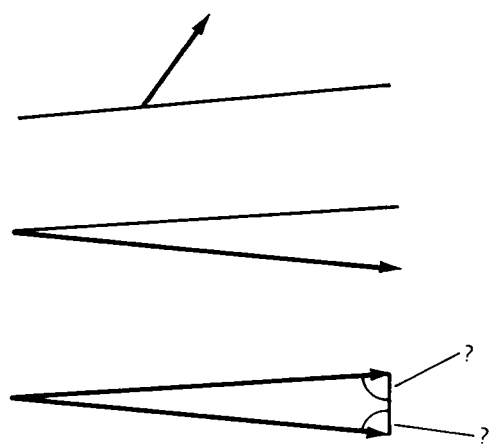
There is another important point about scale drawings. From time to time in laboratory work, students will record angles and distances directly as scale drawings. After recording data in a drawing, some students who are comfortable with trigonometry will measure the drawing and turn to trigonometric computation. The accuracy will never be better than the original drawing! In most cases the computations can be carried out more easily by continuing graphically.

When $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, $\overrightarrow{AC}$ is called the vector *sum* of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ (or *resultant*).

Early in your class discussion you may want to consider some problems involving the sum of more than two vectors. The vectors can be given pictorially or by specifying length and direction. Each vector is moved parallel to itself so that its tail is at the head of the preceding vector. Notice that the vector sum is independent of the order of addition. Thus:

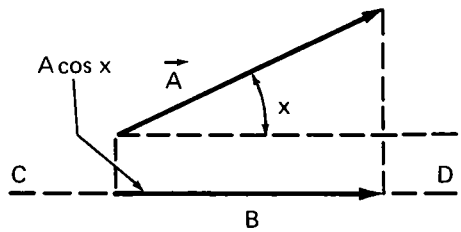
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{F}$$
$$\vec{B} + \vec{A} + \vec{C} + \vec{D} = \vec{F}$$
$$\vec{D} + \vec{B} + \vec{C} + \vec{A} = \vec{F}, \text{ etc.}$$

Vector components are important, but should not take too much time because they are not difficult. It is more important for students to master the graphical constructions introduced earlier than to develop facility with algebraic methods. The test is whether, given a vector and a line, as in Figure (a), a student can sketch the vector component of the given vector along the



given line. If pressed, he should be able to tell which angles are right angles, and through what points construction lines are drawn. You will sometimes find a student who, when given a line and a vector as in Figure (b), will say that the vector component of the vector along the line is as shown in Figure (c), and he will not be sure which of the two angles is the right angle, because the original vector and line were so nearly parallel.

Students who have had trigonometry will see that if the angle between a vector $\vec{\mathbf{A}}$ and a line CD is x, then $\vec{\mathbf{B}}$ is the component of the vector $\vec{\mathbf{A}}$ along the line CD and $B = A \times \cos x$, where A and B are the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, respectively.



When your class is considering the multiplication of a vector by a scalar, emphasize that when k has units, the vector $k\vec{\mathbf{A}}$ is a different physical quantity from $\vec{\mathbf{A}}$; and that only vectors of the same kind should appear on the same vector diagram.

It is important for students to grasp constant vector acceleration before going on to changing acceleration (Section 6). Use plenty of problems. A number of problems suitable for class discussion are given below. If your students' understanding is less than complete, do not abandon hope. They will meet vector acceleration again in later chapters.

To introduce acceleration as a vector quantity, a multiple-flash photograph of the fall of a ball is analyzed. The ball starts out with an initial horizontal velocity so that it falls along a curved path. The velocity vector of the ball changes in both direction and magnitude.

The change of velocity of the ball *must* be regarded as due to a *vector* acceleration since both the direction and the magnitude of the velocity change. A vector equation is therefore developed: $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$. The case described in the text involves constant acceleration and is drawn from common experience. The fact that this is a body falling under the force of gravity should not be stressed or developed since the behavior of bodies under the action of forces is intentionally postponed until the next chapter.

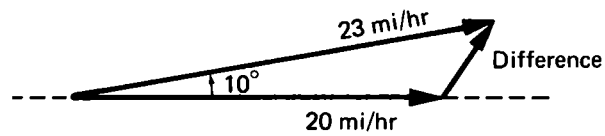
Comment

The following paragraphs suggest supplementary examples in case students need more than the example in the text to grasp clearly the concept of vector acceleration. (There are no problems in HDL for this section.)

Supplementary Examples

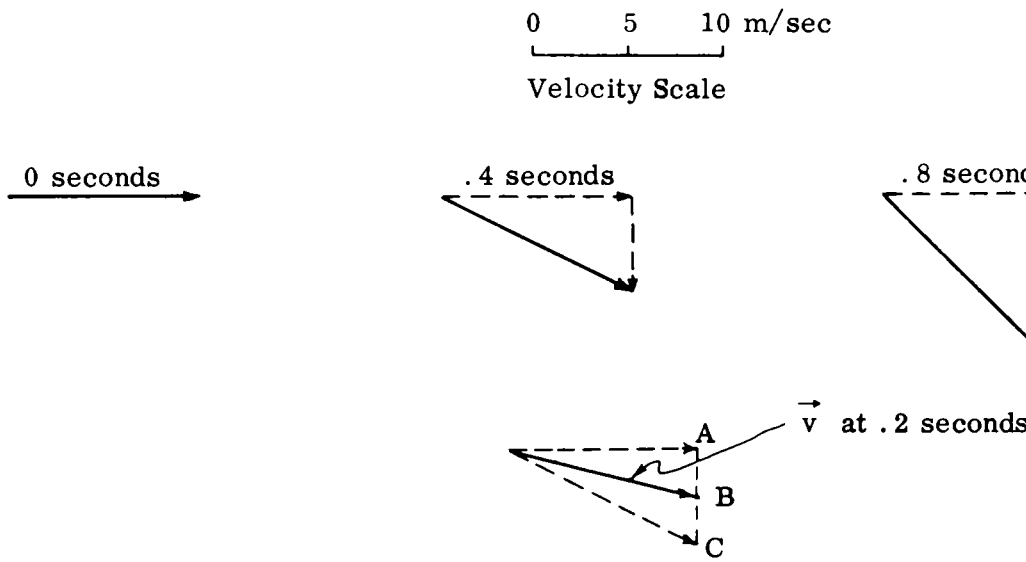
(1) If a car were going 20 mi/hr north at 3:13:33, and 23 mi/hr north at 3:15:45, by how much did its velocity change? 3 mi/hr north. Then, if a car

were going 20 mi/hr due east at 3:15:33 and 23 mi/hr 10° north of east at 3:15:45, by how much did its velocity change? The answer is their *vector difference*. The answer to the proposed problem is shown graphically below. Note that the difference is a *vector*. It has a magnitude (which is *not* 3 mi/hr, but 4.8 mi/hr) and a direction. The velocity of the car changed by 4.81 mi/hr, 33.5° east of north. It is often useful to know how rapidly the *velocity* is changing. In the foregoing example, the velocity changed by 4.81 mi/hr, 33.5° E of N, in 12 seconds: or, 4.81/12 = 0.40 mi/hr-sec, 33.5° E of N.



(2) The equation $\vec{v}_f = \vec{v}_i + \vec{a}t$ is of key importance. The students will have little grasp of its meaning unless they go through graphical steps to develop it themselves. The graphical work will pay great dividends in helping them to grasp the next section. Here is one suggestion.

Suppose an object has an initial velocity of 10 m/sec to the right and that the velocity changes by adding 5 m/sec down every 0.4 second. Have the class draw the velocity vector at the end of 0, 0.4, 0.8 second. Now ask them to draw the velocity at 0.2 second. They should recognize that the tip of the velocity vector comes to the point *B* on the diagram midway between points *A* and *C*, which represent 0 second and 0.4 second respectively. That is, 0.2 second is halfway between 0 and 0.4 second, and consequently we add only 0.5 (5 m/sec, down) to the initial velocity at 0 second. You can ask them for other fractional times, as well as for times such as 1.0 second (1.0/0.4 = 2.5 basic time intervals; hence add 2.5 vectors down).



Now ask for a formula for t seconds. They should now be able to suggest that you add $(t/0.4)$ vectors of 5 m/sec length downwards.

Ask if they can give you an equation stating this. In all likelihood they will flounder badly, but some may say something such as

$$v_f = 10 + t/0.4 \times 5,$$

leaving out the directions. Of course you can show them this equation is wrong by having them compute v_t from the equation for $t = 0.8$ second (20 m/sec) and compare it with their graphical answer (14 m/sec).

You will then be able to present them with the equation:

$$v_f = (10 \text{ m/sec, right}) + (t/0.4)(5 \text{ m/sec, down}).$$

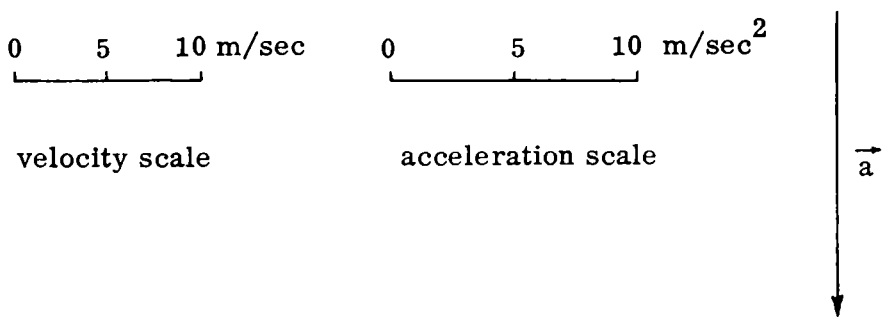
We may consider $t/0.4$(5 m/sec, down) to be a scalar, $t/0.4$, times a vector, 5 m/sec, down, and rewrite this as ($t \times 5/0.4$ m/sec, down); or (12.5 m/sec$^2 \times t$, down); or $t \times$ (12.5 m/sec^2, down). Therefore $\vec{\mathbf{v}}_f = (10$ m/sec, right) + t(12.5 m/sec^2, down), which is of the general form

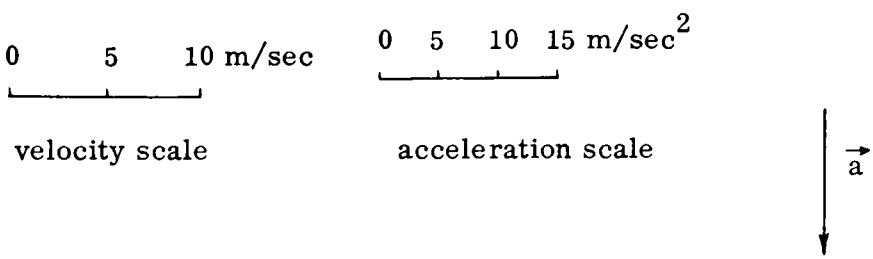
$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t.$$

Be sure to emphasize the difference between the above equation and that for the one-dimensional acceleration of Chapter 9:

$$v_f = v_i + at.$$

Now have your students plot the vector (12.5 m/sec^2, down). To do this they may need to specify the scale they will use for acceleration. Some may not realize either that they must choose a scale or that they are free to make any choice they want. Thus both of the solutions below are correct:



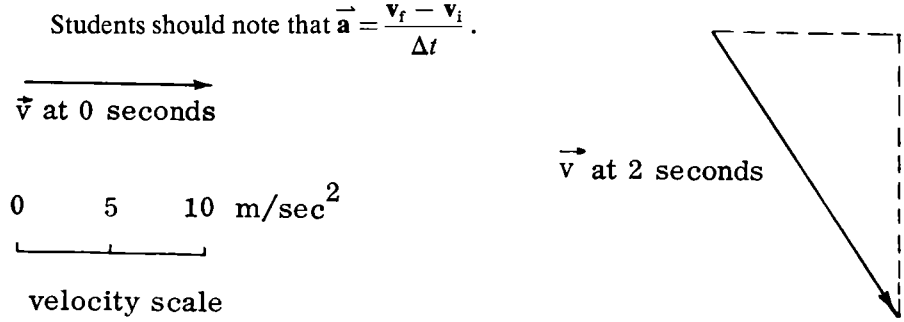


The development suggested above is intended to explore a number of hidden points which may trouble students in Section 10–6 on changing acceleration. In particular, troubles arise because we are dealing with vectors, and because the acceleration has different units from velocity and must therefore be plotted on its own scale.

At this stage you should turn the problem around and have the class compute the constant acceleration $\vec{a}$ that would change $\vec{v}_i$ into $\vec{v}_f$ in the time t. Once again introduce the problem with a graphic example. Several new points now come up. You might first ask the class to find the velocity vector which has been added to the initial velocity to get the velocity at 2 seconds.

For our example, it is seen to be (15 m/sec, down). This vector must be $\vec{a}t$. Therefore, since t is 2 seconds, $\vec{a}$ has a magnitude of $(15/2)\text{m/sec}^2$ and a direction down. To plot it we must again choose our acceleration scale. We then draw a vector down and of length corresponding to 7.5 m/sec^2 on the acceleration scale.

Students should note that $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$.



An example of a speedboat speeding up on a curved path is used to develop the idea of a changing acceleration vector, of the average acceleration over an interval, and the instantaneous acceleration.

The concept of acceleration as a vector is of key importance. All of the ideas in this section, particularly instantaneous acceleration, may not sink in on this first encounter, but students will have a chance to dig further into these ideas in later chapters. At this point, the most valuable thing students can do is individual work on graphical solutions.

Work out, or preferably have the students work out, an accurate and detailed analysis of the speedboat problem in the text, using data measured from Figs. 10–17 and 10–19. Try to get them to obtain fairly accurate tables of the average value of the acceleration $\vec{a}$ during each 10-second interval shown on Fig. 10–17 and the average value of $\vec{a}$ during each 2-second interval shown in Fig. 10–19. The two tables show approximately the same value for the acceleration during the same intervals, but the second table should show some detail of the way $\vec{a}$ is changing.

Students should perform a similar graphical analysis of the circular motion portrayed in Fig. 10–21. Other examples are given in Problems 14 and 15. Graphical analysis of these two problems is more instructive at this point than trigonometric solutions.

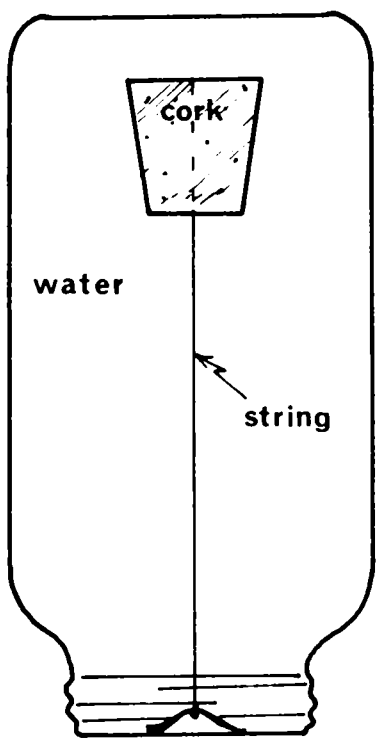
Do enough to make the concepts of average and instantaneous acceleration clear. The major treatment of uniform circular motion is in Chapter 12. Here you may wish to use it simply as one example in which you can state a simple rule for the velocity vector at different times. Avoid being drawn into a complete discussion of circular motion. *Above all, do not let forces creep into the*

discussion. If you find you are using forces to deduce the direction of an acceleration, you are doing the wrong thing. Students have a natural desire, in fact an almost irresistible urge, to talk about centrifugal force, but you must resist attempts to get into such matters. Acceleration, like displacement or velocity, is a geometrical concept. For example, from the fact that an object moves at 5 m/sec in a circle of radius 2 meters, we can compute its instantaneous acceleration without any mention of forces. In fact, in Chapter 12 we shall wish to deduce the instantaneous acceleration, $\vec{\mathbf{a}}$, from measurements of velocity and time, and compare it with the force acting on a body of known mass. When the force is measured separately we can then study Newton's Law $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, and see that it is really a vector law. The logic is spoiled if the students get the incorrect idea that $\vec{\mathbf{a}}$ can only be found from the force.

Demonstration

Cork in Bottle: An effective demonstration showing the direction of the acceleration vector in uniform circular motion can be made with a Mason-jar accelerometer. The accelerometer is constructed by soldering a wire loop to the inside of the jar lid and fastening a cork to the loop with a short length of string. The jar is filled with water and should be sealed under water to eliminate all air bubbles.

Place the jar, lid down, on a roller-skate lab cart and ask the students which way the cork will move, relative to the jar, as the jar is accelerated across the bench top. The cork moves in the direction of acceleration. Hold the jar at arm's length and ask which way the cork will move as the jar is revolved in uniform circular motion. The cork will point toward the center of the circle, and since the cork moves in the direction of the acceleration, the acceleration in circular motion is toward the center of the circle.



CHAPTER

11 Newton's Law of Motion

This chapter introduces Newton's law of motion, which is fundamental to the study of mechanics.

Do all you can to insure that the students get the material by their own efforts. Have them perform Experiments 20 and 21 early; 20 can very well precede the first textbook assignment, and 21 should follow shortly afterward. The post-laboratory discussions of these experiments should give a firm and adequate basis for the formulation of the law. From time to time in subsequent work, use the law in the form $\vec{\mathbf{F}}\Delta t = m\Delta\vec{\mathbf{v}}$, since this form will be used on occasion in later chapters. In view of the variety and complexity of the phenomena that can be unraveled with this law, its elegant simplicity is almost surprising. To appreciate the scope and power of Newton's law, students should become familiar with its application. Examples are given in the HDL section. However, too early or too great emphasis on the formality of applying Newton's law, especially in some of the more complicated problems, may detract from students' basic understanding of the relations between force, time, velocity, and mass. For this reason, the formal statement of Newton's law is deferred in the text until after the analysis of experiments which, holding two of these variables constant, display the relation between the other two. This approach is intended to avoid the simple memorization $F = ma$ without a real understanding of the fundamental relationships between force, time, velocity, and mass. Early laboratory work will contribute to this purpose.

The text begins with a presentation of Galileo's law of inertia after students have been encouraged to think independently about force-free motion. The emphasis is on Galileo's intellectual achievement, not only in the original enunciation of the law of inertia, but even more in the development of a method of reasoning in which he combined the observation of real experiments with idealizations of these to simpler and more tractable situations. Some schools may be fortunate in having air tracks or air tables and pucks which can be used to demonstrate nearly force-free motion.

During the development of Newton's law, the concept of inertial mass is introduced. It is expected that students will already have some familiarity with the equal-arm balance and with gravitational mass, but the differences between determining mass on the balance (the gravitational mass) and determining mass through dynamics experiments (the inertial mass) should be emphasized, as well as the similarities in their properties. Experiment 22 should be performed in connection with this topic; it is important that the students have the experience of making measurements of both kinds of mass on the same objects so that the points at issue will be clear.

Be sure the students understand that the relationship $F = ma$ provides a meaningful, nonarbitrary basis for defining a unit of force. Arbitrary units have already been introduced for the measurement of the fundamental quantities of length, time, and mass; no arbitrary definition of a unit of force—such as the force required to produce a given extension of a standard spring—is necessary.

Newton's law of motion is based on a wide range of experiments and observations, which are described in this and succeeding chapters. Sections 11–8, 11–9, and 11–10, for completeness of presentation, cite properties of the law which will not be verified by actual comparison with experiment until Chapter 12. The HDL problems relating to these sections are, however, very useful. They serve as valuable reviews of earlier vector work, and as introductions to mechanics. Subjects in the text for which the number of appropriate HDL problems may appear to be insufficient (inertial and gravitational mass, varying forces) will be found to reappear in later chapters, to be treated in more detail.

EXPERIMENT

Changes in Velocity with a Constant Force 20

This experiment and the following one (The Dependence of Acceleration on Force and Mass) are very important and should not be omitted. Since they lead the student to discover Newton's law, they should be done before Section 11–3.

As a result of this experiment the student will discover that, within his experimental error, a constant force acting on an object changes its velocity at a constant rate—that is, its acceleration is constant. The unit of force in this experiment is that produced by a rubber loop stretched a constant amount, and the velocity is expressed in meters per 10 "ticks." Notice that the units used are not important in discovering or expressing the fundamental law.

We do not use gravity as a pulling force on the cart because it raises too many questions that cannot be answered at this stage of the course. The student has no real way of knowing that gravity supplies a steady force to a moving object; and, in the next experiment, where masses must be measured, it is hard for him to see that the mass of the falling weight must be included as part of the total mass being accelerated. Furthermore, the rubber loop gives the student a feeling for what a constant force really is.

The experiment is best done on a smooth, level table about 2 meters long. If the table is not level, this will show up when the tapes made by a coasting cart are examined. The distance covered between two successive ticks of the timer at the beginning and end of the tape can be compared by holding the two ends of the tape side by side. There is no need here for accurate quantitative measurements. It is sufficient to see that a rapidly coasting cart moves at a reasonably constant velocity on a horizontal surface.

The timing tape should pass smoothly through a timer fastened to the table with a C clamp. The total frictional drag on the cart will be about 0.3 newton, a small force compared with the force applied through the rubber loop. The tapes should be labeled for future identification as soon as they are made. If the laboratory period is short, the analysis of the tapes can be done as homework.

Practice in using the apparatus is necessary before making any quantitative runs. It is not easy to keep the rubber loops stretched to the same length at the

start and while the cart is accelerating at high velocities near the end of the run. In starting the cart, it is important to synchronize the towing with the release.

A final word of warning: The rubber loops are ideal for shooting paper clips and miscellaneous missiles about the lab!

Answers to Questions

The velocity of a coasting cart is more uniform at high than at low speed.

Within experimental error, the data when plotted on the graph will show that a constant force produces a constant acceleration [Fig. (a)]. The deviations from a straight line result primarily from the difficulty of keeping the rubber loops stretched a constant amount.

The force exerted by the rubber loops is not the only force acting on the cart. The drag of the timing tape and the friction between the wheels and the table exert forces opposite to the motion. These forces produce negligible velocity changes compared with those produced by the applied force, as we can conclude from the coasting run.

The acceleration of the larger mass is less than that of the smaller one with the same force acting on the cart. Quantitative measurements of the effect of mass on acceleration will be done in the next experiment.

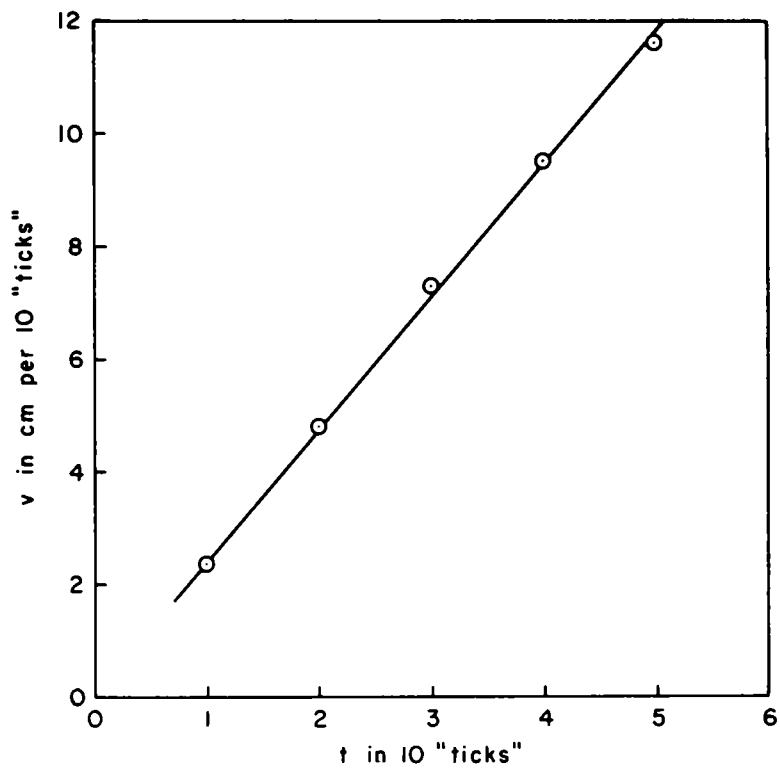


Figure (a)

Apparatus

1 Dynamics cart
1 Rubber loop
4 Bricks

1 Timer
Paper tape
Carbon-paper discs
2–3 C clamps
1 1½-volt dry cell
1 Sheet graph paper
Meter stick
Table stop
Cellulose or masking tape

EXPERIMENT

The Dependence of Acceleration on Force and Mass 21

The technique is the same as in the previous experiment, and the same precautions should be taken. Extra care is needed in the first part, where large forces are exerted. If it is necessary to improvise rubber loops to replace the standard ones supplied, five No. 19 rubber bands chained together are approximately the equivalent of one loop of the supplied strand.

In analyzing the motion the student may assume, after doing Experiment 20, that the acceleration is constant throughout each run.

Answers to Questions

The plot of the acceleration as a function of force comes close to a straight line and shows that force and acceleration are proportional [Fig. (a)]. From the plot, we can say that the ratio of force to acceleration is constant in this part of the experiment. With no friction, the curve will pass through the origin. With friction, the graph will show zero acceleration for a positive value of force because the applied force is equal and opposite to the friction force, and the *net* force is zero. In this experiment, the friction is so small compared to the pull of the rubber loops that failure to maintain a constant pull may obscure the force of friction. As a result, the curve may cross the force axis slightly to the right or left of the origin and cannot be depended upon to measure the force of friction [Fig. (a)].

The plot of the ratio of force to acceleration as a function of the number of bricks comes close to a straight line [Fig. (b)].

The intercept of this straight line with the "number of bricks" axis gives the mass of the cart in units of bricks. This is only approximately true, since the wheels of the cart are given rotational motion, and their rotational inertia may show up on the graph as equivalent to a mass of about 0.2 kg. The experimental data will not be accurate enough to show this.

Notice that at this point we have not yet discussed the relation of inertial and gravitational mass. If you do not plan to assign Experiment 22, you can use this experiment to introduce the discussion of inertial vs. gravitational mass by asking the students to compare the ratio of the gravitational masses of the cart and a brick with the ratio of their inertial masses found from the graph [Fig. (b)].

To find the inertial mass of a chunk of lead or a heavy stone, pull the unknown mass on the cart with one rubber loop. From the ratio of the force to acceleration, the mass can be found by interpolation (or extrapolation) from the "f/a vs. m" plot.

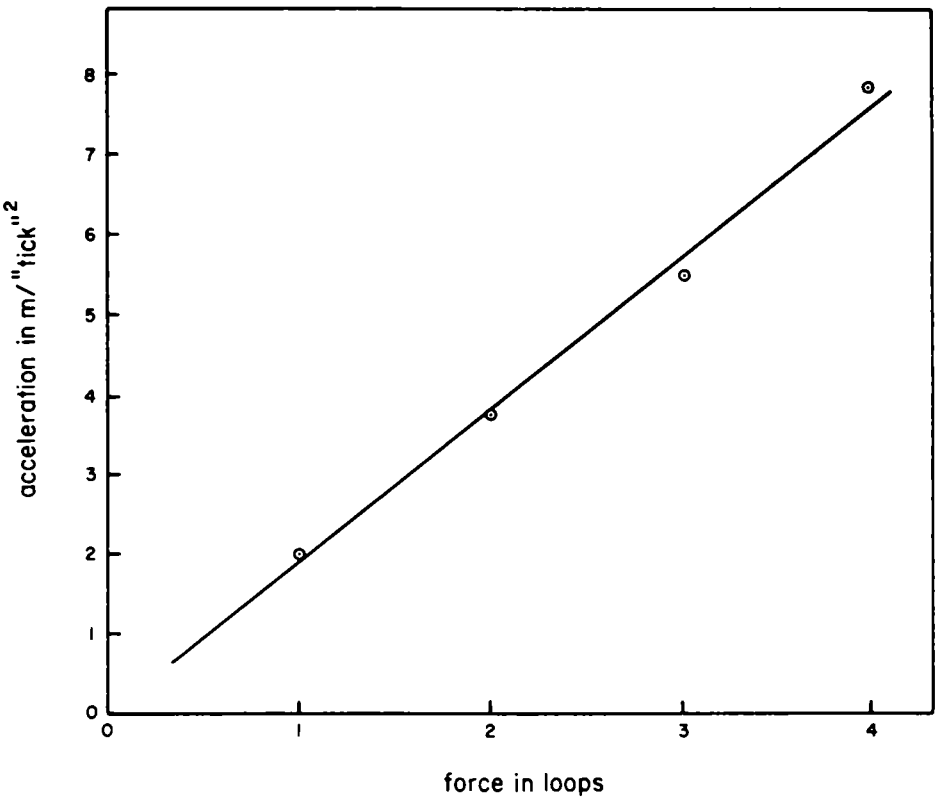


Figure (a)

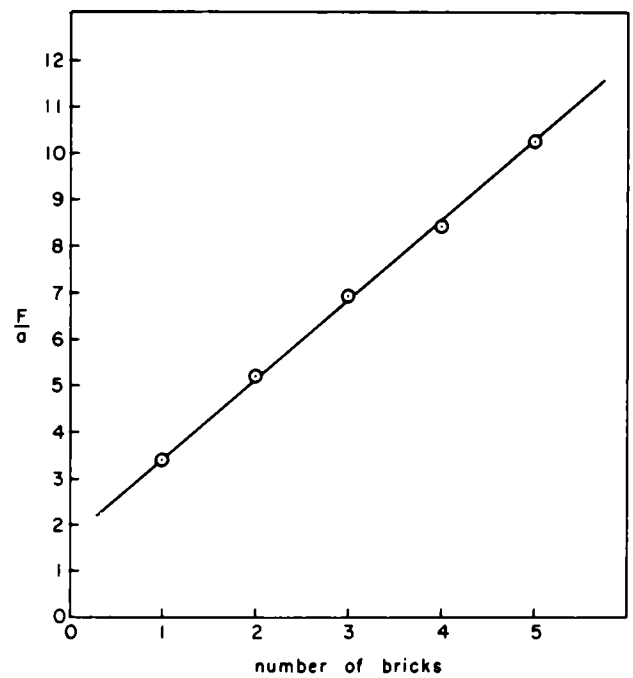


Figure (b)

Apparatus

1 Dynamics cart
1 Timer
Paper tape
1 Meter stick
4 Rubber loops
Graph paper
C clamps
1 1½-volt dry cell

EXPERIMENT

Inertial and Gravitational Mass 22

This experiment gives quite accurate results and will help clarify the relation between inertial and gravitational mass. It is best done after the discussion of Sections 11–5 and 11–6.

The experiment shows that gravitational and inertial mass are conceptually two different things. This is seen from the ways in which they are measured. Nevertheless, given two objects, the ratio of their inertial masses equals that of their gravitational masses.

The apparatus used in this experiment gives results accurate to about 1 percent.

Two-inch C clamps are convenient units of mass and can easily be attached to the platform of the balance. This gives acceptable results, since the clamps are reasonably uniform. If time permits, you may want to adjust the clamps to weigh the same to about 1 percent. To do this, take the lightest clamp as a standard and file or grind the others to match.

Other objects of similar size, shape, and composition can be used as standard masses. A piece of sandpaper glued to the platform of the inertial balance prevents these masses from sliding when the balance vibrates. If they slide, the balance will come to a stop before an adequate count can be made.

One of the things for the student to learn from this experiment is that the standard unit for inertial mass can be completely independent of the gravitational unit of mass. Therefore, it is advisable not to adjust the units of inertial mass to exactly 100 grams.

Any object of different material from the standard can be used as the unknown mass. But, to allow for accurate interpolation, it should have a mass that does not fall too near either end of the range for which the balance has been calibrated (100 to 600 grams). The unknown mass can be clamped to the balance with a unit C clamp.

In finding the period of the unloaded balance, it may not be possible for the student to count enough vibrations to get the period with an error of only about 2 percent. This less accurate value is still worth plotting on the graph, but it should not be given too much importance when the curve is drawn.

When the load on the balance is too large (above about 750 grams) there is a visible up-and-down motion as well as horizontal motion. This makes the plotted relationship between T and m diverge from $T = 2\pi\sqrt{\frac{m}{k}}$, the relationship that holds for simple harmonic motion. (Do not discuss this relation in class at this time, as it will come up later in Chapter 12 of the text.) The

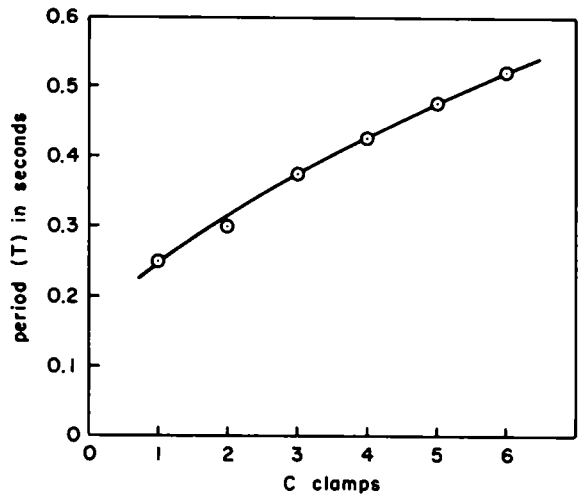


Figure (a)

more complicated graph [Fig. (a)] that one actually gets is still perfectly good as a calibration curve.

Unlike a simple pendulum, the frequency is independent of amplitude over a very wide range of amplitudes.

Answers to Questions

For larger masses, the period is greater.

Your students may be confused as to how to measure the period of the inertial balance with less than 2% error. One procedure is to estimate the percent timing error and the percent counting error and add these two to get the total percent error expected.

The real problem is to make as accurate measurements for fast vibrations as for slow ones. For a fixed number of vibrations, the period of a slow rate can be determined more accurately for two reasons: first, the time interval is longer; and second, it is easier to see when a vibration has been completed.

You can ask the students if it is better always to measure the same number of vibrations in each trial, or always to count for the same time duration. Taking equal time durations is the better way. The timing error will then be the same in all cases. When the vibration is high, there will be many vibrations to count, and the chance of making an error is greater; but the ratio of the counting error to the total number of counts would be more nearly equal for all trials.

The platform will easily make 100 vibrations, so the student has a wide choice of time durations. By repeating a count on the platform alone for the time duration he initially selects, he can determine the spread of error and can see if the majority of counts fall within 2%. He can then use the same time duration for the remaining trials.

The gravitational mass of the "unknown" is found from its inertial mass m_i:

$$\frac{(m_g)\ \text{unknown}}{(m_i)\ \text{unknown}} = \frac{(m_g)\ \text{standard}}{(m_i)\ \text{standard}}$$

In our experiment (m_i) standard = 1 by choice.

Inertial and gravitational mass are proportional within the experimental error. They are not equal, since different units (C clamps and grams) have been used. They are, of course, equal in most physical work since the same standard piece of matter, the standard kilogram, is used for both scales.

On the moon we would get the same results. Neither gravitational nor inertial mass changes in going from earth to moon.

Apparatus

1 Inertial balance
6 C clamps, 2″ (can be shared to some extent)
1 C clamp, 3″
1 Metal slug
Cord
Ringstand with clamp or crossbar
1 Sheet graph paper
Stop watch or clock with sweep-second hand
Pan balance
Several unknown masses (mass of 2 to 4 units)

Films

"Forces," by Jerrold R. Zacharias of M.I.T., is an introduction to the mechanics part of the course. The film examines the general nature of gravitational and electric (both electrostatic and magnetic) forces and touches briefly on nuclear forces. The gravitational force between relatively small objects is demonstrated with a Cavendish balance. Running time: 22 minutes.

"Inertia," by Edward M. Purcell of Harvard University, develops the idea of inertia, considers frictional effects, and introduces the "frictionless" puck which has a gas bearing. From stroboscopic photographs of the puck moving under a constant force, the proportionality of force and acceleration is found. One of the best ways to use this film is as the basis of a summarizing discussion of the ideas of Sections 11–1 through 11–4. Running time: 27 minutes.

"Inertial Mass," by Edward M. Purcell of Harvard University, is a continuation of "inertia." Using the same techniques as in the earlier film, acceleration is found to be inversely proportional to mass. Thus, with a constant force, acceleration is a measure of inertial mass. The proportionality of inertial and gravitational mass is shown. This film may be used in connection with a discussion of Sections 5 and 6. Running time: 20 minutes.

Demonstration

1. A Dry Ice puck is a very useful device to demonstrate uniform motion in the absence of forces. The construction of the puck is shown in Fig. (a). Since the layer of gas between the puck and the table is very thin, it is necessary to have a table with a smooth surface of glass or metal. It is essential that the surface of the table, as well as the surface of the puck, be very clean and dry. Wipe the table and the puck with a dry cloth to remove the condensation accumulated during the demonstration.

If the table is level, the puck will slide the length of it at very low velocities without slowing down. If the table is not level, the puck will accelerate from rest. Experimenting with these pucks makes Galileo's law more real.

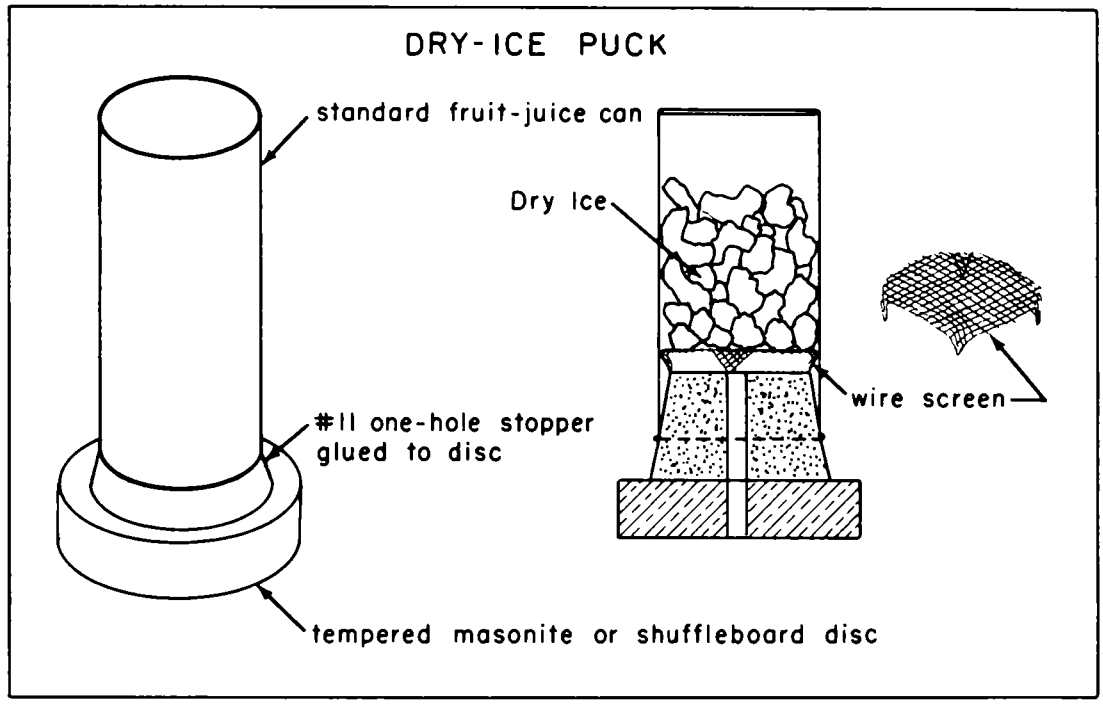


Figure (a)

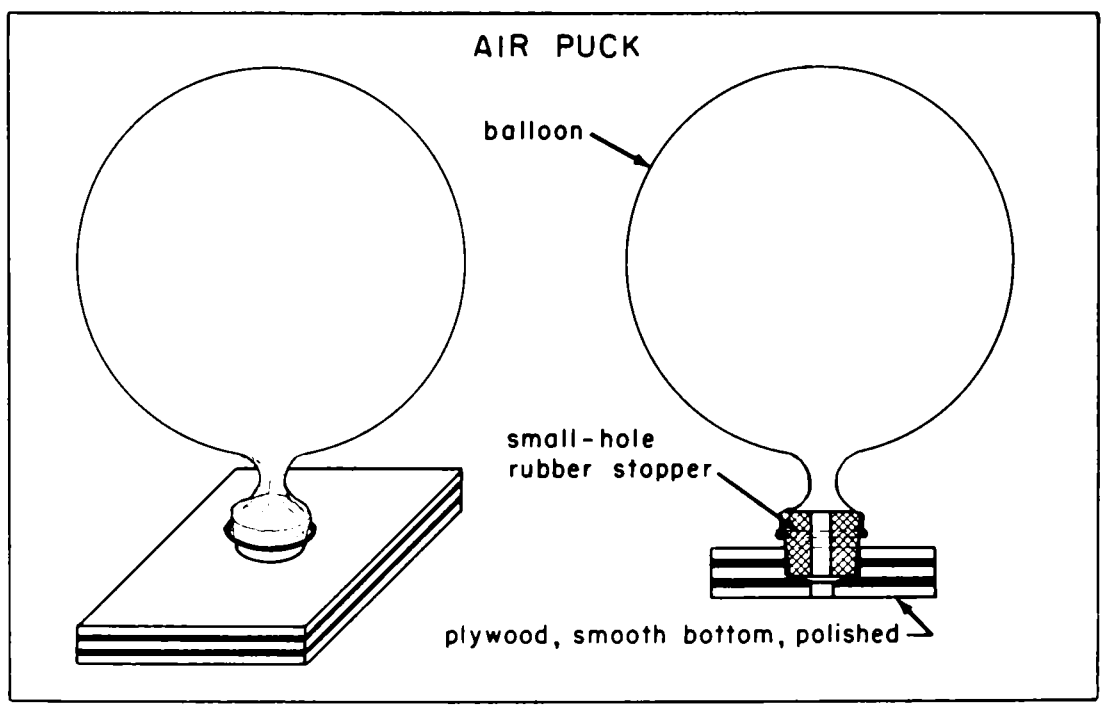


Figure (b)

If Dry Ice is not available, an air puck made as shown in Fig. (b) can be used. Its performance is inferior to that of the Dry Ice puck, but it is useful nevertheless. It can even be used with fair success on a table with a Masonite or Formica top.

2. A weight of several pounds is suspended with a length of thread. (The weight could be some reasonably heavy tool such as a pipe wrench.) Tie another piece of thread below the weight. Now pull gradually, harder and harder, on the lower piece of thread. Ask students to guess which thread will break. Eventually the thread above the wrench will break. (Look out for your knuckles!. The upper thread breaks first because the force on it will be the applied force plus the weight of the wrench.

Next, suspend the wrench as before, but this time starting with no tension in the lower thread, pull with a sharp jerk. If you do this quickly enough, the lower thread will break and the top one will remain intact. The explanation is that in order to transmit force to the upper thread, (since the thread is somewhat elastic) the wrench must move, increasing the stretch of the thread. The tendency of the wrench to remain stationary prevents the force of your jerk from being transmitted instantaneously to the upper thread. Before the wrench has time to move far enough to transmit this force, the lower thread breaks.

CHAPTER
12

Motion at the Earth's Surface

After a small amount of class discussion the HDLs may be used for covering the material in Sections 12–1 through 12–4. Notice the introduction of $\vec{\mathbf{g}}$ as the gravitational field vector in Section 12–1, with units of newtons per kilogram. By our choice of units of mass, we make the acceleration of a freely falling body numerically equal to $\vec{\mathbf{g}}$, but the students will calculate gravitational forces more easily if you consistently treat $\vec{\mathbf{g}}$ as a field vector rather than an acceleration. This section also helps to prepare the students' minds for the electric field around a point charge.

Both in discussing the text and in solving problems, students should be reminded of the three distinct steps involved in determining the motion which results from the action of one or more known forces:

1. Determine the net force, the vector sum of all the forces acting.
2. Using Newton's law, find the acceleration.
3. With this known acceleration, a, find the motion by applying the kinematic equations.

This separation is particularly worthwhile because it stresses the tremendously wide applicability of Newton's law. Sometimes the forces may be complex; indeed the steps are often reversed so that the force is found from the motion.

In dealing with projectile motion the independence of the components of force should be emphasized. This is the key to the vector nature of the force. One reason for stressing free fall and doing practice problems is to get students to appreciate the great simplification which is possible when the components are separated. Taking components is also essential in relating simple harmonic motion to circular motion.

Students are so often instructed to "neglect friction" or "neglect air resistance" that they may think only simple, idealized motion can be analyzed. In Experiment 23 Forces on a Ball in Flight the student can see how an application of what he has learned about simple projectile motion is the starting point which leads to an understanding of more complex motion. That is the most important thing for him to learn from this experiment. He also makes a discovery for himself; by analyzing Fig. 1 he learns that air resistance acts in a direction opposite to the motion and that it increases as the speed increases. This experiment, although important, involves no experimental equipment and can be done as a homework assignment after projectile motion has been studied in the text.

The first stage in understanding circular motion involves an appreciation of the qualitative idea that an acceleration which occurs in the direction perpendicular to the instantaneous velocity does not change the magnitude of the velocity. This idea is frequently a difficult one for students and some detailed discussion or review is usually in order. The development rests on the idea that vector acceleration is defined as rate of change of *vector* velocity. The magnitude of the velocity vector may remain unchanged. Also, many students must be reminded that when the centripetal force is removed, as in Figure 12–11, the subsequent motion is linear and tangent to the circle. Intuition sometimes fools one here because one may suppose that if the force pulling toward the center is removed, the object will move radially outward. Ask students about this, preferably in conjunction with a demonstration.

The purpose of Experiment 24 Centripetal Force is to find the dependence of the centripetal force on the frequency of revolution, mass, and radius of an object moving in a circle. The experiment is best done before the relation is derived in the text (Section 12–5). As a result of doing the first part of the experiment, the student will conclude that the centripetal force is proportional to the square of the frequency. If time permits and some of the students wish to go further, they will conclude from experimental evidence that the centripetal force is proportional to the mass and to the radius of the circle.

If you are crowded for time you can omit Section 8 completely without sacrifice of background for the work which follows. Experiment 25 Simple Harmonic Motion may be done after the text has been discussed. At the beginning of this experiment students make predictions using the equation describing simple harmonic motion and then verify these predictions by taking measurements. After their reliability has been established, students will use the equations without further verification.

The last three sections of this chapter on frames of reference and fictitious forces answer a number of questions which come up in daily life and prepare the way for the chapters on relativity. Spend a day on them if you can and supplement the discussion by showing the film "Frames of Reference," by Donald Ivy and Patterson Hume. It makes points visually which are extremely difficult to establish in words.

The perception that we sometimes invoke "fictitious" forces to explain familiar phenomena may lead some students to ask penetrating questions about the fundamental nature of force. After all, we cannot really "see" gravity or electric and magnetic forces, only their effects. Might not these, or some of these, forces also be fictitious in some frame we have not yet thought of? As yet we do not know enough about the causes of gravitational or electrical forces to know whether these "action-at-a-distance" forces are real or "fictitious." Einstein, in his theory of general relativity, has formally accounted for the force of gravity in terms of the properties of space and time, but as of now we do not have a "unified field theory" to account similarly for electric and magnetic effects. Thus, at least at the present stage of development in physics, we treat these action-at-a-distance effects as real forces. We simply do not know of other frames of reference which we can apply (as we can with the "centrifugal" force) which will make electric and magnetic forces vanish.

As they have progressed from free fall, to projectiles, to circular motion, to simple harmonic motion, students have seen a wide variety of situations in which Newton's law applies. When you finish the chapter, it may be worth while as a review to list the characteristics of the forces and motions which have been studied:

TYPE OF MOTION	MAGNITUDE OF FORCE	DIRECTION OF FORCE	MOTION
Free Fall	constant	constant in line of motion	uniform acceleration
Projectile	constant	constant but not in line of motion	parabolic, uniform acceleration
Circular Motion	constant	varying to be perpendicular to motion	uniform circular motion
Simple Harmonic Motion	proportional to displacement	restoring, i.e., opposite to displacement	oscillatory

EXPERIMENT

23 Forces on a Ball in Flight

Centimeter graph paper provides a convenient scale, although any tracing paper will suffice. Fig. (a) is a sample analysis of Fig. 1. Students may have trouble reducing the change in velocity resulting from $\Delta\vec{v}_g$ to a vector they can

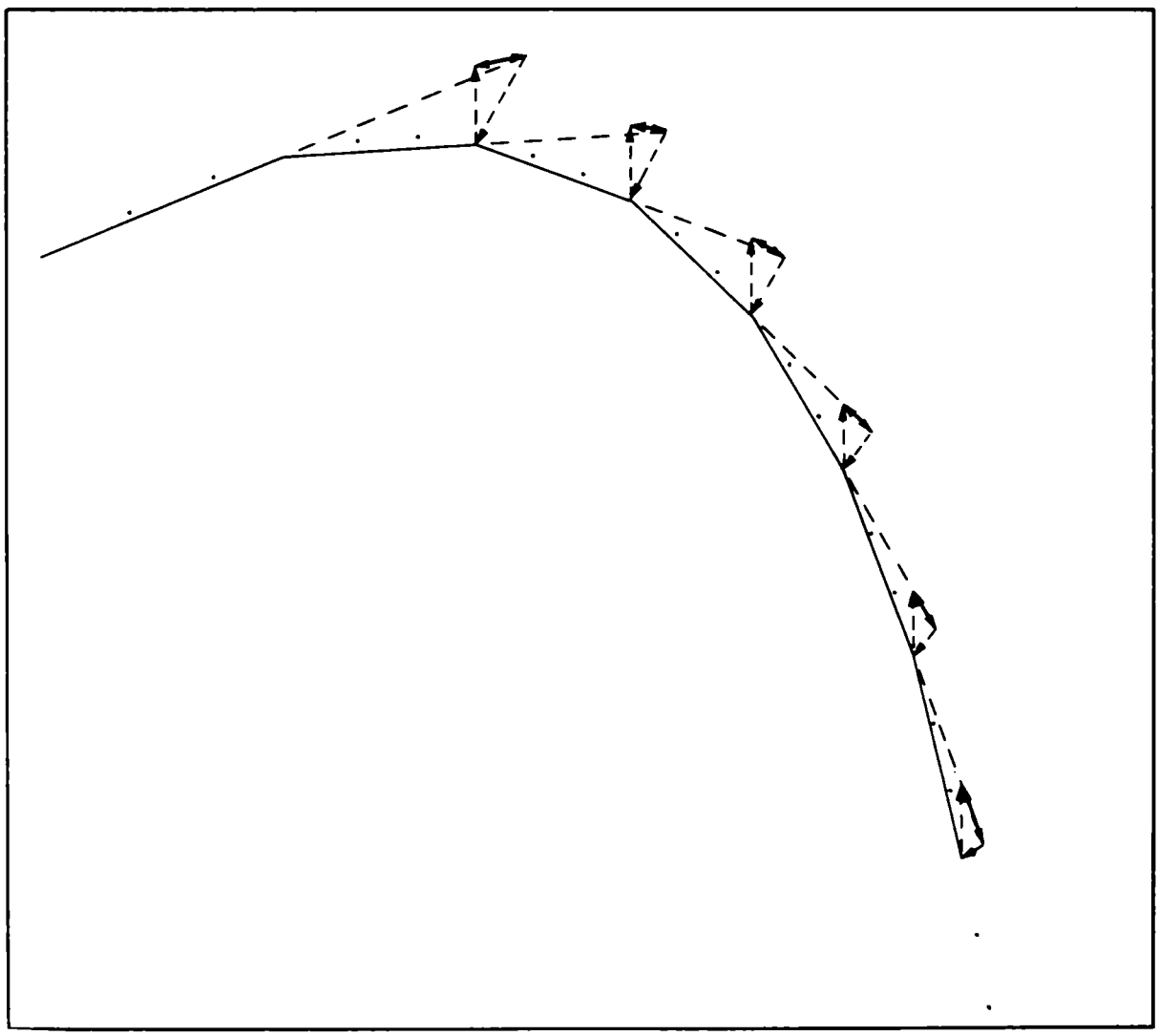


Figure (a)

subtract on their diagram. The first step is $\Delta v_g = g\Delta t = 9.8$ m/sec^2(0.1 sec) $= 0.98$ m/sec. This must then be converted to meters per *tenth* of a second. This will be $0.98(0.1) = 0.098$ m/0.1 sec. This is equivalent to converting the acceleration of gravity from meters per second squared to meters per (tenth of a second)^2,

$$9.8 \text{ m/sec}^2 = \frac{9.8}{100} = 0.098 \text{ m/(0.1 sec)}^2.$$

This value is then reduced to the scale of Fig. 1. Since the photograph is $\frac{1}{10}$ actual size, the length of the velocity vector due to gravity Δv_g that must be subtracted will be $\frac{0.098}{10} = 0.0098$ m. This change is, of course, in the vertical direction and is subtracted from the velocity change $\Delta\vec{\mathbf{v}}$ by adding its negative as shown in Fig. 2(b).

If a student is interested in analyzing Fig. 3 and 4, he should check the scale reduction by measuring the meter stick in the photographs.

Answers to Questions

A qualitative examination of the photograph shows that the horizontal velocity decreases from left to right. One concludes from this that in addition to the force of gravity there is a force that has a horizontal component.

The total velocity changes found by subtracting velocity vectors on the diagram vary from interval to interval in both direction and magnitude. The magnitude of the $\Delta\vec{\mathbf{v}}$ decreases during the motion and the direction slowly becomes more horizontal. Since velocity changes are in the direction of the forces which produced them, there is at least a horizontal component of the net force acting on the sphere. Without further analysis it is impossible to tell how the unknown force changes.

The change in velocity of the ball is 0.1 sec caused by gravity is 0.98 m/sec and it acts vertically downward. In meters per *tenth* of a second it is 0.098 m/0.1 sec.

When the effect of gravity is subtracted, the residual velocity changes due to the remaining force have both a vertical and a horizontal component and are opposite to the direction of motion. The larger the velocity, the larger is the opposing force. The retarding force is the result of air resistance. This force is proportional to the square of the velocity under the conditions of our experiment. The total variation of the velocity, however, is not great enough to show this proportionality by a graph of the force as a function of $\vec{\mathbf{v}}^2$, particularly with the limited accuracy with which one can measure the force in the analysis of the data.

The ball must have a very large area-to-mass ratio (a very low density) to show such large effects of air resistance at relatively low velocities. If the ball had been the same size but much heavier, it would have followed a path more like the parabolas illustrated in the text. The ball used in all three photographs was a 0.05-gm sphere 1.5 cm in diameter cut from Styrofoam.

If gravity had been the only force acting on the sphere, its trajectory would have looked like Fig. (b) where the average velocity in the first interval was taken as the initial velocity.

The ball in Fig. 1 was thrown by hand with little or no spin. Fig. 3 and 4 were made with the same flash rate as Fig. 1, but the ball was given a spin.

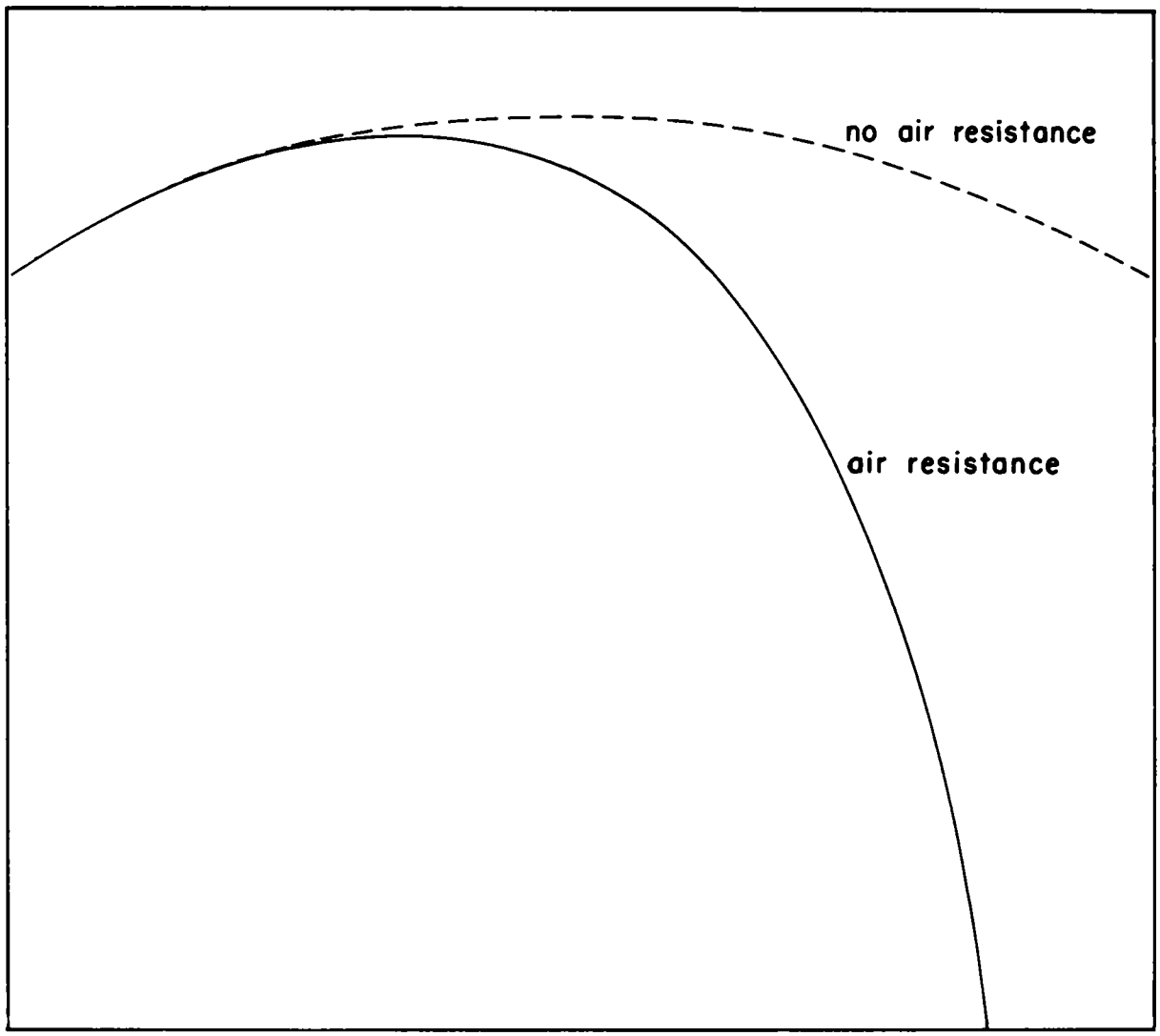


Figure (b)

In Fig. 3 the ball was given a counterclockwise spin about an axis perpendicular to the page. In Fig. 4 the spin was clockwise. The analyses of Fig. 3 and 4 are shown in Fig. (c) and (d).

The analyses of the spin photographs and the plotting of the trajectory with no air resistance should be considered supplementary to the main experiment and done only by particularly interested students. In Fig. (c) the force which remains after subtracting the effect of gravity is no longer opposite to the direction of motion but has a component at right angles to the path. This component becomes smaller as the rate of spin decreases during the motion. Fig. (d) shows a similar component perpendicular to the path but in the opposite direction.

A complete mathematical treatment of this spin motion (or even of the motion with air resistance and no spin) is a formidable task not even to be attempted.

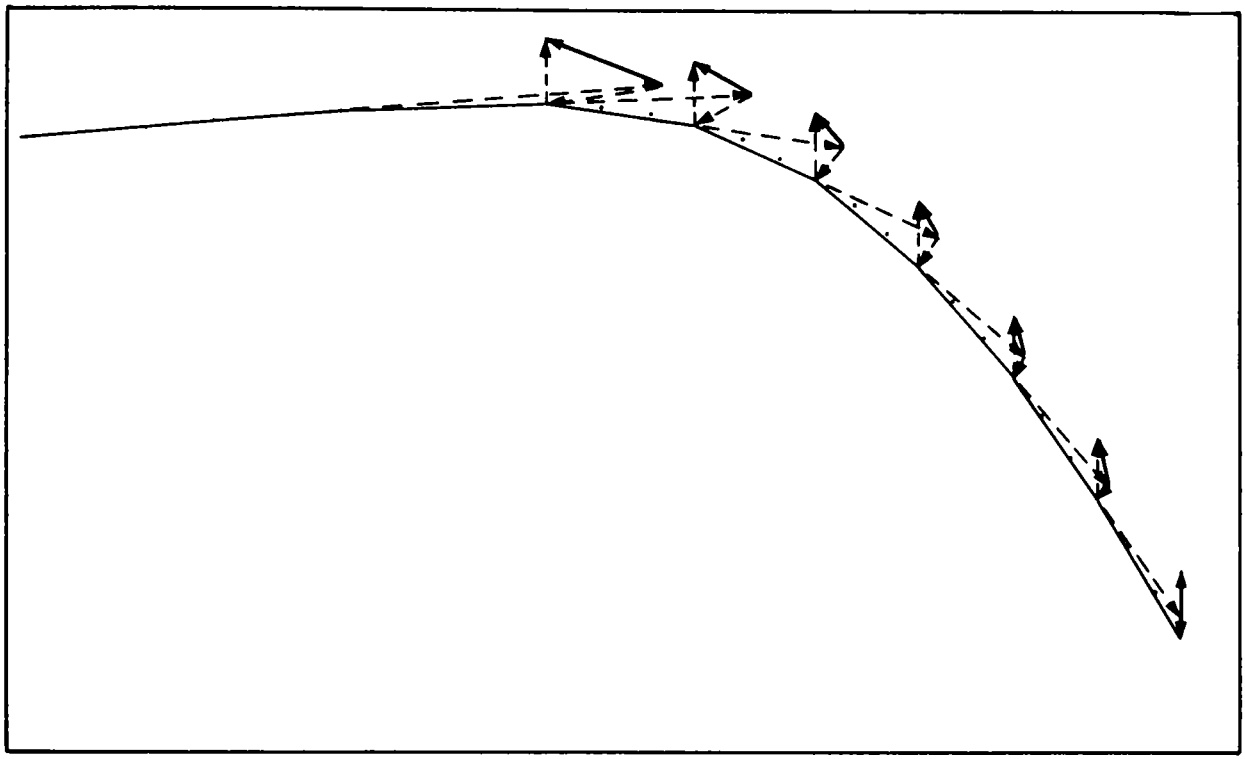


Figure (c)

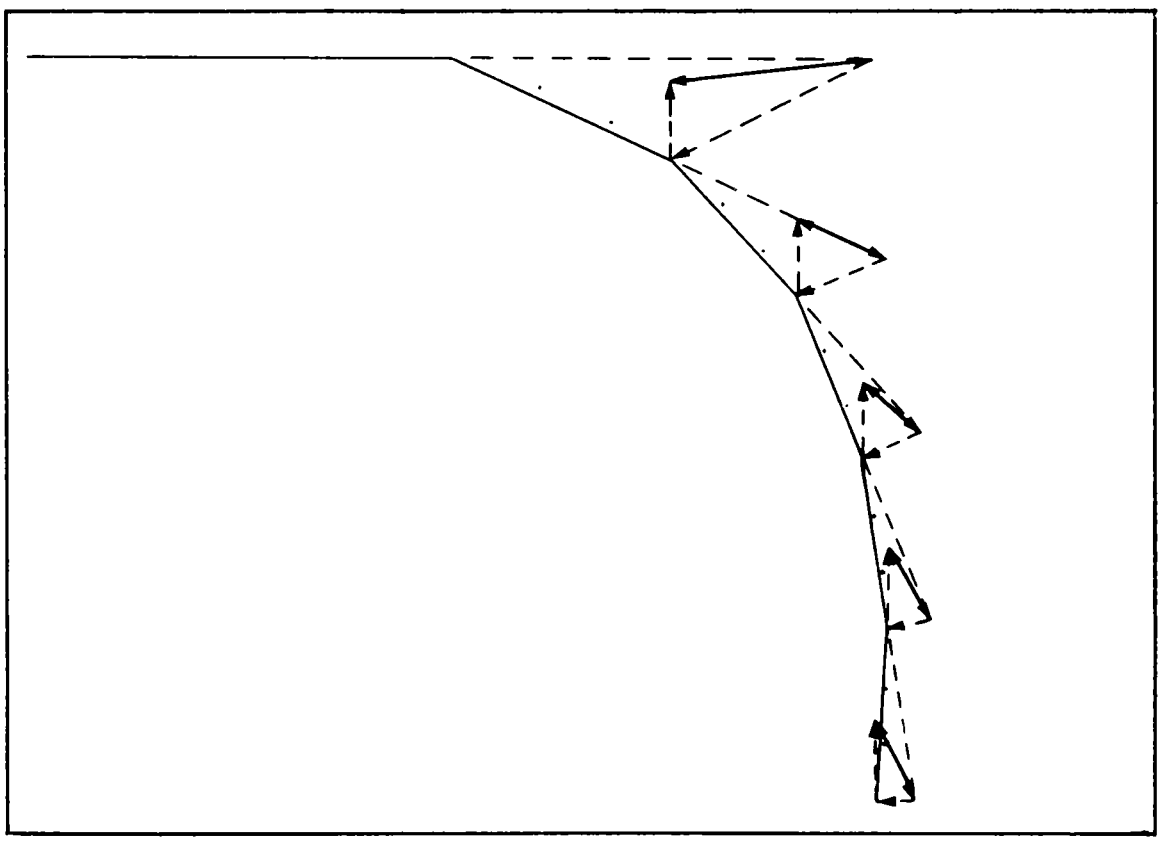


Figure (d)

In all three photographs, the velocity of the ball near the end of its trajectory is nearly constant. The ball is close to its terminal velocity. When the ball reaches its terminal velocity, the force of gravity on the ball equals the air resistance, the *net* force is zero, and it moves at constant velocity.

Students may wish to fire Ping-Pong balls with a spin. A cardboard mailing tube makes a good gun. Put the ball in the tube and swing it in an overhand arc or an underhand arc, depending on the desired direction of spin. This method was used in making Fig. 3 and 4.

Apparatus

Translucent centimeter graph paper or tracing paper
Ruler

Hints on Strobe Photography

A camera with a motor-driven strobe disk in front of the lens records on film the position of an object each time a slot in the strobe disk moves past the lens. This technique makes it possible to analyze two-dimensional motion which cannot be measured with a tape timer.

In most instances, exposure rates of 5 to 30 per second will suffice. A low-speed motor such as a 300-rpm synchronous clock or instrument motor is the most versatile, since higher exposure rates can be obtained by having several equally spaced slots in the disc. A synchronous motor is best for driving the disc, since its rate of rotation, and therefore the exposure rate, is known. However, in many cases the unit of time is arbitrary and therefore a cheaper motor will be adequate.

The speed of a non-synchronous motor can be measured and checked for constant speed by photographing, through the rotating disc, a 78-rpm or $33\frac{1}{3}$-rpm phonograph disc marked with a white spot. If a synchronous motor is overloaded, it will not run at its rated speed. One should be sure, therefore, that the disc is not so large that air resistance against its surface will slow it down. When a synchronous motor is overloaded, a neon or fluorescent lamp operating on 60-cycle A.C. will flicker when observed through the rotating slotted disc.

The rotating disc should be as close as possible to the camera lens without touching it. Both the camera and the stroboscope should be held rigidly.

The individual exposure of a moving object can be varied by adjusting the disc-slot and lens apertures. Open the basic lens aperture as wide as possible. The lens should then be taped so that its aperture has the same shape as the slot and has a width equal to or smaller than that of a slot in the disc. Sharper images are obtained by decreasing the slot widths.

The total exposure time can be controlled better by using the bulb setting rather than the time-release setting of the camera. If a cable release, instead of the hand, is used to operate the shutter, there will be less chance of the camera vibrating.

A film with an A.S.A. emulsion speed of 160 or higher is adequate. In some cases, a better image can be obtained by overdevelopment and/or printing on high-contrast paper.

The plane of the film in the camera should be parallel to the plane of the motion, to prevent unequal magnifications of the different areas of view. If a

scale is used to give absolute measurements, it should be placed in the same plane as the motion, so that there is a minimum error due to parallax.

For best results, erect a dark-colored backdrop of cloth, paper, or other material approximately 2 meters square. If it is necessary to paint the surface, a flat paint is best to prevent glare and reflection. The object to be photographed should be mounted to move parallel to and about a meter in front of the backdrop. The camera and stroboscope are then placed approximately $1\frac{1}{2}$ meters in front of the object.

The moving object can carry its own light source (a flashlight bulb powered by two AA dry cells) or be illuminated with two ripple-tank light bulbs (150-watt, clear glass, with shields). For objects moving horizontally, mount the two bulbs on either side of the camera, about a meter apart and pointing inward at an angle of approximately 40° to the plane of the object. To photograph objects moving vertically, mount one bulb on the floor pointing upward at an angle of 80° to the object plane, and the other bulb about two meters directly above pointing downward at an angle of 80°.

If it is desired to show a scale on the finished print, place a meter stick in the plane of the object.

Satisfactory prints using a Polaroid camera have been made with the arrangement described above and with the following specifications:

Lens aperture	*f*4.7
Shutter setting	B
Strobe wheel	2, 3, or 6 slits open
Film	Polaroid, No. 47 (3000 ASA)

Parts and Materials

Any camera with a bulb or time exposure
Motor (preferably synchronous): 300 to 1800 rpm (mounted)
Disc (6″ to 10″) of heavy paper, thin metal, plastic, or similar material
Several high-wattage lamps with sockets, cords, and supports

EXPERIMENT

Centripetal Force 24

As in many of the experiments in this volume, it should be emphasized that the discovery of fundamental laws does not require the use of standard units of measure. Here we use arbitrary units of force and mass: the unit for measuring centripetal force is the force of gravity on a steel washer; the mass unit is the mass of a No.4 rubber stopper.

The experiment can be done by students separately in the laboratory or at home, but it is best done by two or three together—one to swing the stopper in a circle, another to count the number of revolutions, and a third to do the timing. If the laboratory is too small for swinging many stoppers in a circle of one meter radius, it is advisable to do the experiment out-of-doors or in the school gymnasium.

The size of the glass tube is not critical, but the bearing surface for the string at the top end must be carefully fire-polished so it is smooth and free from bumps to keep friction at a minimum. The forces are such that the tubes will not break unless it has a flaw. To prevent injury, wrap the tube with tape so there will be no danger from jagged pieces of glass if it should break.

The cord used is critical if friction is to be kept low. Fisherman's hard nylon *braided* casting line, 12-lb test, works well.

The stopper size, radius of the circle, and size and minimum number of washers have been carefully chosen to give good results. Large variations from the recommended values may make the graph of force as a function of velocity look like a straight line.

To keep the stopper moving at constant frequency, the top of the tube must be moved in a small circle. This provides a small component of force along the path to counteract friction. However, this circle should be less than a centimeter or two in diameter or the circle described by the stopper cannot be accurately measured by the length of the cord. The radius of the circle described by the stopper should be measured from the middle of the stopper near its center of mass. Since the dimensions of the stopper are small compared to the radius of the circle, measurement from this point will give a value for the radius which is close to the correct value.

The Laboratory Guide specifies a minimum number of washers so the cord will be horizontal enough to permit the use of its length as a reasonably accurate measure of the radius of the circle. Actually, using the "slant" radius given by the length of the cord leads to correct results no matter how large the angle between cord and the horizontal. This can be verified by analyzing Fig. (a). The force of gravity $m_w g$ acting on the washers is transmitted along the cord and acts on the stopper along L. The horizontal component of this force provides the centripetal force F_c along R. We see that $R = L \cos \theta$ and $F_c = m_w g \cos \theta$. Writing the equation for the centripetal force in the form

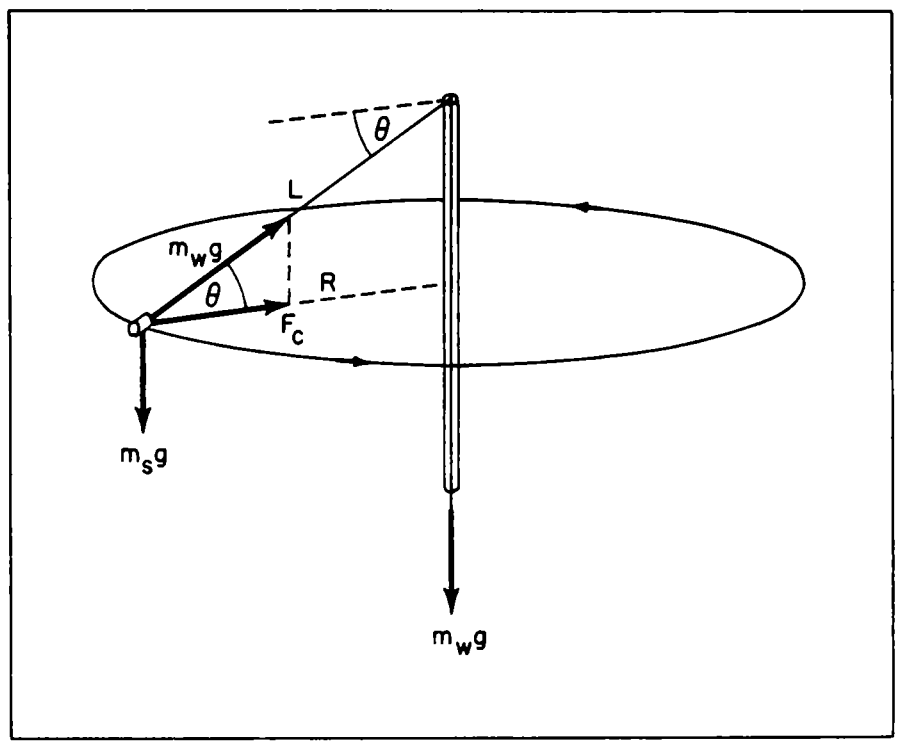


Figure (a)

$$F = 4\pi^2 m_s f^2 R = \frac{4\pi^2 m_s R}{T^2},$$

we have by substitution

$$m_w g \cos\theta = \frac{4\pi^2 m_s L \cos\theta}{T^2}$$

$$m_w g = \frac{4\pi^2 m_s L}{T^2}$$

This means that the force exerted by the washers is inversely proportional to the period squared for a constant length of cord L. Since the student assumes that R stays very nearly equal to L, he measures the period of rotation and assumes this to be inversely proportional to the speed v. There is no error due to the changes in the angle θ, because θ drops out of the calculation. You will note from Fig. (a) that θ is determined only by the weight of the washers, since $\sin\theta = \frac{m_s g}{m_w g}$. Using less than the recommended number of washers with a No. 4 stopper will therefore make θ larger.

Answers to Questions

The faster the stopper rotates, the harder one must pull down on the string. If one lets go, the stopper ceases to move in a circle and pulls the string up the tube behind it. By exerting the force himself, the student becomes aware that a force at right angles to the motion is necessary to make an object move in a circle.

A graph of the frequency of the stopper as a function of the number of washers looks like Fig. (b). It is, of course, not clear what function this graph represents, although one might suspect that it is a parabola. The student may think, if his points are somewhat scattered, that the curve is a straight line which does not pass through the origin, but if he is asked what the frequency should be with zero force, he should see that it would have to be zero, that is, the stopper must be stationary. Hence the curve must pass through the origin, curving down from his assumed straight line.

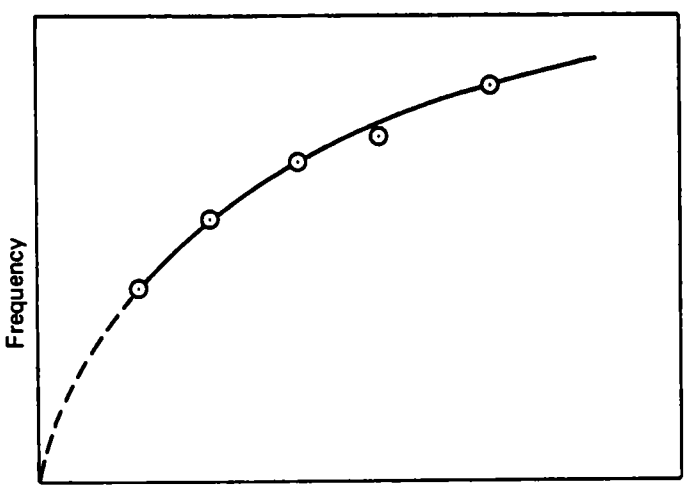


Figure (b)

A plot of f^2 as a function of the number of washers will be a straight line passing through the origin, showing that the centripetal force varies as f^2.

The student can conclude, from Newton's law $F = ma$, that doubling the mass whirling in a given circle with a given centripetal acceleration should require twice the force to keep it moving at the same speed in the circle. If he has time, he can try this.

It is difficult to find out how the force depends upon the radius, because it is hard to whirl the stopper at the same frequency each time the radius is changed. A better way of getting the relation between force and radius is to repeat the first part of the experiment several times, using different radii and only one value of centripetal force for each radius. Then a family of curves of f^2 as a function of F can be drawn [Fig. (c)]. Since we know from the first part of the experiment that these curves are straight lines, the single value of the centripetal force for each radius suffices to establish the straight line through the origin.

For a constant value of f^2, represented by the line AB, the various values of the force (F_1, F_2, F_3, F_4,) lie on curves corresponding to different values of R. When these values of F and R are plotted, they give curves which resemble Fig. (d). [Note that Fig. (c) and (d) do not represent actual data taken with the apparatus. They are merely approximate sketches of the results that would be obtained.] Students who are well versed in "ham" radio may recognize this method of analyzing a family of curves as similar to the method of finding the

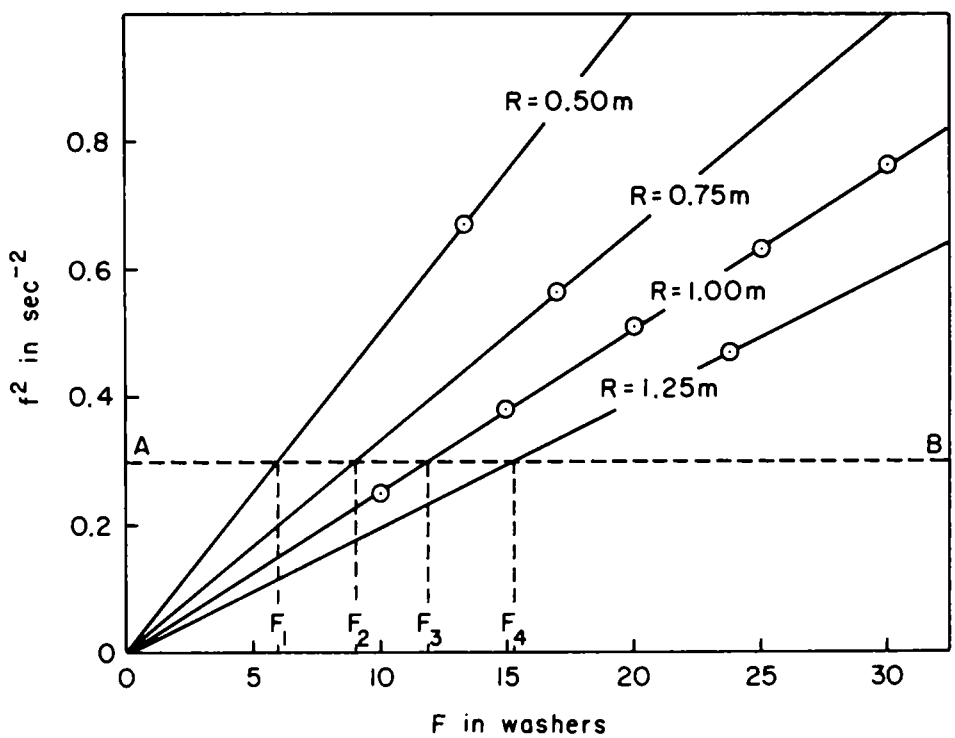


Figure (c)

amplification factor of a vacuum tube from a plot of the plate current as a function of the plate voltage.

If there is not enough time to investigate experimentally the effect of mass and radius on the centripetal force, it may be worthwhile to do this part of the experiment as a class demonstration.

Figure (e) shows all the forces that act on the apparatus. $\vec{\mathbf{F}}$ is the resultant force that the operator exerts on the apparatus (neglecting the weight of the tube). Note that the gravitational component of this force ($m_w\vec{\mathbf{g}} + m_s\vec{\mathbf{g}}$) is constant in direction while the horizontal component $\vec{\mathbf{F}}_c$, the centripetal force, is constantly changing direction in the horizontal plane.

Apparatus

- 1 Glass tube, fire-polished and wrapped with tape
- Cord, 1.5 m (fisherman's hard nylon braided casting line)
- 20 Washers
- 1 Alligator clip, or other suitable marker
- 1 Rubber stopper
- 1 Paper clip
- 2 Sheets graph paper
- 1 $\frac{1}{4}$" dowel or pencil
- Meter stick
- Stop watch or clock with sweep-second hand

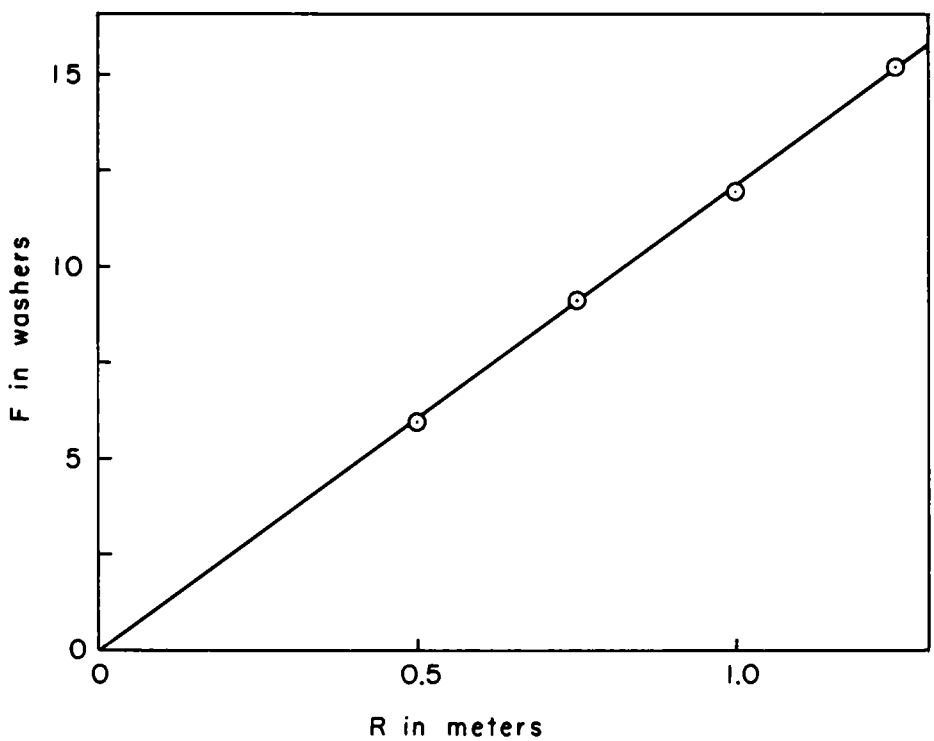


Figure (d)

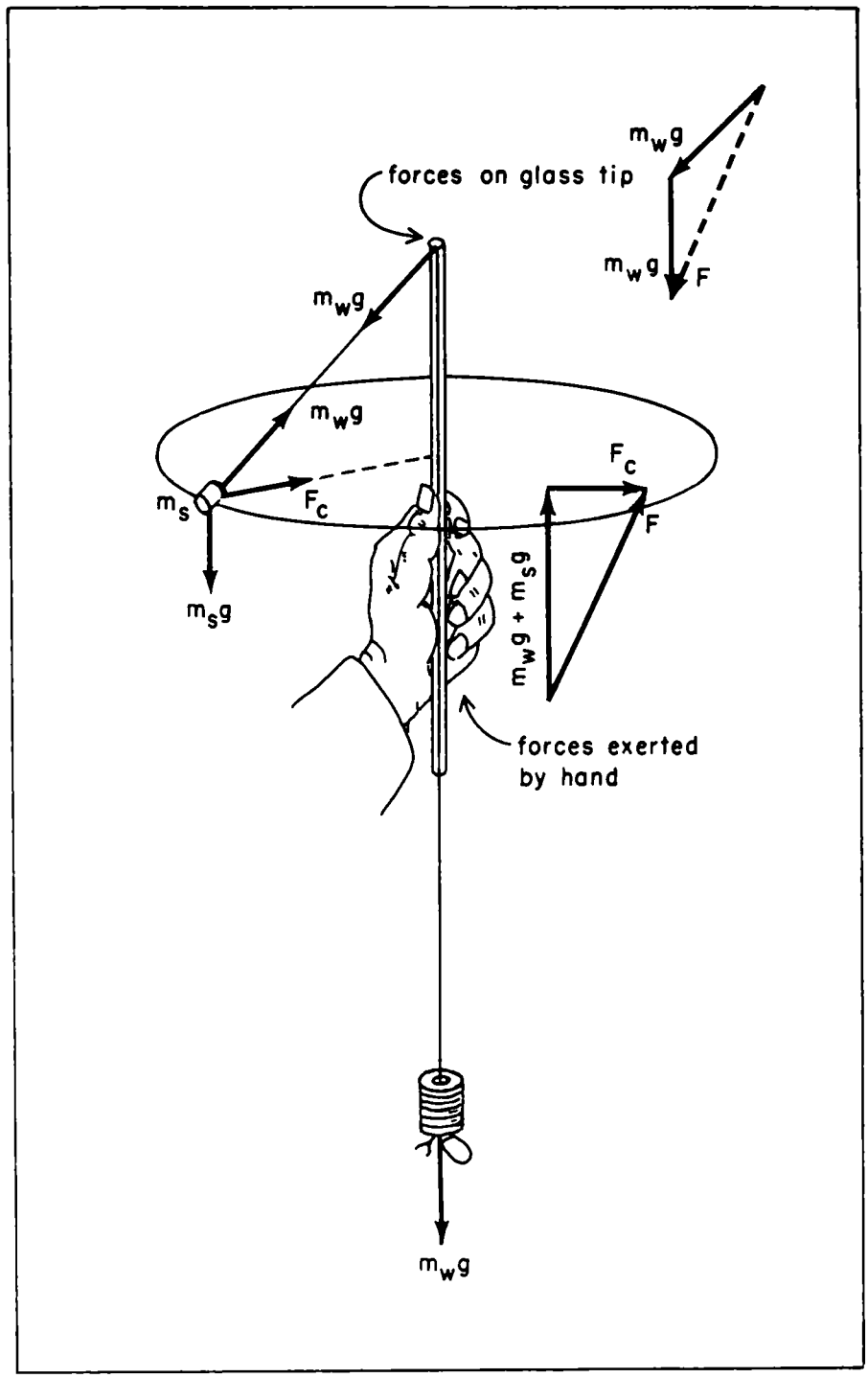

forces on glass tip
$m_w g$
$m_w g$
F
$m_w g$
$m_w g$
F_c
m_s
F_c
F
$m_s g$
$m_w g + m_s g$
forces exerted by hand
$m_w g$

Figure (e)

If a new spring is being used and the coils touch, a mass which just pulls the coils apart should be suspended on the spring when k is being determined; this mass does not have to be accounted for. If it is not used, the first extension will be smaller than it should be.

It is recommended that students time 20 vibrations when they are measuring the period and that they take the average from several trials. By this time students should do this automatically, but might need reminding on an individual basis.

A 5-inch C clamp may be used for the unknown mass. It can be suspended from the spring by a piece of string.

Students are not required to determine the spring constant of the second spring by making a graph of the extension of the spring as a function of the stretching force. However, they may use this method as a check if they want to. This is also true for the two springs hung together.

In order to predict the period of a known mass when the two springs are hung together, students will have to decide what the spring constant of the combined springs will be. They may determine it by making a graph or through reasoning since they know k for each spring. Whatever their method, this is a good place to let them decide what to do. They can then actually measure the period to see if their spring constant is correct.

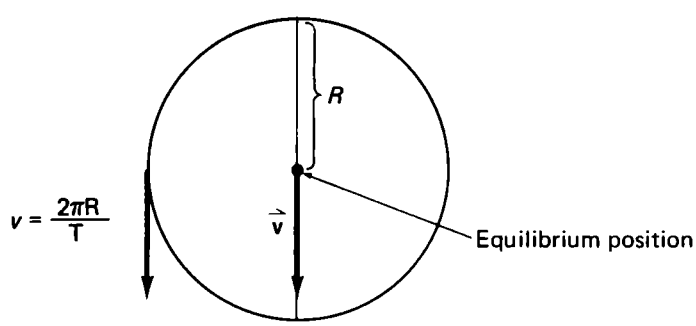
The maximum velocity is $\frac{2\pi R}{T}$ where R is the distance the mass is displaced from its equilibrium position. Using a reference circle, the maximum velocity will occur at the center of the motion when there is no component of the velocity perpendicular to the motion.

The tape timer should be mounted below the vibrating mass so that it will pull the tape up through the timer when it is released.

The tape timer may be calibrated by pulling the tape through for a known amount of time and then counting the number of marks on the tape. A stroboscope may also be used if it is available. It will be necessary for each student to calibrate his own timer because its speed depends on the battery used.

Answers to Questions

The period is not affected by the amplitude of the vibration; it is only dependent upon the suspended mass and the spring constant of the spring.



The inertial mass of the C clamp is found by measuring its period of vibration when suspended from a spring.

It has been assumed that the mass of the spring does not affect the period of vibration.

Sample Data

Spring constant =nt/m
Period of 500 gm mass
  calculated 0.80 sec
  measured 0.85 sec
Determining mass of an object (5-inch C clamp)
  Measured from T m = .79 kg
  Weighed m = .74 kg
Force constant of second spring from mass and period
  K = .34 nt/m
Determining T for two springs connected in series; K = 17 nt/m
  Prediction 1.08 sec
  Measured 1.16 sec

Apparatus

1 Helical Spring Kit
1 5-inch C clamp
1 Spring (if second spring is not included in Helical Spring Kit)
Stop watch
Meter stick
Support stand for suspending springs
Graph paper
1 Tape timer
1 $1\frac{1}{2}$-volt dry cell
Tape for timer

Films

"Deflecting Forces," by N. H. Frank of the Massachusetts Institute of Technology, shows how a force acting at a right angle to the direction of motion changes the direction of motion but not the speed. A constant deflecting force (perpendicular to the direction of motion) results in circular motion with uniform speed. Using vectors, $F = \frac{mv^2}{R}$ is derived from $F = ma$. The same magnitude of force is used to produce accelerations in both circular and straight-line motion. The equality of the magnitude of the accelerations shows the validity of Newton's law in vector form. This film is related to Section 5. Running time: 29 minutes.

"Periodic Motion," by Donald Ivey and Patterson Hume of the University of Toronto. An object in simple harmonic motion is used to trace a displacement vs. time graph. Analysis of this graph shows that $\vec{\mathbf{a}} \propto -\vec{\mathbf{x}}$; from Newton's law it follows that $\vec{\mathbf{F}} \propto -\vec{\mathbf{kx}}$. This is verified by an experiment. The relationship between simple harmonic motion and the vertical component of uniform circular motion is observed and analyzed, leading to the conclusion that the

period for simple harmonic motion is given by $T = 2\pi\sqrt{m/k}$, where the motion results from a force whose force constant is k. This relationship is then tested and confirmed by experiment. This film is for use with Section 8. Running time: 30 minutes.

"Frames of Reference," by Donald Ivey and Patterson Hume of the University of Toronto, demonstrates how different frames of reference lead to different descriptions of motion. The motion of a ball which falls from a moving cart, when the cart moves first with constant speed then under constant acceleration, is observed both from a reference frame which moves with the cart and from a fixed frame. These observations are used to distinguish between inertial and noninertial frames of reference, and to show how "fictitious" forces arise in a noninertial frame. Experiments with a Dry-Ice puck on a rotating turntable show the fictitious nature of centrifugal force in a rotating frame of reference. The film closes with a brief study of the evidence provided by the Foucalt pendulum for the rotation of the earth in a Newtonian frame of reference. This film can be used to good advantage with Sections 9, 10, and 11. Running time: 26 minutes.

Demonstrations

1. First take a book and a sheet of paper (a little smaller in area than the book); hold them side by side and drop them simultaneously. The force of air resistance is comparable to the gravitational force on the paper, while it is much smaller than the larger gravitational force on the book. Hence, the paper soon has a smaller net force acting on it and falls more slowly. (The paper quickly reaches its terminal velocity whereas the book continues to accelerate.)

Ask for suggestions for explaining the lack of constancy of the acceleration and for making the paper fall faster. Two common suggestions involve dropping the paper either after holding it edge down or after having crumpled it. Try these.

If no one suggests it, ask about holding the paper above or below the book. In either case they will fall together. Putting the book above the paper is not convincing because the book obviously pushes the paper down. But when the paper is placed on the book (without protruding beyond the edges), the book merely shields the paper from the resistive air force. The two will drop together with the paper remaining on top of the book throughout the fall.

2. It will be helpful to use one or more demonstrations of the fact that two identical objects will fall together independent of their horizontal motion. Begin by asking students what they expect. Many students expect intuitively that the object with horizontal velocity takes longer to fall because "it has farther to go." The equal fall times can be demonstrated easily and effectively; such demonstrations can be used to introduce a detailed development of the analytic consequence of the vector nature of Newton's law of motion. Start by using common objects rather than a piece of "demonstration equipment." Coins can be used. More elaborate apparatus demonstrating independence of fall time can be used after you have shown the simpler example.

3. You can show that the projection of circular motion is the same as simple harmonic motion. Mounting a turntable in the vertical rather than the horizontal plane enables a comparison of the projection of circular motion with a mass oscillating vertically on a spring. If you have a relatively frictionless

setup, you might be able to use a mass oscillating horizonally. This is comparable to the demonstrations in the film "Periodic Motion."

If you do not have a standard demonstration of this type, you may want to use the type of apparatus described below.

Simple harmonic motion can be demonstrated by observing the shadow of an object moving in a circle at constant speed. The shadow will move with simple harmonic motion back and forth on a screen. A comparison of the motion of a pendulum with the motion of the shadow of the object moving in a circle shows that a pendulum with small amplitudes also performs simple harmonic motion. The apparatus in Figure (a) can be used to demonstrate this.

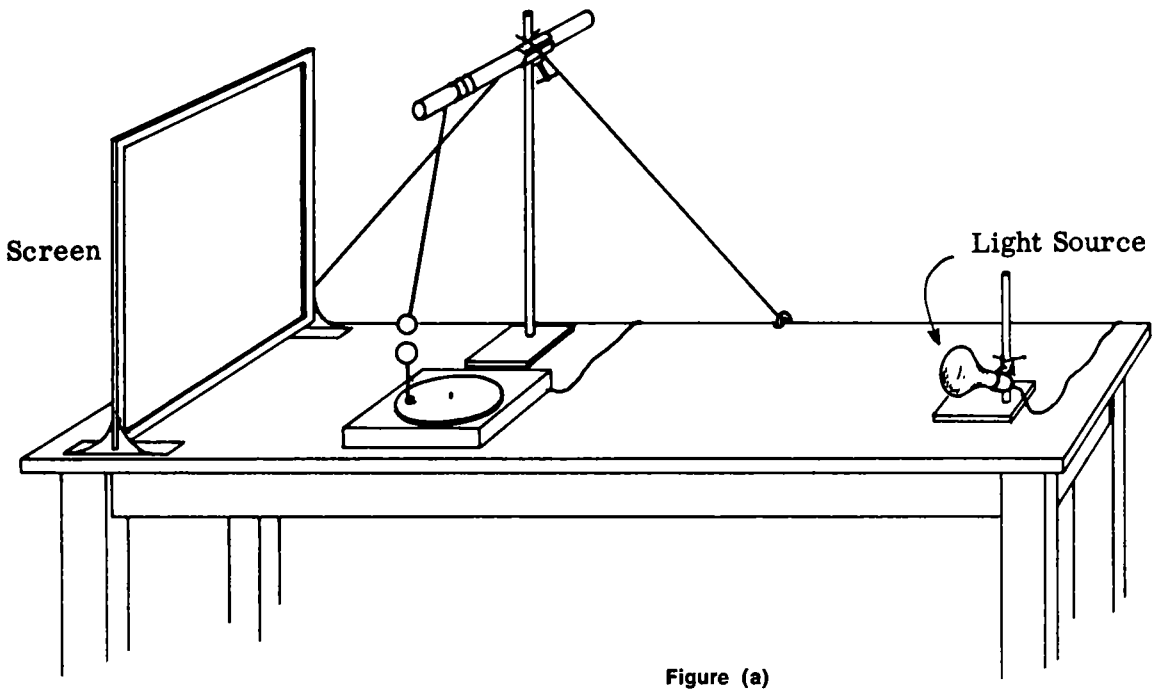


Figure (a)

A Ping Pong ball is glued to one end of a thin stick while the other end of the stick is taped to a magnet which adheres to a phonograph turntable (Figure b). From a tall, sturdy support, suspend a simple pendulum with its length adjusted so that it will swing with the same frequency as the rotation of the turntable. The length of the pendulum should be about 80 cm for 33⅓ r.p.m.'s. This turntable frequency is best since a higher frequency necessitates a short pendulum which swings through too large an angle. Hang the bob above the turntable so that the bob is slightly higher than the Ping Pong ball (as shown in Figure b).

A dowel or rod clamped to a ringstand provides the pivot for the pendulum. This pivot can be rotated (Figure c) to adjust the length of the pendulum until the period is the same as that of the turntable.

The light source should be as far from the pendulum and turntable as possible to reduce the effects of parallax due to the changing distance between the light and the rotating ball.

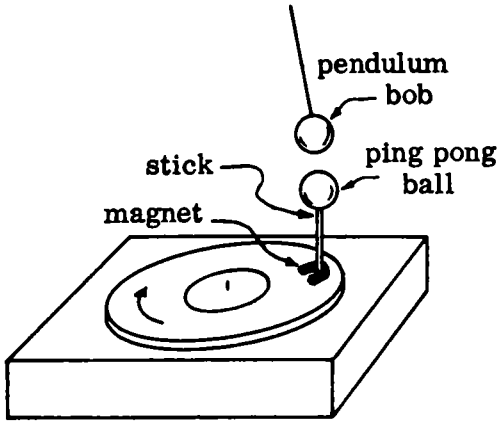


Figure (b)

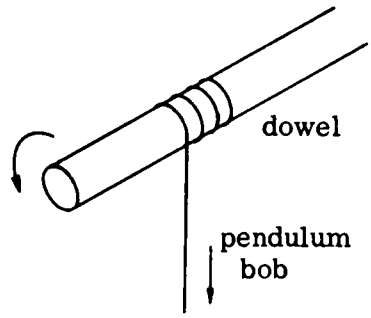
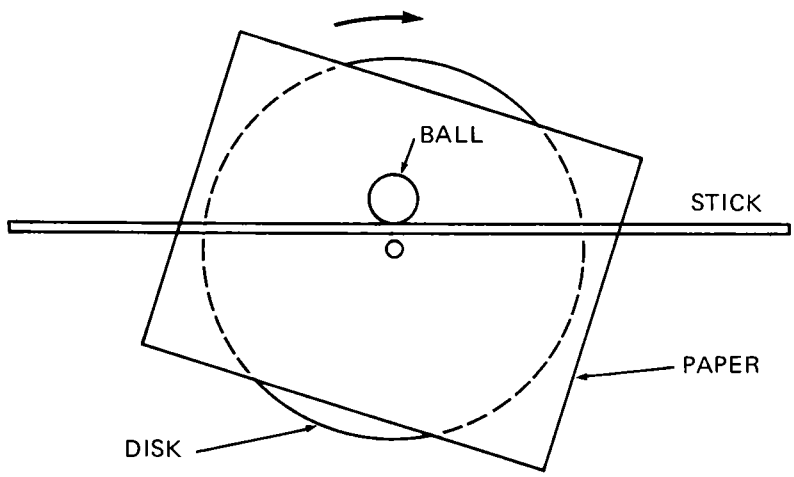


Figure (c)

Initially, the amplitude of the pendulum should be greater than the radius of the turntable so the amplitudes will be the same after the bob and the ball have been synchronized.

Release the bob at an instant so that its moving shadow lags a little behind the shadow of the ball. The motions may then be synchronized by holding a finger against the rim of the turntable, slowing it down until the two shadows coincide.

4. The paths of the relative motion in a fixed and in a rotating frame of reference can be demonstrated using a horizontal turntable or disc (such as the PSSC angular momentum apparatus) which rotates freely about a vertical axis, and a steel ball about an inch in diameter.



The disc should be smooth and level. Place a piece of paper on the disc as shown in the diagram and cover with a piece of carbon paper (carbon down). Fasten the papers to the disc. The stick shown in the diagram is used to keep the ball moving in a straight line in our frame of reference.

With the disc spinning at about 2 revolutions per second, drop the ball in the position shown from a height of about 5 mm. The ball will roll along the stick in the direction of rotation of the disc. After the ball has rolled to the edge of the disc, stop the disc and position it as it was when the ball first hit it. Draw a line from the point of contact of the ball parallel to the stick. This is the path of the ball in the fixed frame of reference.

Figure (a) shows the central portion of these paths.

The curve in (a) will be similar to an involute. To produce an involute, however, the ball would have to move from the point of contact along the straight line with a constant velocity, and the disc would have to revolve with a constant angular speed. In our case, the disc drives the ball and is thus slowed. Since the ball was given no horizontal velocity when we dropped it on, it must have accelerated along the stick, at least in the beginning, to get off the disc.

Repeat the demonstration with the stick removed. Watch the motion of the ball in the fixed frame. With the disc held in the position where it was when the ball first hit, draw the approximate path you observed in your fixed frame. Figure (b) shows the central portion of these paths.

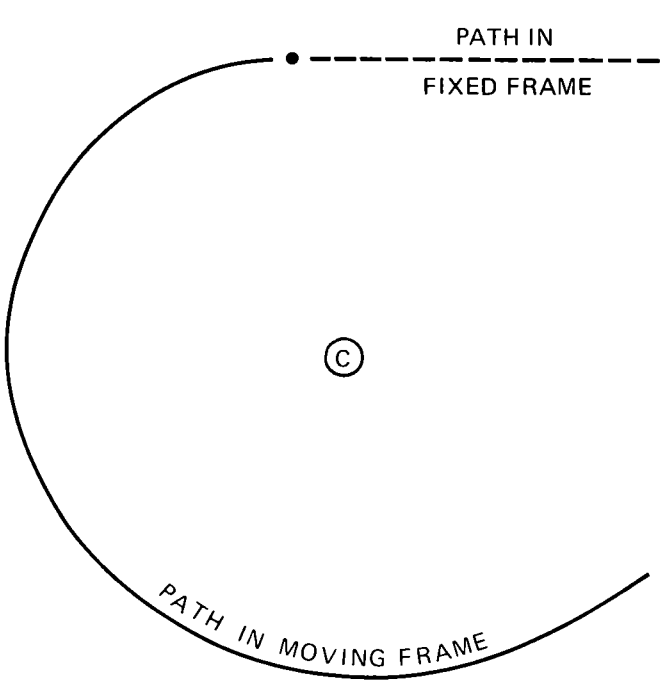


Figure (a)

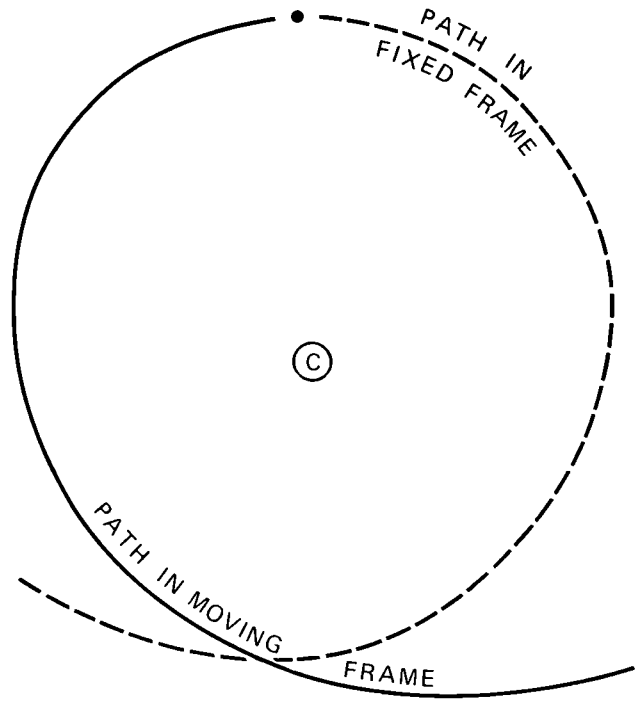


Figure (b)

Questions

Give the class some simple practice problems in projectile motion which are designed to have easy solutions if the components of the motion are separated. A suggested sequence of problem types is given below. In each case an object leaves a table with a horizontal velocity, v_{0x}, at $t = 0$. In early problems the initial vertical velocity should be taken as zero.

Making the numbers simple so that the arithmetic is easy will help students concentrate on the nature of the motion. (Using $y = -5t^2$ is most helpful for this simplification.)

1. The very simplest problems are one-step problems involving the elapsed time, t.
   (a) Give two of the quantities in $x = v_{0x}t$ and ask for the third.
   (b) Give one of the unknowns in $y = -4.9t^2$ and ask for the other.
2. Give two of the three quantities v_{0x}, x, and y, and ask for the third.
   (a) Begin by asking first for t, and the other unknown next. For example:
   If $x = 20$ m, and $v_{0x} = 2$ m/sec, what is t? What is y?
   If $y = -4.9$ m and $x = 10$ m, what is t? What is v_{0x}?
   If $y = -4.9$ m and $v_{0x} = 3$ m/sec, what is t? What is x?
   (b) Later you can give two of the three quantities, v_{0x}, x, and y, and ask for the third without explicitly asking for t. However, students should get into the habit of finding t as an intermediate step. If y is given, t can be found immediately. If x and v_{0x} are given, t can be found.
3. You may want to extend these problems to include an initial vertical velocity.

In choosing problems it should be remembered that the most important aspect of projectile motion is the independence of the components. The best way to learn this independence is to use it often in solving problems. This independence deserves much more attention than does the trajectory of the motion.

CHAPTER
13 Universal Gravitation and the Solar System

You have two important tasks to perform in this chapter. One is to make sure that your students can use Kepler's three laws and Newton's law of universal gravitation as they apply to the planets and to their natural and artificial satellites. The other is to help them appreciate the feats of scientific imagination which gave us our modern view of the solar system and its motions.

You may want to point out at the end of the chapter that Kepler's law of equal areas requires that the force which holds the planet in its orbit be directed toward the sun. With more powerful mathematics, the elliptical shape of the orbit about one focus requires that the force have an inverse-square dependence on distance from the sun. Taking the inverse-square force as given, Kepler's third law may then be interpreted to mean that the ratio of inertial mass to gravitational mass is the same through the whole range of masses of the solar system.

If you are pressed for time, Chapter 13 can be handled largely as a reading assignment with a bare minimum of class discussion and problem solving, without loss in continuity in progressing through the volume.

However, an overly rapid survey of this chapter will miss some of the most exciting parts of physics. A study of science cannot be divorced completely from a study of the history of science, nor can an appreciation of the contribution of great scientific figures be achieved without a knowledge of the facts of science. The professional scientist's enthusiasm for his work can be traced in part to his appreciation for the fact that he is a member of a team—a team that began many centuries ago and which will continue indefinitely. Just as the historical figures were able to contribute to the understanding of problems that disturbed them, so does the modern scientist, drawing on their successes and failures, hope to contribute to the clarification of problems of contemporary interest. Modern physics derives from the work of men like Galileo and Newton. Mechanics, viewed from the scientific environment of its great innovators, can be vital and exciting. Hopefully, it is this flavor which your students can derive from the present chapter. If your students can find the time for it, a little outside reading at this point can contribute hugely to their perception not only of scientific principles, but of the human nature of science, and the simply tremendous intellectual accomplishments of men like Newton, Kepler, and Galileo. Further, planetary physics is not simply a matter of historical interest. A realistic perception of the problems of space travel is possible only through a grasp of the dynamics involved. In this course, the ideas return in the Rutherford model of the atom.

Students should not feel that gravitation is a completely settled issue. There are a number of unresolved basic questions about gravity, even years after Einstein's theory of general relativity:

(1) Why does the gravitational field follow an inverse square law?

(2) What is the speed of propagation of the gravitational field?

(3) Does gravity theory hold for very large and very small distances?

(4) Does the law of gravitation hold everywhere in the universe?

(5) Is the gravitational "constant," G, really constant, or does it change slowly with time over billions of years?

(6) Is the gravitational force between two bodies independent of the distribution of matter in the rest of the universe?

We have no answers to these "simple" questions, or for many others of great fundamental interest. If gravity is a field of investigation which is currently somewhat static, it is not through exhaustion of problems, but of physicists. It is extremely difficult to conceive and execute experiments to test details of gravity theory on a terrestrial scale, and neither technology nor the economy is yet ready to support an intergalactic astrophysical year!

Films

"Universal Gravitation" by J. N. P. Hume and D. G. Ivey of the University of Toronto reconstructs the logic behind the law of universal gravitation through imagining how the law might have been discovered on "Planet X" in another solar system which has only the one planet. The film points out how the frame of reference adopted affects a kinematic description. Observation of the orbits of artificial satellites enables determination of the inverse square law of force, the constancy of R^3/T^2 and eventually leads to a proper kinematic description of Planet X's solar system, and to the law of universal gravitation. This film is closely related to Sections 6, 7, and 8.

"Elliptic Orbits" by A. V. Baez. Beginning with an elliptic orbit and Kepler's law of areas, gravitational force is demonstrated to obey an inverse square relation. The derivation is almost geometric. This film can be used in connection with discussion of Sections 7 and 8. Running time: 18 minutes.

Questions

1. An earth satellite, S_1, moves in a circular orbit with radius four times the earth's radius. Compare its period of revolution with that of a second satellite, S_2, which is very close to the earth's surface. What is the ratio of their speeds? $\left(\frac{R^3}{T^2} = K.\right.$ Therefore, $\frac{R_2^3}{T_2^2} = \frac{(4R_2)^3}{T_1^2}$, and $\frac{T_1^2}{T_2^2} = \frac{64R_2^3}{R_2^3}$. Compared to satellite S_2, the satellite S_1, which orbits at $4R$, will take 8 times as long to complete one circuit around the earth. The ratio of speeds is given by:

$$\left.\frac{v_2}{v_1} = \frac{2\pi R_2}{T_2} \times \frac{8T_2}{2\pi 4R_2} = 2. \quad \text{Hence, } v_2 = 2v_1.\right)$$

2. Can two different frames of reference give equally valid kinematic descriptions of the motion of an object? (Yes; but one may be much more complicated than the other.)
3. How does Newton's law influence the choice of a frame of reference? (The kinematical description should lead to the simplest and easiest force analysis.)

4. Describe the motion of an artillery shell fired horizontally from the viewpoint of
   (a) an observer moving along in a jet plane which has a horizontal velocity equal to the horizontal component of velocity of the shell;
   (b) an observer in free fall (no air resistance).

(a. The shell drops straight down with acceleration = g. b. The shell appears to be going away in straight line horizontal travel with constant speed.)

Addenda

General Relativity According to the basic ideas of the dynamics of Galileo and Newton, the laws of motion are valid only in an "inertial frame," that is, a system which is at rest or moving with constant velocity. These two possible states are indistinguishable, because forces are detected only when a body is accelerated. Newtonian dynamics begins to break down for bodies moving with speeds near that of light, but Einstein's theory of special relativity has satisfactorily extended the basic notions of classical dynamics to the range of all possible speeds.

The existence of gravity, however, complicates matters considerably. We know that the surface of the earth is not an inertial frame in the Newtonian sense. If we project a body upward, it does not continue to move up but reverses direction and falls back to the floor. We generally describe the situation by saying that we are in an inertial frame, but that there is a force—the "force of gravity"—also acting on the body. Indeed, it is impossible to isolate any body from the gravitational forces exerted by the mass of the rest of the universe. Thus, the definition of an inertial frame in terms of a state of "rest" or "uniform motion" is only an approximation in a universe which contains mass, since all bodies in the universe interact with one another to a certain extent.

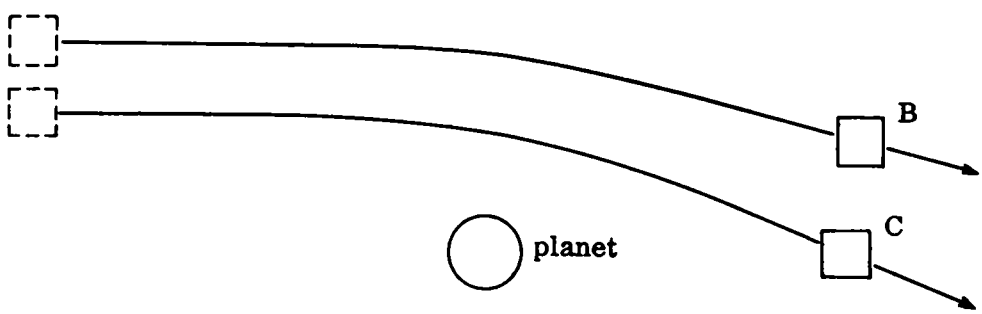
Einstein, in his theory of general relativity, attempts to restate the laws of physics in a manner which takes into account the equivalence of gravitation and acceleration. This theory is, as is well known, of enormous mathematical complexity, and it is hardly to be recommended as an exercise for high school students. However, the basic concepts of the theory are simple enough to be illustrated by a few examples.

Suppose we were drifting along in space in a rocket ship in the "weightless" condition which every modern student understands. In this state we could perform dynamical experiments on ourselves and the surrounding objects floating in our room and quickly decide that we were in a true inertial frame, since the laws of motion would be obeyed exactly. If we jumped up from the floor, we would most certainly hit the ceiling, no matter how gently we jumped. Now imagine, during the course of these experiments, that this "weightless" interlude suddenly comes to an end. We would find ourselves standing on the floor, feeling our own weight. All of the surrounding objects which a moment earlier had been floating around the room would suddenly fall to the floor. What could we conclude had happened?

Without looking out the window of the rocket ship (this, incidentally, wouldn't really help), we could conclude that one of two things must have happened. Either our rocket ship had suddenly accelerated upward or we have just *come to rest* (with the floor "down") on the surface of some planet. In either event, it would be obvious that the laws of physics had suddenly changed and we were no longer in an "inertial frame." We would be hard put to devise

an experiment to tell whether we had in fact been accelerated or had entered a gravitational field, since either occurrence would have had the same effect on our observations. This is precisely what is meant when we say that the "inertial mass" of a body and its "gravitational mass" are inextricably interrelated. It is this "principle of equivalence" which lies at the heart of general relativity—that is, that the effects of an acceleration *are in principle* indistinguishable from the effects of a gravitational field. But what about our inertial frame? We had a perfectly good way of measuring this earlier, simply by observing the validity of the laws of motion. Evidently, during this time interval, when our rocket ship was an "inertial frame," we must have been in uniform motion in the absence of a gravitational field (impossible in our universe), or *in "free fall" in a gravitational field.* Thus, the true inertial frame is not one which is in uniform motion in our space, but rather one which is in free fall.

General relativity, then, is simply the description of nature as viewed by an observer in a true inertial frame—patiently doing experiments in a freely falling box. To this observer, the laws of dynamics would be exact within his box, but he would quickly conclude that the space outside his box had some very peculiar properties. To illustrate, imagine that we are in free fall in such a box, moving at a fairly high speed through space. Through our window we observe another person moving along parallel to us at the same speed and also doing dynamical experiments. We could compare results by flashing messages back and forth and agree that our results were identical and both of us were in inertial frames. If we then arrived in the vicinity of a large planet (in such a manner that we didn't conclude the experiment by hitting the planet), both boxes would be deflected by the gravitational field, as shown above.



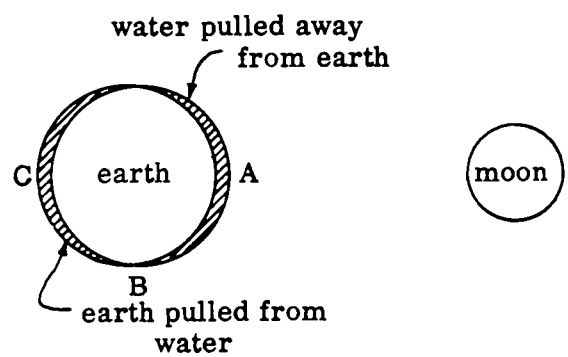
Our box, labeled *B* in the figure, would be deflected somewhat, and the other box, labeled *C*, would be deflected rather more since it is closer to the planet. Since both we and the observer in *C* are in free fall, within the confines of our own boxes neither of us would feel or be able directly to measure any effects due to the acceleration. We are both still in inertial frames as far as our internal measurements can tell us. However, since we are now obviously drifting apart, we would have to conclude that the distance between us suddenly started to increase without either of us apparently having accelerated. Instead of describing what has happened in terms of some "force" which we could not detect, we might rather describe the occurrence in terms of a distortion in space and time which we could observe.

This is the essence of the theory of general relativity. The gravitational field, according to this viewpoint, causes a "warping" of space and time in the vicinity of a mass. Gravity then enters not as a "force" but as a property of space and time.

This whole procedure would be rather fruitless if it did not permit the extension of the laws of physics and the prediction of certain observable events. In fact, general relativity predicts certain facts which are not correctly explained either by Newtonian dynamics or special relativity. One of these involves the curvature of light in a gravitational field. In general relativity, the "shortest distance between two points" is not a straight line but is a "geodesic," the path which light would follow in traveling between the two points, bent by the gravitational field. According to special relativity, light has energy, and since mass and energy are equivalent, light will be deflected by a gravitational field. However, the amount of deflection is not given correctly by this simple argument because the force of gravity on a moving object depends on its speed. We can actually observe the bending of light which travels from distant stars and passes very close to the sun during a solar eclipse. The effect is very small, about two seconds of arc, but it is accurately measurable, and agrees precisely with the predictions of general relativity.

The theory of general relativity is of considerable importance in physics, but is more the working tool of the cosmologist in his study of the universe. The cosmologist is concerned with such questions as "Is the universe finite?" —that is, would a geodesic directed outward from the known universe go on forever; or traveling always out, return to its starting point? Much of the effort of modern astronomers is directed toward answering this question. At present, we do not know whether the space we know is finite or infinite, but it is certainly "warped," in the relativistic sense.

Tides Students living in the middle of the continent may not be familiar with tides. Although the details of tidal motion are complicated by local variations in the shore line and ocean floor, the principal motions are caused by gravitational attraction of the moon for the earth. This phenomenon is illustrated below. (Scale enormously exaggerated.)



Now that students know that the force between the moon and earth results in an acceleration, they may be able to better understand the tides. Essentially, the moon and earth freely fall (accelerate) toward one another. The force per unit mass is the parameter which establishes the acceleration of a particle. Since the gravitational force field of the moon varies as the inverse square of the distance from the moon's center, the acceleration toward the moon of a particle at A tends to be greater than that of a particle at B, which in turn tends

to be greater than that of a particle at C. If the earth were rigid, no tidal motion could result. The external force differences would be compensated by internal forces. When water is the substance at A or at C, since it is free to flow, changes in level will occur. Thus the moon's attraction produces two bulges in the ocean on opposite sides of the earth. Since the earth rotates once in 24 hours, there are two high tides per day at any ocean location, although they may not be of exactly equal height. While the influence of the sun must also be considered in calculating the exact size of the tides, its effect is considerably smaller than that of the moon.

The force exerted by the sun upon a unit mass anywhere on the earth is about 100 times greater than the corresponding force exerted by the moon. Nevertheless, the tidal effect of the moon is about twice that of the sun. This is because the difference between the force of the moon on a unit mass on the earth's surface at A and on another at D, is more than twice as great as the difference in the sun's force at these points (this is because the fractional change in the inverse square of the distance from the moon to these points is almost 400 times greater than in the fractional change in the inverse square of the distance from the sun to these points.

The highest tides occur about twice a month approximately with the new moon and full moon, when the sun, earth, and moon are all in positions along the same line. Tides may vary considerably in magnitude from place to place along a coast because of details of the local topography. The largest tides are most frequently seen in partially enclosed bays. In the Bay of Fundy, for instance, the difference between high and low tide is over 50 feet.

Fluctuations in Gravitational Force To first approximation, the acceleration of gravity is constant over the surface of the earth. To second approximation, it varies from point to point on the surface because of variations in height of the terrain and the fact that the earth is not exactly spherical. To third approximation, the acceleration of gravity varies with the distribution of masses in the earth's crust at the particular location. This fact is of great importance to geologists, since hidden features of the earth's crust may be detected by accurate measurement of local variations in g. If g is measured above a large cavern which may be filled with oil, it will have a smaller value than if measured over a region of solid rock, and a still smaller value than if measured over a large deposit of lead or gold. The value of g also depends on the rotation of the earth, as discussed in Problem 23.

Composition of the Earth Cavendish's experiment enabled him to find the *mean* density of the earth, and determined that this was about $5\frac{1}{2}$ times that of water. Since the surface rocks have a density of only 2 to 3 times that of water, the interior of the earth must obviously be made of a material of higher density than the surface crust. The molten basalt, or lava, which underlies much of the earth's crust is itself far too light to account for the measured density of the earth. A good deal of the earth's mass apparently resides in a relatively small heavy "core" at the center of the earth, of density about ten times that of water. This core is presumably composed of metals like iron and nickel rather than rocks. The core, as determined by seismic measurement, is essentially spherical in shape and about 2000 miles in radius. The core itself is apparently not uniform in density but shows a distinct rise in density, increasing as the center of the earth is approached, starting at about 1000 miles from the center. At the center of the earth the density is about 16 times that of water.

CHAPTERS

Laws of Conservation 14-17

Chapters 14–17 develop two important conservation laws of mechanics, conservation of momentum and conservation of energy.

Chapter 14 establishes the conservation of momentum from experiment, thus developing it as a new physical law, not as a deduction from Newton's "second" and "third" laws as is sometimes done. In fact, at the stage where conservation of momentum is treated, Newton's third law has not yet been introduced. It is important that your treatment be consistent with this point of view. A mixture of the two possible approaches is almost sure to cause confusion for the students. The validity of conservation of momentum, like all other physical laws previously established, is based on the synthesis of a number of experimental observations. There are good reasons for this approach. To get direct evidence for Newton's third law requires the measurement of velocities as a function of time during a collision process. This is possible only for slow collisions of macroscopic bodies. It is impossible on the atomic scale. Furthermore, not all forces of interaction obey Newton's third law; the best-known example is the magnetic interaction of moving electric charges. However, momentum is conserved in all known interactions, and it takes only measurements before and after the interaction to demonstrate it.

Chapters 15–17 provide a broad treatment of the concept of energy. These chapters depart rather sharply from the standard development that has been used in many introductory courses. The text makes no attempt to give a specific, short definition of energy. There is none! Rather than beginning with a definition, a better eventual comprehension and appreciation of the significance of the idea of energy can be achieved through building an understanding of how the concept of energy can be applied to many phenomena.

CHAPTER

14 Momentum and the Conservation of Momentum

Beyond the recognition that momentum is $m\vec{\mathbf{v}}$, students should see that any two *objects which have the same momentum,* no matter how it was acquired, *have a common property.* Their common property is that both could be stopped by the same impulse, $\vec{\mathbf{F}}\,\Delta t$. Students should see that it is the product $m \times \vec{\mathbf{v}}$ that is important. Any given momentum can be produced by an infinite number of combinations of masses and velocities. *All* the combinations resulting in the same momentum have a common property. They can all be stopped by the same impulse, $\vec{\mathbf{F}}\,\Delta t$.

Although two objects might have equal velocities, there is no simple relation between their velocities and the impulses needed to stop them; the impulse would depend on their masses.

Some students will accept the vector nature of impulse without going through the detailed development given in the text. The best way to present this to your class depends upon how much time you devoted to vectors in Chapter 10. For example, if your students know that the product of a scalar and a vector is another vector, the fact that $\vec{\mathbf{F}}\Delta t$ is a vector is obvious. Force always has direction. Including the time the force acts does not change its directional nature. (Incidentally, the equation $\vec{\mathbf{F}}\,\Delta t = m\,\Delta\vec{\mathbf{v}}$ would be meaningless if $\vec{\mathbf{F}}\,\Delta t$ were not a vector and $m\,\Delta\vec{\mathbf{v}}$ were a vector. If we did not specify the direction of the force, we would not know the direction of the resulting change in velocity.

Bring in Experiment 26 Momentum Changes in an Explosion as early as possible, since it provides experimental evidence on which the conservation of momentum is based. It is most appropriately done as soon as impulse and momentum have been studied in the text, and preferably before taking up momentum changes when two bodies interact (Section 14–3).

Since Experiment 27 The Cart and the Brick comes after the student has discovered conservation of momentum in the explosion experiment, it is not important whether it is done before or after momentum conservation is taken up in the text. A more complex experiment than 26, it gives the student a deeper insight into momentum conservation. Aside from the practical utility of momentum conservation as a tool in solving specific problems of mechanics, this idea takes its place among the conservation laws as one more entity that remains constant despite the whirling and dancing of submicroscopic particles and the often complicated motions and interactions of bulk matter—one of the fixed intellectual vantage points from which man organizes his attack on the problems of extending his control of the physical world.

There are two fundamental bases of student understanding of momentum conservation. The most obvious and most important is the idea that when only two objects interact, the total momentum of the two does not change. This is extremely general and does not depend on the interaction; the interaction can be simple or complex, it can be caused by chemical explosion, elastic forces, muscular pushes, etc. As long as only two objects are involved, momentum is conserved. This aspect has been treated up to this point. The second aspect is more subtle. It involves the question of when two objects are indeed isolated. You will have to spend some time discussing this or your students may think that momentum conservation applies only in the physics course and not in nature.

Before going into a detailed discussion of isolated systems, let us indicate the source of student confusion. When students are given an artificial problem involving, say, the collision of two balls, they may apply momentum conservation to get the answer they are after. But, they know that a single ball will not maintain its velocity; it will slow down. Hence, instead of being a universal, powerful law, momentum conservation may seem to be an expedient to be used to get the correct answer for a particular class of problems. Considerable class discussion will be needed to relate momentum conservation to nature as the students see it day after day.

Comments on Isolation

In dealing with momentum problems, the isolation of a system (what to include in the system, and what to exclude) is not always simple. The only case in which near-perfect isolation occurs is in an interaction somewhere in outer space. Such cases are somewhat artificial, but easy to understand.

A second kind of isolation is illustrated by smooth objects on a frozen lake (assuming the ice is frictionless), or billiard balls on a table (again neglecting friction). Here external forces are not small. Gravity pulls down, while the ice or the table pushes up. However, because the surface is rigid, the forces *automatically adjust* themselves to be equal and opposite, and the *net* force up or down on the object is zero. In such cases, then, there is a kind of pseudo-isolation with large external forces acting, but adding up to zero.

A third and very useful case (and one which students should understand in order to appreciate the broad utility of momentum conservation) involves isolating an interaction in *time*—applying momentum conservation during the time interval that just covers the interaction. Consider the question of changes of momentum between two billiard balls colliding on a billiard table. No one doubts that a billiard ball set rolling on the table will eventually come to rest because of friction. Hence, two billiard balls on a table do not form a really isolated system. But consider just the instant of collision. The table, an object external to the system, exerts frictional forces on the balls during the collision. These forces may not be small. The important thing is that the impulse imparted by these forces during the brief instant of the collision is negligibly small *compared* to the impulses of the forces of interaction between the balls during the collision. Therefore the momentum of the two-ball system may be said to be conserved *during the collision* even though it is not conserved if longer times are considered. If we need to consider frictional effects we can apply momentum conservation to the interaction, then consider the effect of friction on these results.

Another illustration of this case is an exploding rocket. Here, gravity acts on the rocket as an external force. However, the explosion takes place in such a short time that the *impulse* imparted by gravity to the rocket during the explosion is negligible and momentum conservation within the rocket system may be used to describe the situation immediately before and after the explosion. We can then, if necessary, consider the effect of gravity on a given mass with a given velocity. Also, the gravitational case has a special property. In an interaction in a constant gravitational field, the accelerations of all bodies are equal so the gravitational force *can* be neglected.

The objective of Experiment 28 A Collision in Two Dimensions is to impress upon the student that momentum is a vector quantity and that it is the *vector* sum of the momenta, not the arithmetical sum, that is conserved. Only by analyzing a two-dimensional collision can this be made really clear. This experiment is important and should not be omitted. It is best done after Section 14–4.

Section 14–6 may be treated briefly or even omitted without seriously affecting the continuity of the course. This does not mean that the idea of center of mass is of secondary importance. On the contrary, it is basic to the understanding of all mechanics beyond the most elementary. If you have plenty of time, you should surely include this section. It may be abbreviated only because most students do not think seriously about the motion of extended bodies but in some way assume that the "average" motion is given by Newton's law, an intuitive feeling which happens to be correct.

It is not necessary to analyze the supernova example in any detail. If you attempt to be precise, you may find some students asking questions which are almost semantic. You can probably never satisfactorily answer any questions since the students simply will not know enough electromagnetic theory (nor can you promise them they will learn enough in later chapters). The whole situation straightens itself out, as mentioned in the text, when the momentum of the light is taken into account. A light beam of energy E, carries a momentum E/c, where c is the speed of light. Some of the students may ask whether light, if it has momentum, has mass. The answer is yes; it has a mass given by $E = mc^2$.

Figures 14–9, 14–11, and 14–20 show billiard balls colliding. Most students think of these balls as rolling on a table. Actually they were suspended on long threads. That is why spots and stripes appear to stay fixed or rotate very slowly.

EXPERIMENT

26 Momentum Changes in an Explosion

The spring exploder can be dangerous if it is released indiscriminately. It should not be cocked without the spring safely anchored to the cart. You may want to caution your students not to release the spring when the cart is pointed in the direction of others.

Springs may differ in their strength. A weak spring acting on a heavy cart may not give it enough velocity to reduce the effect of slight variations in the table top.

The optimum distance between the two bumpers is about 1.5 m. If the distance the cart must travel is too great, the cart may, because of friction, come to a stop before reaching the bumper. On the other hand, if the distance traveled is too short and the velocities are reasonably high, the coincident timing method will not be accurate.

Both carts should be loaded with at least one brick, since some of the force of the exploder applies torque to the wheels, making them rotate. This appears as a loss of linear momentum. As long as the mass of the wheels is small compared to the total mass, this effect is insignificant.

The starting positions of the carts can be marked on the table with chalk. Make sure the student places the starting mark for each cart at the end closest to the bumper, as shown in Fig. 2.

If both carts have the same load, friction tends to cancel out, since the force of friction acts for the same time on both carts, giving equal and opposite impulses to the two carts and decreasing their momentum equally. There will, of course, be different forces of friction on two carts loaded with different masses, but the effect of friction is small and can be neglected in this experiment.

If the table is not level, one cart will gain momentum during its run, and the other will lose momentum. If the table cannot be made level, it may be necessary to run two trials for each loading of the carts, reversing their positions for the second trial and comparing the average momentum change of each cart.

Answers to Questions

If the spring is released with the cart at rest on the table, the cart does not appear to move. The student will conclude that the momentum of the cart before and after the explosion is zero. Changing the load on the cart will not change this result.

With various loads on the carts, it can be observed qualitatively that the lighter cart moves faster as a result of the interaction. Some students may guess that the momenta of the carts have the same magnitude but are in opposite directions.

In calculating the ratio of momenta, there is no need for a standard unit of momentum. The momenta can be expressed in kilogram-meters per "clank." (A "clank" is the unit time interval, the time taken by both carts to reach the bumpers in a given run.) Of course, changing the loads on the carts will result in different speeds and hence different time and momentum units, but the time unit will be the same for both carts. Without trying to find a specific unit, the student can answer that the change in momentum of one cart is equal in magnitude and opposite in direction to the change in momentum of the other cart. (The ratio of the momenta is minus one to about 5 percent.)

The total momentum before and after the explosion is zero, and hence momentum is conserved.

Apparatus

2 Dynamics carts
4–5 Bricks
4 C clamps
2 Table stops (wood bars)
Meter stick
Pan balance
Wedges
Chalk or masking tape

EXPERIMENT

27 The Cart and the Brick

The experiment is not difficult to perform, but there are several techniques to be noted. Heavy cord should be used to support the brick, because it is much easier to pinch a thick string to the support bar as shown in Fig. 1. Several practice runs should be made before recording the motion of the cart.

Since some friction will always be present, the cart will slow down throughout the run. To analyze the interaction properly, the velocity of the cart should be measured just before and just after the interaction. At least one brick should be on the cart when measurements are made, to prevent the final velocity from being so low that table irregularities will cause it to fluctuate too much. Good judgment is needed in finding the velocities before and after the interaction, to choose an interval long enough to be meaningful and short enough to have fairly constant velocity.

To prevent damage to table tops from falling bricks that miss the cart, wrap the bricks in paper.

An alternate analysis of the data can be made by plotting velocity as a function of time. By extrapolating the two lines which represent the slowly changing velocities before and after impact, the actual velocities can be determined at the time of impact [Fig. (a)]. On smooth tables, practically no extrapolation is necessary. In any case, only the better students should be encouraged to use this method of analysis.

Answers to Questions

The cart will slow down when the brick is dropped on it. If the mass of the cart is increased, it will appear to slow down less.

The decrease in the momentum of the cart equals the increase in the horizontal momentum of the brick, but the changes are in opposite directions.

The total horizontal momentum of the system of the cart and suspended brick before interaction is just that of the cart. Since, as a result of the collision, the cart loses the same momentum that the brick gains, the total momentum of the system of cart and brick after the interaction is the same as before the interaction. Momentum is conserved within experimental error (about 2 percent).

The horizontal impulse applied to the falling brick is equal to its change in horizontal momentum. The length of time of the interaction can be estimated by carefully examining the tapes and estimating how many intervals show a rapidly changing velocity. In general, the whole interaction will occur in one or two ticks.

To estimate the horizontal force (in newtons) applied to the brick, "ticks" must be calibrated in seconds. This is best done by pulling a tape through the timer for a known time (4 or 5 sec) and counting the dots. Accuracy in calibrating the timer this way is not important, since the inspection of the tape will only give, to an order of magnitude, the number of intervals during which the interaction took place.

Using the known impulse applied to the brick, the average force of the interaction can be found to an order of magnitude from $F = \frac{I}{\Delta t}$ where I is the impulse and Δt is the interaction time. This is a very rough determination, and it is worth pointing out that this is an average force; in actuality, the force

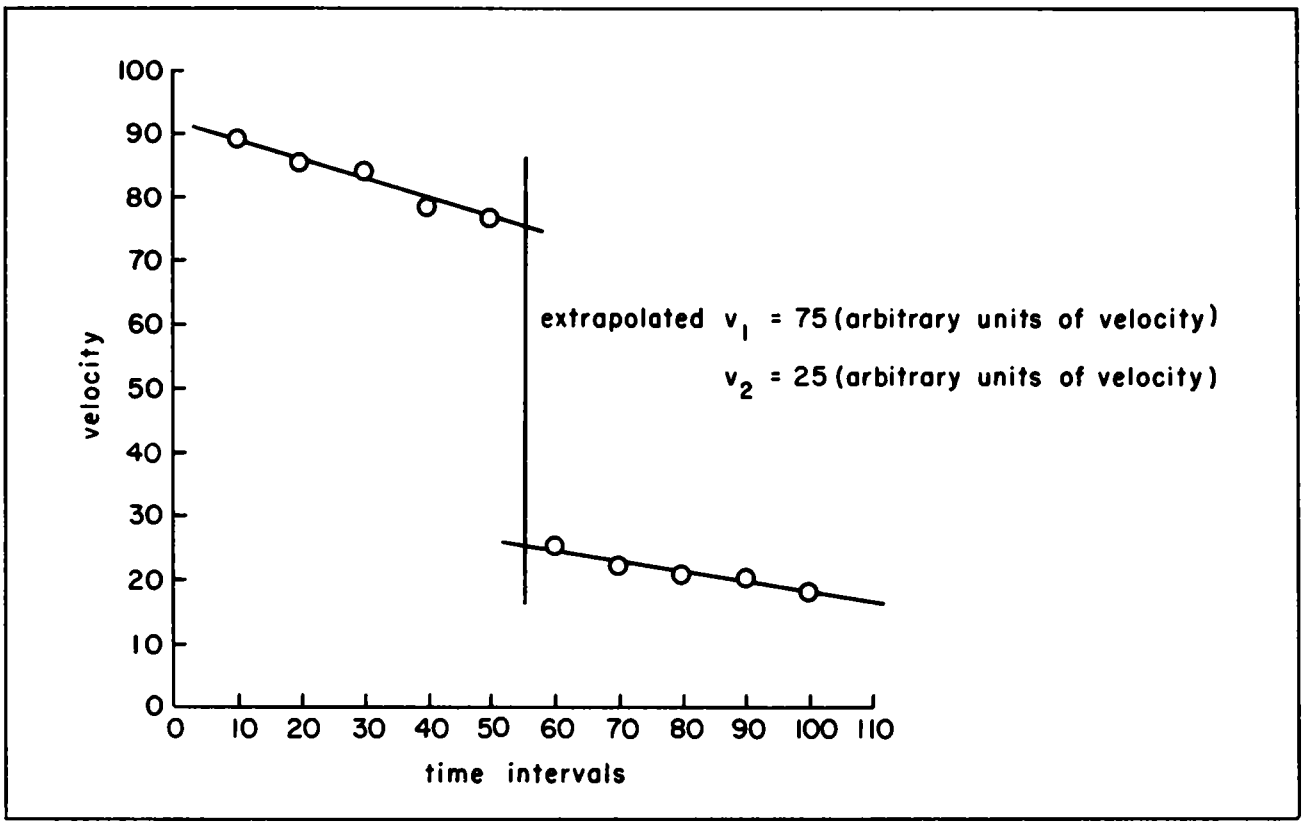


Figure (a)

probably varies widely over the interaction time in some unknown manner.

The force applied to the cart is equal but opposite to the force applied to the brick.

The vertical momentum of the falling brick is transferred to the earth via the cart and the table. Momentum is a vector, and none of the vertical momentum is transferred to horizontal motion. Theoretically the magnitude of the velocity is of little importance in this experiment.

If the experiment is done by dropping sand slowly into a box on the cart, the cart slows down but the duration of the collision is greater than when the brick is dropped. If the sand was allowed to run out of the cart, however, the velocity of the cart would not change. It must be kept clearly in mind that our system consists of cart and sand at all times. Only the sum of momenta of both remains unchanged. When we drop the brick on the cart there is a friction force acting between them until they both move with the same velocity.

When we let the brick or sand fall off the cart, both cart and sand have the same horizontal velocity and will continue to have the same velocity until the sand touches the table, which is not part of our system. One may not include or exclude any object from the system during the interaction, because this will lead to errors. Dropping the brick on the cart is misleading in this respect. Only because the brick has no horizontal momentum at the beginning can we say that the mass of the cart increased and hence its speed had to decrease.

The reverse of the experiment of dropping the brick on the cart is to shoot the brick off the rear of the moving cart so that its velocity relative to the table is zero; the cart then speeds up and the brick drops vertically.

Apparatus

1 Dynamics cart
1 Timer
Paper tape
Carbon-paper discs
1 1½-volt dry cell
3–4 Bricks
1 Ringstand with crossbar and clamps
3 C clamps
Heavy cord
Sheet of masonite or other protective cover for tables
Pan balance
Wedges
Cellulose or masking tape

EXPERIMENT

28 A Collision in Two Dimensions

Tape timers can be used to measure velocities and momenta only in collisions occurring along a straight line. Collisions in two dimensions on a table can be analyzed by studying stroboscopic photographs, and it is done this way in the textbook. Without this equipment, some other method must be used to time the motions. The constant horizontal velocity of a projectile provides a simple way of measuring velocities in any horizontal direction.

When a ball rolls slowly over the edge of a table, the table gives it a small horizontal impulse; the faster the ball moves, the smaller the impulse, and above a certain speed the impulse is zero. For this reason the target ball is mounted on a screw away from the table edge, so that the incident ball will not be in contact with the table when the collision occurs.

Some students may feel this is really a collision in three dimensions, since the spheres not only move in different directions in the horizontal plane but also fall vertically. You may remind students that they are finding the velocities just before and just after the collision, when the objects are moving in the horizontal plane, and that the following vertical motion is used merely as a means of measuring the velocities in the horizontal plane.

Once the apparatus is set up, a large amount of data can be taken quickly, but the analysis is long and will probably have to be done as homework. Don't require too many collisions to be analyzed. (The calculations of the vector components of the final momenta in a direction at right angles to the initial momenta is interesting, but not an essential part of the experiment.)

Answers to Questions

When the incident sphere is rolled down the incline ten or fifteen times with the target removed, the points where it lands on the paper will be scattered over a small area that can be enclosed by a circle whose diameter is about 3

percent of the distance traveled by the ball. Therefore, the initial velocities in the different trials vary from the average by about 1.5 percent.

The momenta in the case of the spheres of equal mass are represented by the displacement vectors on the paper. If the experiment is carefully done, the vector sum of the two final momenta will be equal in both magnitude and direction to the initial momentum of the incident sphere, within the experimental accuracy as determined by the scattering of the points. A sample set of data and vector diagrams is shown in Fig. (a) and (b). Momentum is conserved within experimental error. Notice that the target sphere has to be lifted

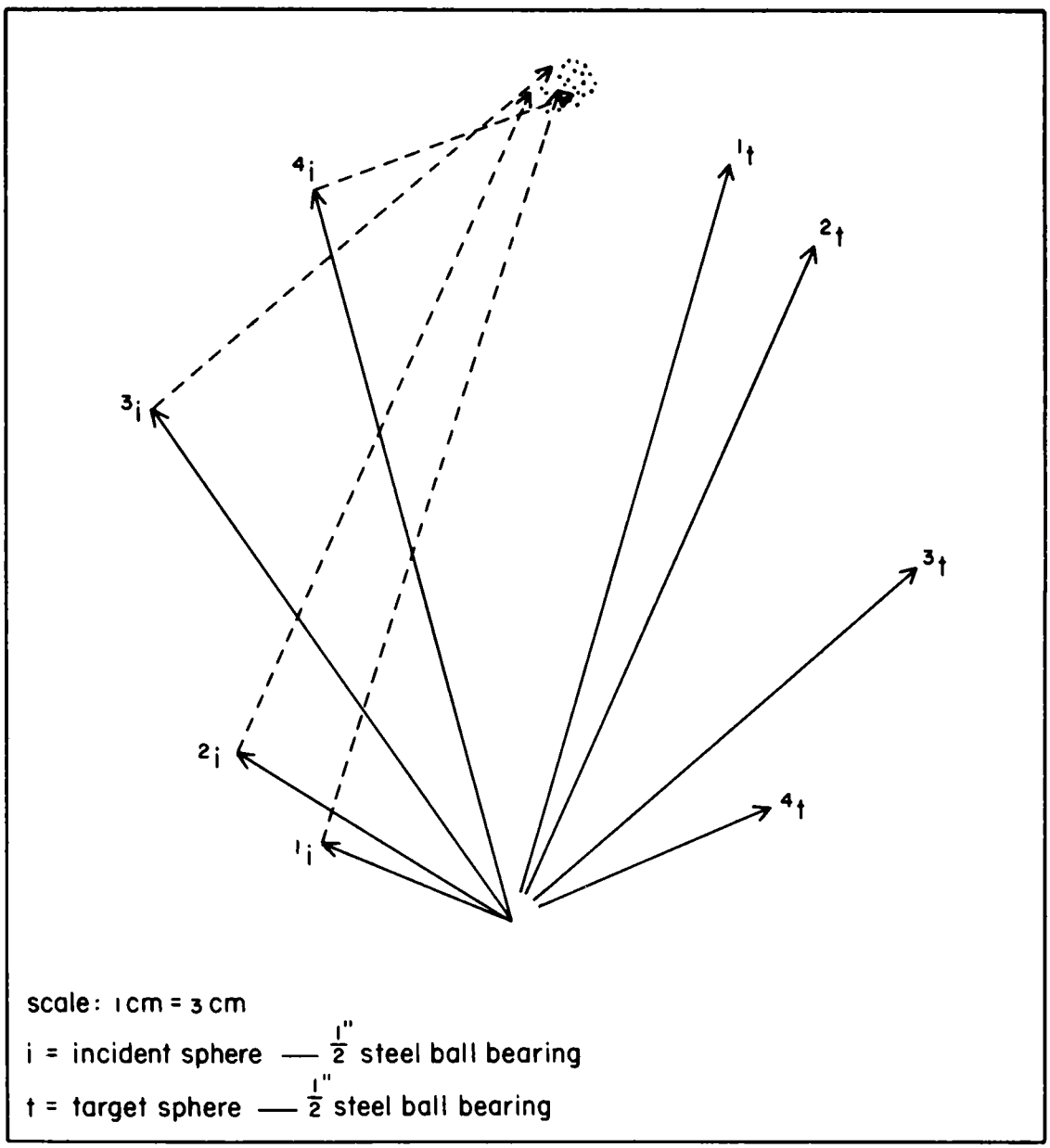


Figure (a)

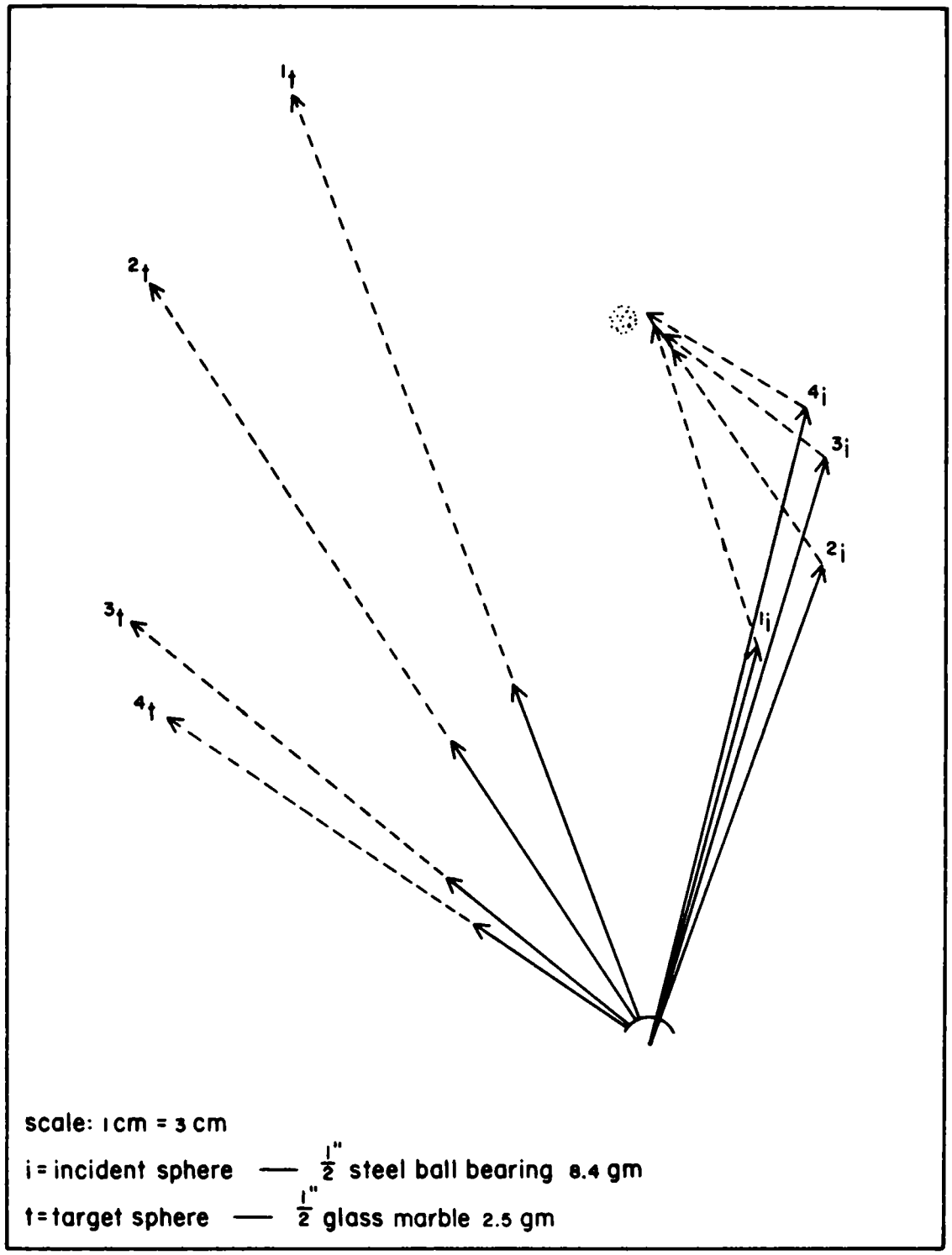

1t
2t
3t
4t
1i
2i
3i
4i
scale: 1 cm = 3 cm
i = incident sphere — 1/2" steel ball bearing 8.4 gm
t = target sphere — 1/2" glass marble 2.5 gm

Figure (b)

out of the slight depression that holds it in place. In the process a small fraction of the momentum given to the target sphere in the collision is transferred to the support. Thus, with the incident sphere hitting the target sphere on the right, the vector sum of the momenta also tends to the right, as seen in Fig. (b).

Though the vector sum of the momenta is constant, the *arithmetic* sum of the momenta magnitudes is not.

When the experiment is repeated with unequal masses, the heavier ball should be used as the incident ball. Otherwise, with head-on or near head-on collisions, the lighter incident ball will rebound and bounce off the table edge.

When unequal masses are used, the displacements on the paper representing velocities no longer represent momenta. The vector sum of the final velocities will not equal the initial velocity.

To convert the velocity vectors to momentum vectors when unequal masses are used, the velocity vector of each ball must be multiplied by the ball's mass. For simplicity we may choose the mass of the incident ball as our unit of mass. The momentum vector of the incident ball then becomes identical with its velocity vector, and the momentum vector of the target sphere is shorter than the corresponding velocity vector by the ratio $\frac{m_t}{m_i}$. Momentum is found to be conserved.

The vector components of the final momenta of the two balls in a direction at right angles to the initial momentum are equal in magnitude and opposite in direction.

Apparatus

- 1 Collision-in-two-dimensions apparatus; this consists of:
  - Frame
  - Plastic ruler with center groove
  - Movable arm (with clamp and target-ball support)
  - Thread
  - Plumb bob (screw eye)
  - 2 Steel balls, ½″
  - 1 Glass ball, ½″
  - 1 Plastic ball with tapered hole
- 1 C clamp
- 4 Sheets carbon paper
- 8 Sheets tracing paper (onion skin)
- 1 Protractor or large pair of dividers
- Meter stick
- Cellulose tape

Questions

1. A force of 10 newtons acts on a body for a total time of 5 seconds. What is the magnitude of the impulse? What is the momentum change of the body? If the body was initially at rest, and has a mass of 2 kg, what is its final speed? (50 newton-sec; 50 kg-m/sec; 25 m/sec.)
2. A golf ball of mass 100 gram is struck by a club. After the impact the ball moves off with a speed of 50 m/sec. If the ball and club were in contact for 5×10^{-3} sec, what was the average force acting on the ball? (1000 newtons.)

3. Two forces act on a 10 kg-mass which is initially at rest. First the body is subjected to a 20-newton force for 3 seconds which acts in a northward direction. One second after the first force is applied the body is subjected to a force of 30 newtons which acts in an eastward direction for two seconds.
   (a) What is the northward component of the impulse? (60 newton-sec.)
   (b) What is the eastward component of the impulse? (60 newton-sec.)
   (c) What is the final momentum of the body? ($60\sqrt{2}$ kg-m/sec, northeast.)
   (d) If the eastward force had been applied initially at the same instant as the northward force, instead of a second later, what would be the final momentum of the body? (Same as (c).)
4. A 2-kg ball moving with an initial speed of 3 m/sec to the right is acted upon by a constant force F to the left for a period of 3 seconds. After the interaction, the ball is moving to the left with a speed of 6 m/sec. What is the magnitude of F? (6 newtons.)
5. A body is subjected to a varying force which increases linearly with time from zero to 10 newtons in 0.1 sec, remains constant at 10 newtons for 0.5 sec, and then decreases linearly to zero in 0.2 sec. What is the total impulse? If the body has a mass of 3 kg and was initially at rest, what is its final speed? (6.5 newton-sec; 2.17 m/sec.)
6. Here are two questions which you might ask students about Figure 14–7 to make sure they are "seeing" the figure properly:
   (a) Estimate when the explosion occurred relative to the flashes.

   Students who estimate this by looking at the positions of the small disc will easily see that the explosion came about halfway between flashes. However, some students who look at the positions of the large disc may think the explosion occurred just a bit before a flash. This common error is based on the fact that the dark space between the first two positions of the large disc appears so small. Ask students to explain this; you can point out that the large disc moved not only through this dark space, but also through a distance equal to the width of the white rim.

   (b) Using only the velocity given in the text, find the size of the small disc.

   This can be done easily and quickly if the students use the fact that the small disc happened to move a diameter in each flash. Since $v = 0.48$ m/sec and since each flash corresponds to $\frac{1}{4}$ sec, the distance traveled in two flashes or the diameter is $\frac{1}{4}$ sec $\times$ 48 cm/sec $= 12$ cm.
7. A 5-kg ball of putty moving at a speed of 6 m/sec strikes a 1-kg block of wood which was initially at rest. The putty sticks to the wood. What is the final speed of the wood? (5 m/sec.)
8. A 10-gram bullet is fired with a speed of 1000 m/sec from a freely hanging 2-kg gun. What is the recoil velocity of the gun? (5m/sec.)
9. A 5-kg body at rest is torn apart by an explosion. It breaks into two pieces. A 4-kg piece is seen to move off to the right at a speed of 50 m/sec. Describe the motion of the remaining 1-kg piece. (Moves to left at a speed of 200 m/sec.)
10. A 10-kg mass moving to the right at a speed of 20 m/sec interacts with a second mass which was initially moving to the left at a speed of 20 m/sec.

After the interaction both masses are found to be moving to the right at a speed of 5 m/sec. How large is the second mass? (6 kg.)

11. Can momentum conservation alone determine motion after a collision? If you have time at this stage you can use this question to motivate the search for other dynamical quantities in later chapters. Ask students whether any other final velocities of the two balls in Figure 14–8 could have kept the total momentum unchanged. Students will be able to answer this more easily if you use specific numerical values. Say that the balls each had a mass of 0.5 kg and that the initial speed was 4.4 m/sec. (Note that this speed corresponds to the ball going a little less than 27 cm in $\frac{3}{48}$ seconds. Note too that the apparent elongation of the ball by about 2 cm combined with this speed implies that the camera shutter was open for about $\frac{1}{220}$ second during each frame.) The students should realize that equal momentum could have been realized by many other combinations. Ask them how fast ball 1 would have been moving after the collision if ball 2 had attained a speed of 3.3 m/sec rather than 4.4 m/sec. They should recognize quickly that ball 1 would have continued with a speed of 1.1 m/sec instead of stopping. What if ball 2 had attained a speed of 4.1 m/sec? Ball 1 would have retained a velocity of 0.3 m/sec forward.

    You can leave students with the question of how ball 2 manages to make a choice with all of these possibilities. Physically it is clear that the detailed interaction at collision must determine the result. In Chapter 15, we shall include energy in our analysis of collisions and thereby eliminate some of the ambiguity.

12. Ask students what effect escaping gases would have on momentum conservation applied to a gun and bullet. When the bullet leaves the gun barrel and the gases do escape, they usually carry a negligible amount of momentum. However, whether or not the momentum of the gases is negligible is not so important as is the realization by students that they are considering this momentum negligible. If someone devised a gun in which the gases carried significant momentum, these gases would have to be included together with the gun and the bullet in calculating momenta.

13. Have students locate the position of the center of mass in each frame of Fig. 14–8 and then calculate the velocity of the center of mass. Finally, determine the velocity of each ball as seen from the center of mass. This is a particularly simple case, requiring minimum time for the measuring but including most of the important ideas. Tracing paper and a centimeter ruler are the only tools needed.

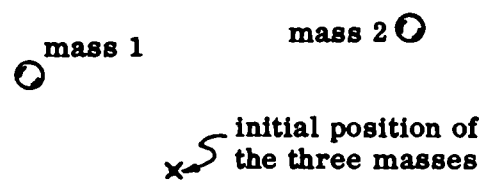
14. You can discuss Fig. 14–17 so that students appreciate how the motion of the center of mass is simple even though the wrench appears to be moving in a complicated way. The following questions can be used to lead students to consider the motion as the resultant of two simple components.
    (a) If the wrench is 20 cm long, what is the average speed of the center of mass? The students should be able to find that the wrench moves a distance equal to its length (or very slightly over that) in two flashes. Hence, $v = 0.20/0.067 = 3$ m/sec. (Note that in doing this students will confirm the fact that $\vec{\mathbf{v}}_{cm}$ is essentially constant.)
    (b) Which way is the wrench moving? The only clue is that the wrench is probably slowing down due to friction. It will take rather careful measurements to show that the wrench is moving from left to right.

(c) Is the picture distorted by the camera? Yes, the wrench size seems different at the two ends of the picture.

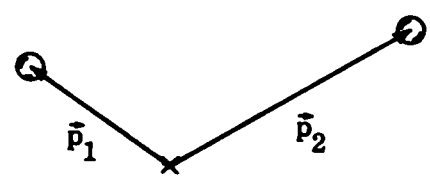
(d) What is the speed of rotation of the wrench about its center of mass? It rotates about once in 14 flashes or has a rotational speed of just more than 2 revolutions per second.

15. A billiard ball, moving directly east at 2 m/sec, hits two other identical balls. After the collision it is found that the original ball is moving north at ½ m/sec, and one of the struck balls is moving east at 1 m/sec. How fast and in what direction is the third ball moving? It will be quite sufficient, even desirable, to do this problem graphically. The final momentum is composed of three momenta and must equal the initial momentum. (Note that this collision is not elastic, because the final kinetic energy is less than the initial kinetic energy.)

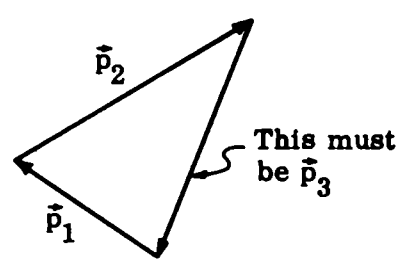
16. Three identical masses are close together and at rest on a frictionless table when an explosion drives them apart. A photograph a moment later shows two masses. Where is the third?



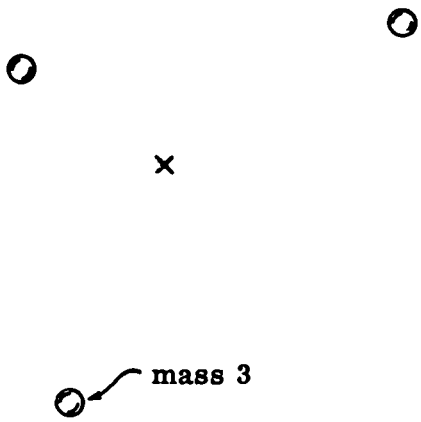
Since the distance traveled is proportional to the velocity and the masses are the same, the momentum vectors are just:



The initial momentum was zero. Therefore, the final momentum must be zero.



The final positions are then:



CHAPTER

15 Kinetic Energy

Chapter 15 introduces energy, one of the most useful concepts in physics. The idea of energy will be extended in the next two chapters, and is used extensively throughout the rest of the course. Application of the law of conservation of energy is a powerful tool for solving a wide range of problems, particularly those in which the forces of interaction are not known in detail (e.g., in collisions). These chapters depart rather sharply from the standard development that has been used in many introductory courses. For this reason it may be worthwhile to discuss briefly the general nature of how energy is developed in these chapters.

The text makes no attempt to give a specific, short definition of energy. There is none! Rather than beginning with a definition, a better eventual comprehension and appreciation of the significance of the idea of energy can be achieved through building an understanding of how the concept of energy can be applied to many phenomena. In pursuing such a development, the text presents the following ideas:

The term energy is applied to many widely-different characteristics of the motion and relative position of matter. A single term is useful because these widely-varying characteristics are all connected through a single conservation law. In order to emphasize the breadth of the energy concept and to stress energy conservation, the text purposely avoids an over-simplified energy definition such as "the capacity to do work." Instead, various forms of energy are introduced and the convertibility of energy from one form to another is discussed. The concept of work takes its rightful place as simply one mode of energy transfer. Work is defined on a quantitative basis; it is then possible to formulate a quantitative description of several forms of energy, starting from the amount of work done to produce them.

The strategy of the text is to introduce the three forms of energy that appear: kinetic, potential, and thermal. With these three forms, it is possible to discuss the transfer of energy from one form to another, showing that the total energy is always conserved.

Chapter 15 contains a great deal of new and important material which will require considerable class time for a full development. Time should be spent on the solution of problems involving work and kinetic energy in order to drive home the quantitative use of these concepts in physical situations. The Home, Desk, and Lab section contains a large number of problems which are suitable for stimulating class discussion.

Students sometimes have difficulty in understanding that kinetic energy is a completely different physical quantity from momentum. This difference should

be clarified. Questions 13 and 14 in the guide to this chapter lend themselves to this purpose.

Students should understand that in elastic collisions between two bodies the total kinetic energy of the system is the same before and after the collision. However, the kinetic energy may change during the interaction. When the force is repulsive, the amount of kinetic energy which "disappears" is given by $F\,\Delta s$, where F is the constant repulsive force of the interaction, and Δs is the change in the separation distance between the two bodies. For a given force of repulsion, when the separation distance decreases, the total kinetic energy decreases. The total kinetic energy increases when the separation distance increases. The criterion for an "elastic" collision is that the force between two objects be a function only of their distance of separation. Then, after interaction is complete, the total kinetic energy is the same as it was before the interaction started.

Some students may not see the importance of studying "collisions." A wide variety of problems in modern physics basically concern what is called the two-body problem. Problems dealing with the orbit of a planet around the sun, the dynamics of a rocket, and the collision between two protons are examples of the two-body problem. Fortunately this relatively simple physical situation, the interaction of two objects, has rather broad applicability. The general problem which involves the interaction of more than two bodies can still not be solved exactly.

Two-body problems dealing with the motions of the planets can be solved by applying Newton's law and thus obtaining solutions for the motion along successive very small intervals of an orbit. This kind of solution is possible because the interaction force is well defined at all points of space by the law of universal gravitation. Even so, the procedure is a tedious one. In the case of the propulsion of a rocket, or of the collision of two protons, a solution by application of $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ is impossible because the interaction force is not known. We can make considerable progress with these problems through the use of the conservation laws. When the collision is elastic, we can use the conservation of momentum and of kinetic energy to solve for the final velocities without knowing the details of the force of interaction. An application of this method is illustrated in Chadwick's discovery of the neutron.

Experiment 29 Elastic Collisions and Experiment 30 Simulated Nuclear Collisions may be done after Section 15–7 has been discussed.

It will be wise to make sure that students do not confuse the permanent loss of kinetic energy during an inelastic collision with the temporary "disappearance" of kinetic energy during an elastic collision. They will learn in the next chapter that, in an elastic collision, "disappearing" kinetic energy is going into potential energy from which it can return. Kinetic energy transferred by heat flow can be recovered only partially.

Experiment 31 Inelastic Collisions can serve as a review of Chapters 14 and 15 in showing what determines the conservation of momentum and the conservation of kinetic energy.

EXPERIMENT

Elastic Collisions 29

There is very little loss of kinetic energy in these collisions, and the students' angle measurements and calculations show that kinetic energy is conserved.

Therefore the collisions are elastic, since it was shown in Experiment 28 that momentum is also conserved.

Answers to Questions

Since the equation $v_1^2 = v'_1{}^2 + v'_2{}^2$ has the same form as the Pythagorean theorem, the angle between v'_1 and v'_2 should be 90°.

If the measured angles are 90°, then the collisions were elastic.

The calculations show that the collisions between the steel ball and the glass ball were elastic.

EXPERIMENT

30 Simulated Nuclear Collisions

This analogue experiment lends itself to a variety of extensions, such as collisions between atomic particles of different masses (coins) and collisions with more complicated systems formed by adding more particles to a target "nucleus." Only students with a special interest are expected to go beyond establishing the range-energy relation and finding the momentum of the "neutral" particle in a collision between two particles of equal mass.

The incline in Fig. 1 must be steep enough (about 70°) to justify neglecting friction on it. To obtain a reasonably sharp definition of the beginning of the range, the radius of the curvature at the bottom of the incline must be fairly small.

Heavy graph paper may be used on the Masonite instead of cardboard. It already has lines on it to identify the initial heights. Use worn nickels; they slide better.

A range-energy relation obtained in an actual experiment is shown in Fig. (a). The analysis of a collision is shown in Fig. (b). The range-energy relation

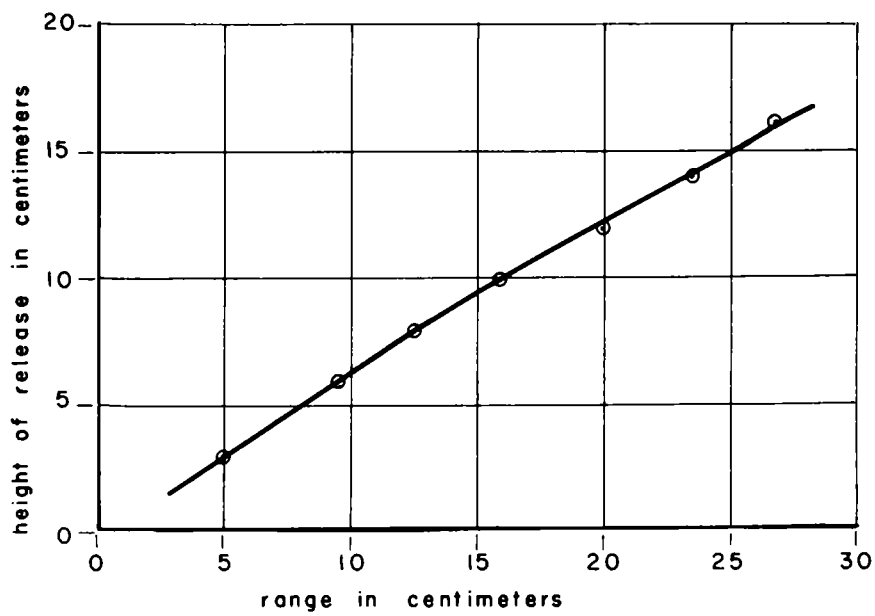
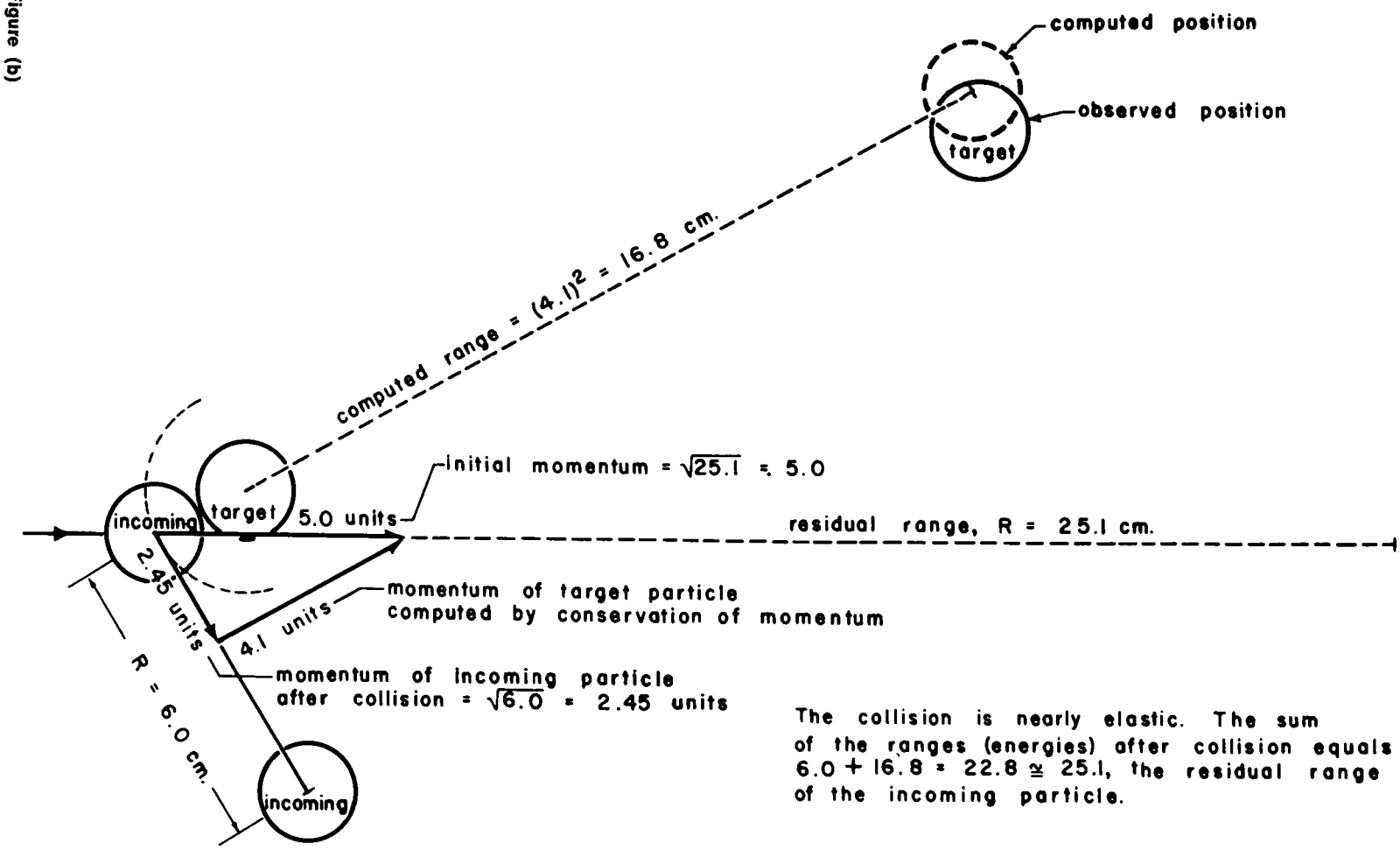


Figure (a)

Figure (b)



The collision is nearly elastic. The sum of the ranges (energies) after collision equals $6.0 + 16.8 = 22.8 \cong 25.1$, the residual range of the incoming particle.

is very nearly a straight line. Hence, by proper choice of energy units we can have the energy numerically equal to the range and measure it in cm. Furthermore, $E = \frac{1}{2}mv^2 = p^2/2m$; by choosing a unit of mass equal to twice the mass of the nickel we obtain numerically $E = p^2$ or $p = \sqrt{E} = \sqrt{R}$. Notice that in this collision experiment the incoming nickel has lost energy along its horizontal path *before* hitting the target. Its energy just before collision corresponds to its residual range, i.e., the range it would have without a collision minus the distance from the bottom of the incline to the target. This is analogous to the situation in a cloud chamber when the incoming particle loses some of its initial energy by traveling a considerable distance before a collision.

In a cloud chamber, the visible tracks give the directions of motions directly. This is not the case in our experiment; here we have to find the relative positions of the nickels at the instant they collide to determine their direction of motion and momentum.

Apparatus

1 Ringstand, 24″
1 Masonite or wood ramp, 15″ × 10″
1 Sheet smooth cardboard 9″ × 12″ (manila folder)
Several sheets mimeograph paper 11″ × 14″
3 Nickels
1 Sheet rectilinear graph paper
1 Ruler
1 Protractor
C clamp or brick
Masking or cellulose tape

EXPERIMENT

31 Inelastic Collisions

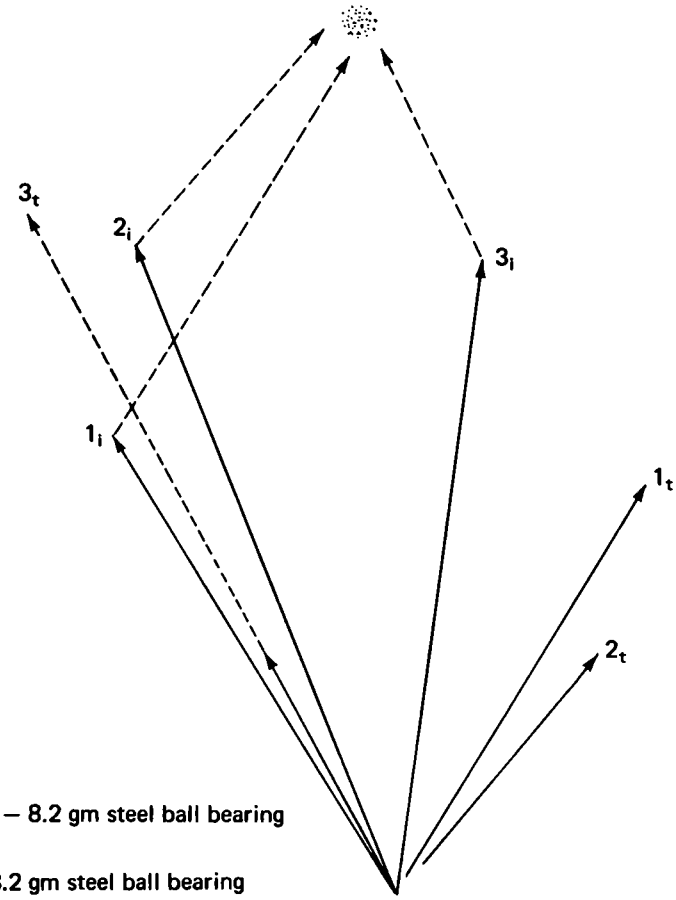
The purpose of this experiment is to show that in collisions the conservation of kinetic energy and the conservation of momentum are not determined by the same factors. In a collision between two bodies, a and b, momentum will be conserved as long as no resultant external force acts on either body; the force that a exerts on b ($F_{a,b}$) must equal the force of b on a ($F_{b,a}$) at every point. For kinetic energy to be conserved the force between a and b must depend only on their separation distance.

$F_{a,b}$ and $F_{b,a}$ at a given separation must be the same before and after the collision. This will not be true if the atoms in either body permanently change their relative positions during the collision, as is the case in this experiment.

Another difference between the conservation of kinetic energy and the conservation of momentum in an elastic collision is that the total momentum remains the same throughout the collision, while the kinetic energy may be transferred into potential energy during the collision, but become kinetic energy again after the collision.

The target ball is placed on the set screw so that the incident ball will hit the taped part. Only a very small piece of adhesive tape is needed.

Since students use the measurement of the angle between momentum vectors to show that the collisions are inelastic, they should verify that the piece of adhesive tape does not significantly change the mass of the steel target ball, as this measurement is only valid for collisions between equal masses.



Scale: 1 cm = 3 cm
i = incident sphere — 8.2 gm steel ball bearing
t = target sphere —
trials 1 and 2: 8.2 gm steel ball bearing
trial 3: 2.6 gm glass marble

Answers to Questions

Predictions: (1) The total kinetic energy after the collision will be less than the total kinetic energy before the collisions; (2) momentum will be conserved; (3) the angles between momenta after the collision will be less than 90°.

The experimental results show that the predictions were correct.

When tape is put on the glass target ball, momentum is conserved, but kinetic energy is not; the collisions are inelastic.

Apparatus

1 Collision-in-two-dimensions apparatus
1 C clamp
4 Sheets carbon paper
4 Sheets tracing paper (onion skin)
1 Protractor or large pair of dividers
Meter stick
Adhesive tape

Films

"Elastic Collisions and Stored Energy," by Dr. James Strickland of Education Development Center.

Elastic collisions with magnetic Dry Ice pucks are demonstrated. From a strobe photo of one of these collisions it is shown that the total kinetic energy is the same after the collision as before; the collision is elastic. The total kinetic energy decreases during the collision. Examination of the data shows that this temporary storage of energy during the actual collision depends only on the separation of the pucks. In another experiment, the pucks are tied together and then "exploded," and the total kinetic energy after the explosion is found to be equal to the predicated energy stored before the explosion. This recoverable stored energy is then defined as potential energy. This film may be used in connection with the end of Chapter 15 or during the first half of Chapter 16.

"Momentum, Energy, and Center of Mass," by Professor Herman Branson of Howard University. This film investigates two-body collisions (magnetic Dry Ice pucks) from different frames of reference: a laboratory frame and the center-of-mass frame, showing the dependence of the total momentum on only the frame of reference from which it is observed. The film further demonstrates that the total energy of a given system may be partitioned into two parts: one part associated with the motion of the system's center of mass relative to a reference frame, which varies with the reference frame; and the other part associated with the motion of the individual parts of the system moving relative to the center of mass—the internal energy, which is the same in all frames. This film could be profitably used associated with the second half of Chapter 15 or later during the first part of Chapter 17. Running time: 25 minutes. Prints of the stroboscopic photographs in Branson's film are available from Macalaster Scientific Company for student analysis.

"Energy and Work," by Professor Dorothy Montgomery of Hollins College demonstrates that the work that goes into raising a heavy ball transfers to kinetic energy when it falls. The relationship between work done and change in kinetic energy is further underscored by a demonstration with a device in which a varying (but reproducible) force acts through a given distance. This is done by finding the work from the area under a force-distance curve and the kinetic energy from measurement of mass and velocity. An amusing "Rube Goldberg" device is used to remind students that *mgh* expresses gravitational potential energy. In closing, the film leads toward heat energy by showing that, as the falling ball is brought to a stop, the loss of kinetic energy is accompanied by an increase in heat. This film may be used in connection with Chapter 15 or 16. Most teachers prefer to use it about the middle or toward the end of Chapter 16. Running time: 28 minutes.

Demonstrations

1. The discussion in the text (Section 15–3) on the transfer of kinetic energy during a collision is somewhat abstract and may be hard for the students to visualize. This demonstration will allow them to watch, almost in slow motion, what occurs during the interaction between two bodies. It is probably best used during the discussion of Sections 15–3, 4, 5 and 6.

Use two loaded carts with "soft" spring bumpers as shown in Fig. (a) and illustrate the collision process shown in the text drawing Fig. 15–5. There is one slight difference, however, between the text collision and this one: there

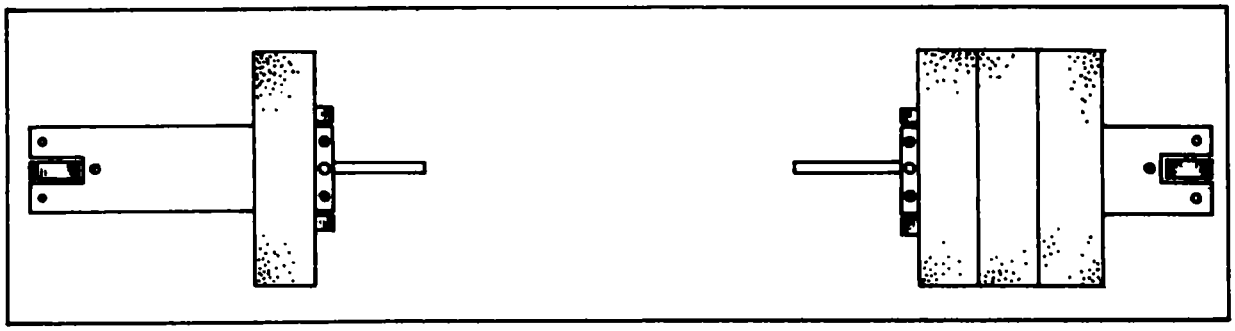


Figure (a)

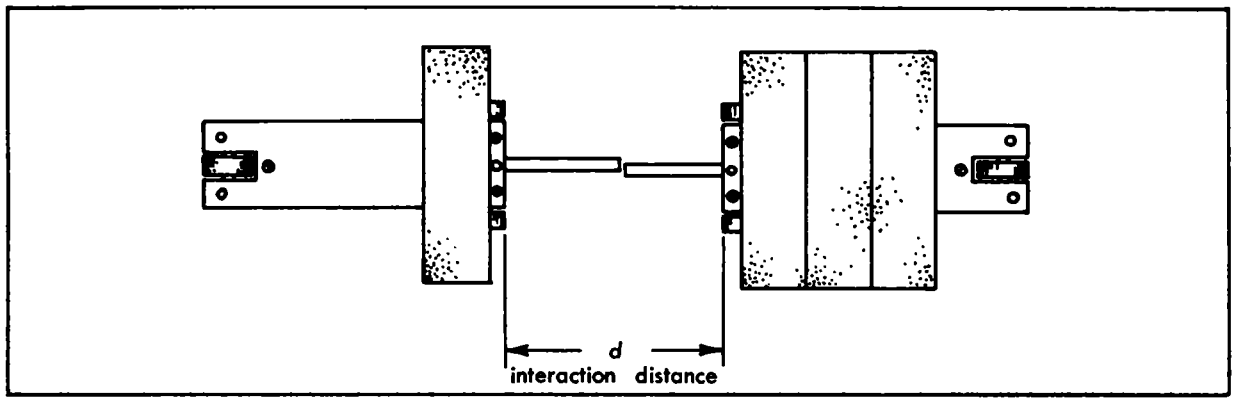


Figure (b)

the interaction force was zero when the separation was greater than d and constant when the separation was less than d. With the spring bumpers, the interaction begins when the bumpers make contact and the carts are separated by the distance d [Fig. (b)]. As the carts get closer together during the interaction the force increases, unlike the constant force described in the text. The overall results, however, are very much the same, although a careful mathematical analysis of the interaction would be difficult. If you put three bricks on the incident cart and one on the stationary cart, you will approximate the mass ration of the example in the text.

Another discrepancy between the abstract collision described in the text and the real collision suggested here is that the collision of the two carts is inelastic. Some of the energy is lost through friction. Frictional effects will be reduced if higher initial velocities are used. Collisions with several initial velocities should be tried.

Point out the following observations by asking leading questions:

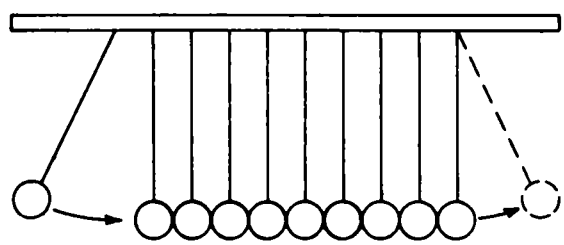
(a) Kinetic energy is lost by the incident cart and gained by the struck cart during the interaction. When the carts are at minimum separation, they both move very slowly, in fact, at the same velocity. It looks as though the total kinetic energy is less at minimum separation than at any other time, but one cannot be sure by mere qualitative observations. However, it is easy to believe that a good deal of energy is stored in the springs at minimum separation, as stated in the text.

(b) When the total mass of the carts is increased, the interaction time increases and the distance of minimum separation decreases for a given initial

velocity. When the velocity of the incident cart is increased, the interaction time appears to be constant in this interaction (because the compression of the springs is very nearly proportional to the force).

(c) If the carts are equally loaded and one is at rest before the collision, the incident cart would end up at rest and the struck cart would move off with all the energy, were it not for energy losses due to friction in the spring bumpers. In reality, the incident cart keeps moving with a small velocity but is brought quickly to rest by friction with the table, sometimes before the interaction is over.

2. A well-known demonstration apparatus consists of a row of elastic spheres in contact supported by two-thread suspensions from a bar. Be cautious about your use of this apparatus. The overall behavior of the spheres is consistent with the laws of conservation of energy and conservation of momentum; but unless you limit yourself to the case of one projectile ball and one target ball, you cannot predict on the basis of these laws alone what the behavior of the system will be. Consider the case of two target balls and one projectile ball. There is no *a priori* reason for believing that the two target balls must have the same velocity after the collision; there would be three unknowns, the velocities of the three balls after the collision. The conservation laws provide only two equations, and hence on their basis alone there would be an infinite number of solutions. A complete analysis is complex, and inappropriate for your classes. See *American Journal of Physics,* Nov. 1960, page 705.



Questions

1. Calculate the work done by a boy who pulls a sled a distance of 20 m while exerting a 100-newton force on a rope which makes an angle of 45° with the horizonal. ($1000\sqrt{2}$ joules.)
2. A man pulls a 20-kg mass up a frictionless inclined plane which makes an angle of 30° with the ground. He moves the mass to a point 5.0 meters in height above the floor.
   (a) How much work has he done? (980 joules.)
   (b) How large a force was required to raise the mass? (98 newtons.)
   (c) How far was the mass moved along the plane? (10 m.)
   (d) How much work would be required to do the same job if the plane were not frictionless, but instead exerted a constant retarding force of 50 newtons along the plane? (1480 joules.)
3. How much work is done by a man who takes a 7.5-kg bowling ball and bag from a shelf 2.0 m high and lowers it to the floor? (150 joules is done on the man by the ball.)

4. How much work is required to carry a 15-kg suitcase a horizontal distance of 50 m? (Zero.)
5. A 60-kg boy pushes a 50-kg girl around a dance floor. He exerts an average force of 80 newtons on his partner. During the dance they move at a rate of 0.50 m/sec for 5.0 minutes. How much work does the poor boy do? (12,000 joules.)
6. A 10-kg mass is moving with constant speed of 10 m/sec.
   (a) How much work must be done to double the speed to 20 m/sec? (1500 joules is transferred to the moving mass.)
   (b) How much work must be done to halve its speed to 5.0 m/sec? ($-$ 375 joules is transferred from the moving mass.)
7. A constant 20-newton force acts for 10 seconds on a 5.0-kg object which is initially at rest.
   (a) What is the object's final momentum? (200 kg-m/sec.)
   (b) What is the object's final kinetic energy? (4000 joules.)
   (c) How much work was done in accelerating the body? (4000 joules.)
8. A force which increases from 0 to 100 newtons at a rate of 10 newtons/sec. 12,500 joules of work are done by this force in accelerating the body from rest to a final speed of 50 m/sec.
   (a) What is the mass of the body? (10 kg.)
   (b) How large a constant force would have been required to give the body the same final speed if the force acted during one meter of travel? (12,500 nt.)
9. A 2.0-kg mass moving at a speed of 20 m/sec has the same momentum as a 10-kg mass.
   (a) What is the speed of the 10-kg mass? (4 m/sec.)
   (b) What is the kinetic energy of each of the masses? (400 joules, 80 joules.)
10. A 10-kg body has a kinetic energy of 500 joules. What is its momentum? (100 kg-m/sec.)
11. A 10-kg body at rest on a frictionless table receives an impulse of 30 newton-sec. What is its final kinetic energy? (45 joules.)
12. With what kinetic energy does a 2-kg rock strike the ground if it falls from a height of (a) 5 m? (b) 10 m? (c) 20 m? (d) Compare the momenta of the rock just before striking the ground for each of the cases (a), (b), and (c).
    (a) 100 joules; (b) 200 joules; (c) 400 joules; (d) $mv(20\text{ m}) = \sqrt{2} \times mv(10\text{m}) = 2 \times mv(5\text{m})$.
13. (a) Let a force of 1500 newtons act through a distance of 200 meters on a 1500-kg car which starts from rest. The work done is

$$W_1 = F_1 x_1 = 1500 \times 200 = 3 \times 10^5 \text{ joules.}$$

To find the final speed of the car after it has traveled these 200 meters, we note that the acceleration was

$$a_1 = \frac{F_1}{m} = \frac{1500}{1500} = 1 \text{ m/sec}^2, \text{ so that}$$
$$v_1^2 = 2a_1 x_2 = 400 \text{ m}^2/\text{sec}^2, \text{ and}$$
$$v_1 = 20 \text{ meters/sec.}$$

The momentum of the car will be

$$p_1 = mv_1 = 1500 \times 20 = 3 \times 10^4 \text{ kg-m/sec.}$$

The kinetic energy will be

$$(E_K)_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}\,1500 \times 400 = 3 \times 10^5 \text{ joules,}$$

which is equal to W_1, the work which was done (see above).

(b) Now let a 1500-newton force act again, in the same direction, through 200 more meters.

$$W_2 = F_2x_2 = 1500 \times 200 = 3 \times 10^5 \text{ joules.}$$

To find the final speed we note that the acceleration again is

$$a_2 = \frac{F_2}{m} = \frac{1500}{1500} = 1 \text{ meter/sec}^2.$$

The final speed v_2, is given by

$$v_2^2 = v_1^2 + 2a_2x_2 = 400 + 2 \times 1 \times 200 = 800 \text{ m}^2/\text{sec}^2.$$
$$\therefore v_2 = 28.3 \text{ m/sec.}$$

The new momentum of the car will be

$$p_2 = mv_2 = 1500 \times 28.3 = 4.24 \times 10^4 \text{ kg-m/sec.}$$

Thus the change in momentum during the second acceleration interval was

$$(\Delta p)_2 = p_2 - p_1 = 1.24 \times 10^4 \text{ kg-m/sec.}$$

Whereas as a result of the initial acceleration

$$(\Delta p)_1 = p_1 - p_1 = 3 \times 10^4 \text{ kg-m/sec.}$$

The final kinetic energy will be

$$(E_K)_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}\,1500 \times 800 = 6 \times 10^5 \text{ joules.}$$

Thus the change in E_K was the same during both acceleration intervals while the change in momentum was quite different. In each case,

$$\Delta(E_K) = W = 3 \times 10^5 \text{ joules.}$$

Why should this be? In the first period, the car accelerated from zero speed to 20 m/sec with constant acceleration. Therefore the elapsed time must have been

$$v_1 = 20 = v + a_1t_1 = 0 + (1 \times t_1),$$
$$\therefore t_1 = 20 \text{ seconds.}$$

The elapsed time during the second acceleration must have been

$$v_2 = 28.3 = v_1 + a_2 t_2 = 20 + (1 \times t_2),$$
$$\therefore t_2 = 8.3 \text{ seconds.}$$

Thus although identical forces acted through identical distances, the *time* these forces were applied was quite different in the two cases. This is because, for the second acceleration, there was a large initial velocity which resulted in the given distance being covered in a much shorter time. As was shown in Chapter 14, change of momentum stems from an impulse. In the two intervals in question, the work done was the same, but the impulse, force × *time,* was quite different.

14. In order to emphasize the difference between the two measures of motion, momentum, and energy, it is often helpful to cite an example such as a 100-gram bullet and a 1500-kilogram car, each striking a block of wood. The bullet might have an initial speed of 300 meters per second; the car, 0.02 meter per second. Thus both have an initial momentum of 30 kg-meters/sec, but the bullet's kinetic energy would be $(E_K)_b = 4.5 \times 10^3$ joules whereas for the car, $(E_K)_c = 0.3$ joule. This difference in kinetic energies results in the car giving the block of wood a slight nudge while the bullet, with the same momentum but much more energy, rips the wood apart.

15. Picture a man riding on a flatcar of a freight train. Suppose it to be moving at 10m/sec. He throws a baseball in the direction of the train's motion, exerting a 50-newton force throughout a 1-meter motion of his hand. The ball attains a speed of 10m/sec relative to the flatcar. The kinetic energy of the ball relative to the flatcar is $\frac{1}{2} \times 1 \text{ kg} \times (10 \text{ m/sec})^2 = 50$ joules. But to an observer on the ground, what appears to have happened? The ball initially was moving with a speed of 10 m/sec as a result of the motion of the flatcar. Its original kinetic energy was therefore 50 joules. After the man threw it, it had a speed of 20 meters per second. Its new kinetic energy was therefore $\frac{1}{2} \times 1 \text{ kg} \times (20 \text{ m/sec})^2 = 200$ joules, and its change of kinetic energy, in the throwing process, was $200 - 50 = 150$ joules. What is the source of the work that produced this energy change? The force which acted on the ball was still 50 newtons. However, the distance through which the force acted is quite different as seen from the ground. It can be calculated as follows:

On the train, the man's hand moved through 1 meter. If his force was constant, the acceleration was constant. His hand's final speed was 10 m/sec. Thus the average speed was 5 m/sec and the time of the acceleration was $t = \frac{s}{v} = \frac{1}{5}$ second. Now in $\frac{1}{5}$ second the train itself moved $\frac{1}{5} \times 10 = 2$ meters relative to a ground observer. Therefore in pushing the ball, the man's arm moved 1 meter as a result of his own motion and 2 meters as a result of the train's motion, a total of 3 meters relative to the ground observer. Thus the work done from the ground observer's point of view was 50 newtons × 3 meters = 150 joules, just equal to his observation of the change in kinetic energy. This is not a fictitious amount of work. The man did only 50 joules in pushing with a 50-newton force through 1 meter. But the man's feet pushed back on the

train, and the train pushed forward on his feet, with this same 50-newton force. While exerting this force on the man's feet the train traveled 2 meters. Thus the train itself supplied 100 joules of the work done on the ball. This work came from the energy liberated from the burning of the train's fuel.

16. A 200-gm ball, moving at 6.0 m/sec, approaches a second ball head-on. the second ball has a mass of 100 grams. While the balls are within 5.0 cm of each other, they repel each other with a constant force of 40 newtons. The second ball leaves the collision with a speed of 8.0 m/sec.
    (a) How much kinetic energy does the second ball gain in the collision? (3.2 kg-m^2/sec^2 or 3.2 joules.)
    (b) What is the kinetic energy of the first ball after the collision? (0.4 kg-m^2/sec^2 or 0.4 joule.)
    (c) How far did each ball move during the collision, i.e., while they remained within 5.0 cm of each other? (8.0 cm.)
17. A block having a mass of 5.0 kg slides on a frictionless surface with a speed of 1.0 m/sec toward a second block which is at rest. The second block has a mass of 1.0 kg. When the separation of two blocks is 10 cm and less, they repel one another with a force of 50 newtons.
    (a) What is the magnitude and direction of the acceleration of each block while they are colliding, i.e., closer together than 10 cm? (a_1 = 10m/sec^2, backward; a_2 = 50 m/sec^2, forward.)
    (b) The beginning of the collision is at the instant when the two blocks first get within 10 cm of each other. From their positions at that instant, how far will each block move in $\frac{1}{60}$ sec? In $\frac{1}{30}$ sec?

$\frac{1}{60}$ sec	$\frac{1}{30}$ sec
$\Delta x_1 = \frac{11}{12} \times \frac{1}{60}$ m = 0.015 m,	$\Delta x_1 = \frac{5}{6} \times \frac{1}{30}$ m = 0.028 m.
$\Delta x_2 = \frac{5}{12} \times \frac{1}{60}$ m = 0.0070 m,	$\Delta x_2 = \frac{5}{6} \times \frac{1}{30}$ m = 0.028 m.

    (c) Calculate the change in the total kinetic energy $\frac{1}{60}$ sec after the beginning of the collision, and $\frac{1}{30}$ sec after the beginning of the collision.

$$\begin{aligned} \tfrac{1}{60}\text{ sec: } (\Delta E_K)_{\text{total}} &= 50(\tfrac{5}{12} \times \tfrac{1}{60} - \tfrac{11}{12} \times \tfrac{1}{60}) \\ &= -50 \times \tfrac{1}{2} \times \tfrac{1}{60} \\ &= -\tfrac{5}{12} \text{ newton-meter} \\ &= 0.42 \text{ joule.} \\ \tfrac{1}{30}\text{ sec: } (\Delta E_K)_{\text{total}} &= 0 \text{ joules.} \end{aligned}$$

18. A 10-kg mass moving with a speed of 5.0 m/sec collides elastically with a 5.0-kg mass which is initially at rest. What is the final kinetic energy of the system? (125 joules.)
19. A 10-kg mass moving with a speed of 5 m/sec collides head-on with a stationary 10-kg mass. The collision is elastic. What is the final velocity of each of the masses? (0, 5 m/sec.)
20. A 2-kg mass moving at a speed of 20 m/sec collides elastically, head-on, with a 5-kg mass initially at rest. Find the final speeds and directions of both masses. (2 kg: $-\frac{60}{7}$ m/sec; 5 kg: $+\frac{80}{7}$ m/sec.)
21. A 10-kg mass moving toward the right with a speed of 5 m/sec collides elastically, head-on with a second 10-kg mass moving initially at 10 m/sec

to the left. What is the final velocity of each of the masses? (10 m/sec left, 5 m/sec right.)

22. A 5-kg mass moving at 5 m/sec strikes a glancing blow on a stationary 5-kg mass. The collision is elastic. After the collision one mass leaves at 45° to the incident direction. What is the final velocity of each mass (magnitude and direction)? ($2\sqrt{2}$ m/sec at $+45°$, $2\sqrt{2}$ m/sec at $-45°$.)
23. A 20-kg mass moving at a speed of 3 m/sec collides inelastically with a stationary 10-kg mass. The two stick together and move off at the same speed.
    (a) What is the final speed of the two masses?
    (b) How much kinetic energy is "lost" in the collision?
    Solution:

(a) $m_1 = 20$ kg
$m_2 = 10$ kg
$v_1 = 3$ m/sec
$m_1 v_1 = (m_1 + m_2) v_2$

$$v_2 = \frac{m_1}{m_1 + m_2} v_1 = \frac{20}{30} \times 3 = 2 \text{ m/sec.}$$

(b) $E_{K_1} = \frac{1}{2} \times 20 \times 3^2 = 90$ joules
$E'_{K_{1+2}} = \frac{1}{2} \times 30 \times 2^2 = 60$ joules.

Hence, 30 joules of energy is "lost" in the collision.

CHAPTER

16 Potential Energy

In this chapter, the concept of energy is broadened to include potential energy. The exchange of energy between the two forms, kinetic and potential, is studied. In using the energy concept we ignore many of the details of the motion, since we do not concern ourselves with interaction forces and accelerations of various parts of the system with respect to one another. However, in ignoring these details we gain in simplicity and in our understanding of fundamentals. The use of the conservation laws is often called an integral approach to problems of mechanics. It relates final conditions to initial conditions independent of the interaction that is involved. Thus, for example, with no precise knowledge of the nuclear force law, we can still make a great many exact deductions about the effects of a given interaction.

While discussing Section 16–1, the details of the non-elastic spring should not be stressed, but it will be wise to make sure that students understand what property is crucial. For the spring to be elastic it must be able to do as much work when released as was done on the spring in compressing it. Students should know that they can compute the potential energy of a spring from the area under the force-compression curve. They are likely to memorize $U = \frac{1}{2}kx^2$ automatically, but the important thing is to understand where it comes from. The key objective is to attain a familiarity with energy conservation and transfer. This can be gained through discussing a number of examples in which the form of the energy (kinetic or potential) is followed, and the signs of the transfer recognized. In this section try to avoid the abstraction of gravitational potential energy or that of any other "action at a distance" force field.

Figure 16–5 may warrant considerable discussion. Here we are representing all the energies involved in the motion of a mass m attached to the end of a spring and oscillating, perhaps on a horizontal, frictionless surface. This is for an ideal spring for which the force is $F = kx$ and the potential energy $U = \frac{1}{2}kx^2$. Thus the potential energy graph resembles Fig. 16–4 except that it is graphed for both compression and extension of the spring. The total energy, $E = \frac{1}{2}mv^2 + U$, must be a constant since the spring is "elastic." When $U = 0$ (at $x = 0$), $E = \frac{1}{2}mv_0^2$ where v_0 is the initial velocity which the mass had the first time it passed through the equilibrium position. The horizontal, straight line which represents E is drawn at this level, the value of the kinetic energy when the compression or extension of the spring is zero. The kinetic energy is given at every other point of the motion by the difference between the horizontal line depicting E and the curve depicting U, $\frac{1}{2}mv^2 = E - U$. Since a negative value of $\frac{1}{2}mv^2$ has no meaning, we draw the line representing

E only out to the point at which it intersects U. At that point, $\frac{1}{2}mv^2 = 0$, and therefore $v = 0$.

In Section 16–3, consideration is given to the potential energy of a system of a pair of bodies which interact at a distance. The mathematical aspects were worked out in Chapter 15, where the kinetic energy during a collision was followed in detail. For action-at-a-distance forces, in apparent contrast with the spring, there is no visible storage place for the potential energy. This may trouble some students. All that is required for introducing the concept of potential energy is the fact that some kinetic energy disappears at one part of the collision and later reappears. One might choose to say "the kinetic energy is destroyed, but later an equal amount is created" or "the kinetic energy is converted to potential energy, and the potential energy is then reconverted to kinetic energy." The second viewpoint has some advantage in that it emphasizes the fact that the total work which the particles could do is the same at any time during the collision, whether we let it appear directly as kinetic energy, or whether we reach in, grab the particles, and make them separate slowly (in which case the energy appears as work against us).

Where is the potential energy located? For the collision of Section 1 we say "in the spring." We could consider the force between two bodies which interact at a distance as having its source in an "imaginary spring" as suggested in the text. Actually, there is no need to "locate" the potential energy, no matter how natural that desire may be. *All that is necessary for the concept of potential energy to be useful is that we be able to use it to predict the work we can realize (or must supply) when bodies go from one separation to another.* In fact, the separation of the bodies gives us as clear an indication of the potential energy of a system as does the extension of a spring.

One may be led into difficulties if he tries too hard to locate the potential energy, perhaps associating it with one or the other of the two bodies. It should be considered a property of the two bodies together, as a single system. The work needed to bring up B while holding A fixed is identical to that of bringing up A while holding B fixed, assuming only that we bring them to the same final separation in the two cases.

Sections 16–4, 16–5, and 16–6 should be treated thoroughly not only because of the interest inherent in the problem itself, but also because familiarity with the potential energy in an inverse square force field will again be required in the study of the Coulomb field and the nuclear atom.

One should not lose sight of the fact that potential energy applies to the *system* of interacting objects. At the outset it is proved that, in most of the situations involving the motion of a mass in the field of the earth, the kinetic energy of the earth can be neglected. For this reason one is apt to think of the mass as having the potential energy, but actually the potential energy belongs to the *system* rather than separately to either of the partners of the system.

Experiment 32 Changes in Potential Energy employs a mass suspended on a coil spring to examine the transfer of energy from gravitational potential energy to the potential energy of a stretched spring. This experiment is best done after Section 4 is discussed.

To appreciate the power of the information obtained from the conservation of energy in connection with Section 16–6, one should ask the question: Does the escape velocity depend on the direction in which the rocket is fired? Using the principle of conservation of energy we see immediately that it does not for the case of a gravitational interaction. Had we attempted to answer the question by using Newton's law directly, the problem would have been much

harder to solve. We would have found eventually that three types of orbits are possible: elliptical (including circular as a special case), parabolic, and hyperbolic. (Straight line motion is a special case of parabolic or hyperbolic motion.) Elliptical orbits correspond to rockets that do not escape (satellites), parabolic orbits to rockets that just escape, and hyperbolic orbits to rockets that escape with energy to spare. The energy argument completely bypasses a fairly involved discussion. Naturally, we do not have quite as much information if we use conservation of energy alone; we know the speed of the satellite at large separation, but we know nothing about the actual orbit. [In the above discussion, as in the text, the escape velocities referred to are relative to the center of the earth, not relative to the surface of the earth. We can take advantage of the eastward motion of the rocket launching path ($\approx$0.5 km/sec at the equator) to save fuel.

The escape energy calculated in the text gives the kinetic energy that a rocket must have (outside the atmosphere) to escape. The fuel requirements are of course, much larger; in a typical rocket most of the work done by the rocket engine ends up as kinetic energy of the exhaust.

The concept of binding energy is one which is often used in discussing atoms. The example given in the text, in which we calculate the energy which has to be added to a bound satellite moving in a circular orbit around the sun, has a very close counterpart in the hydrogen atom, in which an electron moves in a circular orbit around a proton.

Section 16–7 is a summary and extension of the ideas that have already been presented. A brief treatment will suffice. The energy concept will be extended further in the next chapter where heat will be considered.

EXPERIMENT

32 Changes in Potential Energy

This experiment presents a situation in which energy is transformed from one form to another with little loss. Gravitational potential energy is converted into the potential energy of a stretched spring. This is an extremely valuable experiment requiring some care.

A rigid ringstand about a meter high is needed to support the spring and masses. If a shorter ringstand is used, it will be necessary to arrange it so the spring and weights will hang freely over the side of the table.

If the laboratory period is short, the plotting of the extension of the spring as a function of the applied force may be deferred until all the data are obtained.

Answers to Questions

The graph of the spring extension x as a function of the force F will be so close to a straight line passing through the origin (over the range investigated) that one may infer that x is proportional to F within this range. The slope $k = \frac{F}{x}$ of the line can be substituted in the potential energy function of a linear spring, $U_s = \frac{1}{2}kx^2$, to give the potential energy stored in the spring for any extension x.

One can think up situations in which the force vs. spring-extension graph would not be a straight line. In that case, the energy stored in the spring can

be found by graphical integration—i.e., finding the area under the force-extension curve from zero to different values of x. A table or graph can then be made of U_s as a function of x for use later in the experiment.

The loss in gravitational potential energy between the two extreme points $-\Delta U_g = -mg(x_2 - x_1)$ equals the gain in potential energy of the stretched spring between the same points $\Delta U_s = \frac{1}{2}k(x_2^2 - x_1^2)$ within 1 or 2 percent.

Notice that from $-\Delta U_g = \Delta U_s$ or

$$\begin{aligned} -mg(x_2 - x_1) &= \tfrac{1}{2}k(x_2^2 - x_1^2) \\ &= \tfrac{1}{2}k(x_2 + x_1)(x_2 - x_1) \end{aligned}$$

follows

$$mg = -k\frac{(x_2 + x_1)}{2}$$

That is, the weight of the mass is equal in magnitude to the force exerted by the spring when it is stretched to the midpoint between x_1 and x_2. This is the point where the mass was originally at rest before it was lifted to star the oscillations. It is the center of the oscillation.

Setting the arbitrary zero point for the gravitational potential energy at $x = 0$ (i.e., at the lower end of the unstretched spring), we have for the total potential energy:

$$U = U_g + U_s = +mgx + \tfrac{1}{2}kx^2$$

Moving the zero point of the x axis to $\frac{x_2 + x_1}{2} = -\frac{mg}{k}$, thereby defining a new coordinate $x' = x + \frac{mg}{k}$, yields $U = -\frac{1}{2}\frac{m^2g^2}{k} + \frac{1}{2}kx'^2$. Finally, setting the zero of the total potential energy at the midpoint of the oscillation reduces U to $U' = \frac{1}{2}kx'^2$ [Fig. (a)]. Thus, the spring in the gravitational field of the earth behaves like a spring free of gravitational effects in a displaced coordinate system.

The sum of the potential energies when the mass is halfway down is less than the potential energy at the start and finish of the fall. The difference equals the kinetic energy of the falling mass.

The change in kinetic energy may be investigated experimentally by taking stroboscope photographs of the falling mass or by having the heavy mass (1 kg) pull a timing tape as it falls.

Apparatus

1 Steel coil spring (2 cm diameter by 15 cm; k approx. 4 nt/m)
2 Weights (0.5 kg, 1.0 kg)
Ringstand with crossbar and clamps
1 C clamp (to support ringstand)
2 Sheets graph paper
3 Clothespins
Meter stick
Pan balance

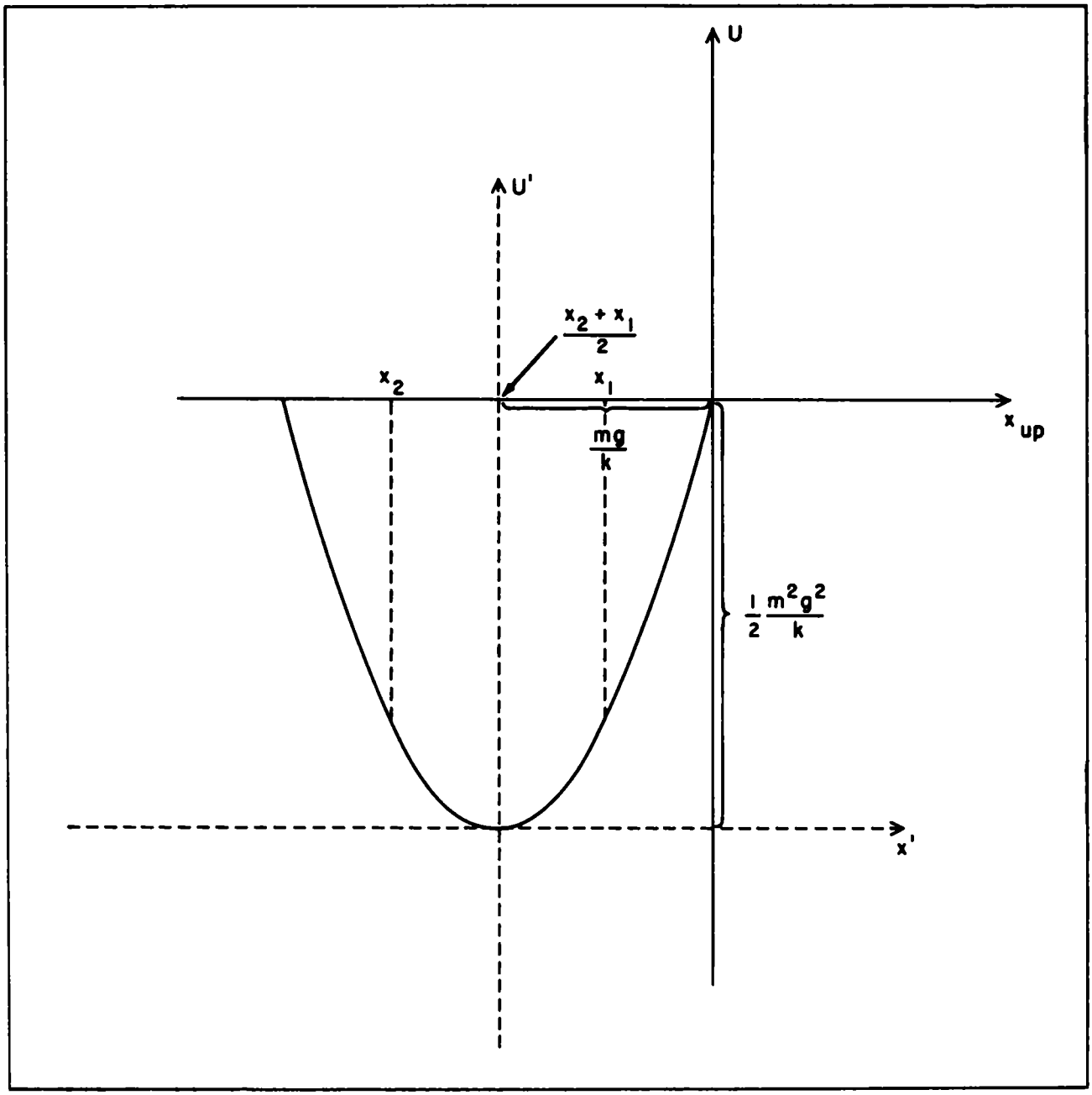


Figure (a)

Films

If either of the films "Elastic Collisions and Stored Energy" or "Energy and Work" (described in the Resource Book for Chapter 15) have not been shown earlier, they can be used profitably during this chapter.

Demonstration

An interesting experiment that you can do as a classroom demonstration shows nicely the conversion of potential energy into kinetic energy and involves some rather nice, simple techniques.

A measurable amount of potential energy is stored in a pendulum bob by swinging it a known distance up from its equilibrium position. The bob is released. At the bottom of its swing the original increment of potential energy will have been completely converted into kinetic energy, $\frac{1}{2}mv^2$. The motion is entirely horizontal. If we could measure this horizontal speed, we could calculate the kinetic energy and check the energy balance. In order to measure this speed, the bob can be hung from a light thread, and a razor blade can be carefully and rigidly mounted so as to cut the thread just when it reaches the vertical. Then the speed can be calculated by measuring the horizontal distance traveled by the bob before hitting the floor. This distance must be divided by the time of flight which is determined, by calculation, from the known height of the bob above the floor when in the equilibrium position.

It is advisable to try this out before doing it in class. The main source of error lies in the energy that is dissipated in cutting the thread. Be sure that the razor blade is *rigidly* mounted and use as heavy a bob as possible. In class you might make several measurements using different values of Δh.

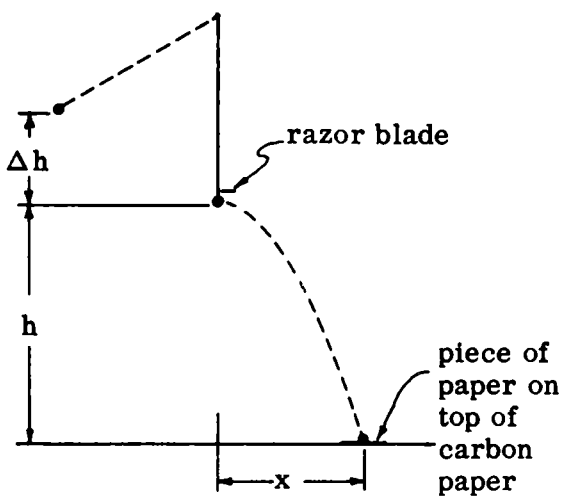
$$U = mg\Delta h$$

$$E_k = \frac{1}{2}mv^2$$

$$v = \frac{x}{t}$$

$$h = \frac{1}{2}gt^2,$$

$$\therefore \quad t = \frac{2h}{g}.$$



Questions

You can discuss many different kinds of interaction between a mass and a spring. For example, let the mass strike the spring, but catch the spring with a ratchet device at the moment of maximum compression. Then the mass stays at rest. If you slowly release the spring, the potential energy of the spring does the work, transferring energy to your hand. (How does the work *your hand* does in *releasing* the spring compare with the original kinetic energy of the mass? It is exactly the *negative* since you "absorb" the potential energy of the spring; it is *minus,* because in releasing the spring, your hand moves in the *opposite* direction from that of the force it exerts on the spring.)

You can let one mass strike and compress the spring, and then after you have caught the spring as before, substitute a different mass. Follow the ac-

celeration of this second mass in detail and show that the final kinetic energy is still the same.

Another variation: Suppose the mass hits the spring and rebounds, but leaves the spring vibrating. How does the kinetic energy of the mass compare with its value before the collision? It is less by just the energy left in the spring. You might ask the class whether the energy left in the spring is potential or kinetic. [It goes back and forth between the two forms, just as the energy of a mass oscillating on a massless spring goes back and forth from kinetic energy (when the spring is unstretched and the mass is going its fastest) to potential energy (when the mass is momentarily at rest and the spring is either compressed or extended the maximum amount).]

Heat, Molecular Motion, and Conservation of Energy

CHAPTER 17

This chapter serves a two-fold purpose. First it provides a major link in the energy conservation chain. In any experiment involving mechanical interactions, a transfer of energy is almost always accompanied by the conversion of some mechanical energy into internal energy. In developing the concept of internal energy, the text accomplishes a second, equally important objective: providing a natural transition from the problems of mechanics to the ideas of atomic structure. This bridge is provided by treating internal energy from the point of view of kinetic theory.

A number of the problems for this chapter deal qualitatively with energy transformations in various types of systems. Discussion of some of these problems will be an effective way to show the importance of internal energy in the overall conservation of energy. If you are pressed for time, discussion of such problems can carry the principal load in developing this chapter. Problems which are well suited to this purpose are numbers 4, 13, 23, and 24. Problems which are interesting if you have enough time are: Problem 11 involving the behavior of an ideal gas, and Problems 21 and 25 concerned with the quantitative conversion of mechanical to internal energy (often called the mechanical equivalent of heat).

While Sections 17–1 through 17–3 contain many new ideas, some rather subtle, the time that you spend on this material can be adjusted to bring your overall schedule into line with your requirements for the remainder of the course. Minimally, students should get some feeling for the possibility of conversion of "organized" kinetic energy into the kinetic energy of random molecular motion.

Some students may see that the derivation given in the text is highly simplified. They may note that it implicitly assumes that all particles or molecules travel with the same speed.

Even if all molecules have the same speed initially, they eventually collide with one another. Even if these collisions are elastic, they produce shifts in speed which would eventually lead to a broad spectrum of speeds. When two such objects collide elastically while their center of mass is at rest with respect to the laboratory system, no change in speeds results in the laboratory system. When their center of mass is in motion (e.g., collisions at right angles) changes in speeds result. A molecule moving with speed $3v$ would have nine times the kinetic energy of one moving with speed v, but would possess only three times

the momentum of the slower molecule. Thus the relation between pressure and kinetic energy must somehow depend on the actual distribution of speeds of various molecules. A rigorous proof is possible (one which takes into account the actual distribution of velocities), and it gives the same result as has been simply derived in this section. The actual distribution of velocities can be shown to be simply the one that is statistically most probable. It is called Maxwell's distribution.

In these sections we are at the very heart of the kinetic theory of gases, one of the great achievements in physics. We show (or at least present plausibility arguments in lieu of more complicated but exact proofs) that the bulk thermal properties of matter can be described exactly in terms of the dynamics of the motion of the individual molecules comprising a substance. Yet some students may feel that, after all, we have *defined* the equivalence of temperature and kinetic energy by our theory—so why should it be surprising that they turn out to be equivalent! The answer is that there is no quantity in dynamics which corresponds explicitly to the temperature. The parameters of mechanics are mass, force, speed, acceleration, momentum, energy, etc. The remarkable accomplishment of the kinetic theory is that it identifies such a complicated macroscopic property as temperature with so simple a microscopic dynamical property as the kinetic energy of motion of the centers of mass of the molecules.

Experiment 33 Completely Inelastic Collisions should be done in conjunction with Section 17–4.

After you have gone over the kinetic theory derivation of the ideal gas law, you will find that this Chapter progresses well under home reading by the students and class discussions based on problem assignments.

EXPERIMENT

33 Completely Inelastic Collisions

This experiment will provide an experimental test of the relationship, $E_k' = \frac{m_1}{m_1 + m_2} E_k$. In verifying that valid predictions can be made with this equation, the student must also reason why the original and final kinetic energies are not the same.

During your pre-lab, students' answers to the first question in the lab might be discussed. At first they may think that two balls can only lose a certain percent of their total kinetic energy (e.g., 60 percent) and still conserve momentum, but they should come to realize that it depends on m_1 and m_2.

If the position of the set screw was not changed after Experiment 31, the screw may be lowered exactly one quarter of an inch to get the plastic ball at the correct height.

By doing the calculations to estimate the rise in temperature of the two balls, students will be made aware of the size of the energy changes which occur in this experiment.

Answers to Questions

The fraction of total kinetic energy which may be lost depends on the masses of the two balls.

Prediction: E_k lost $= E_k - E_k'$

$$= E_k - \left(\frac{m_i}{m_i + m_t}\right) E_k,$$

where m_i = mass of the incident ball and m_t = mass of the target ball.

$$= E_k \left(1 - \frac{m_i}{m_i + m_t}\right)$$

$$E_k \text{ lost} = E_k \left(\frac{m_t}{m_i + m_t}\right)$$

The "lost kinetic energy" appears as an increase in the internal energy of the balls.

The prediction is verified by the experimental results. See Sample Data and Figure (a).

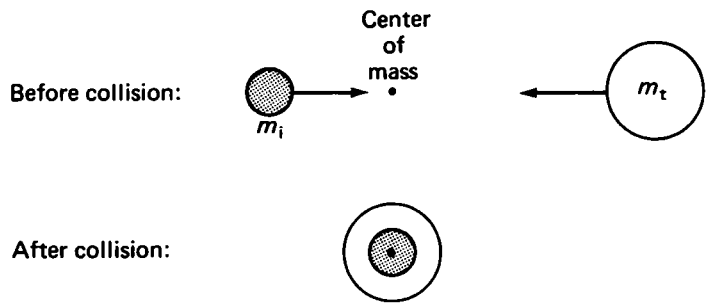


Figure (a)

A completely inelastic collision, as when the balls stick together, produces the greatest loss in kinetic energy.

The rise in temperature will be approximately 10^{-4} °C.

Apparatus

1 Collision in Two Dimensions apparatus (See Experiment 28)
1 C clamp
4 Sheets carbon paper
8 Sheets tracing paper (onion skin)
1 Protractor or large pair of dividers
Meter stick
Cellulose tape

Sample Data

$m_i = 8.3$ gm
$m_t = 6.0$ gm

Prediction: E_k lost $= E_k \left(\frac{m_t}{m_i + m_t}\right)$

E_k lost $= .42E_k$

Experiment: E_k lost $= .48E_k$

Scale: 1 cm = 3 cm
$m_{initial}$ = 8.3 gm
m_{final} = 14.3 gm

Film

"Mechanical and Thermal Energy," by Jerrold R. Zacharias of the Massachusetts Institute of Technology. This film views energy conservation microscopically, relating random molecular kinetic energy to the macroscopically observed variables. Using models, this film shows the interconnection between energy of bulk motion and thermal energy of random motion, how random motion can average out to a smooth effect, and how thermal conduction occurs. This film is best scheduled with Sections 17–4 through 17–8. Running time: 22 minutes.

Questions

1. A 20-kg mass traveling at a speed of 10 m/sec strikes a 5-kg mass which was originally stationary. The two masses stick together after the collision.
   (a) What is the final speed of the combination?
   (b) What was the initial kinetic energy of the system?
   (c) What is the final *total* energy of the system?
   (d) How many calories of heat were produced in the collision?

   (a) $p_i = 20\text{ kg} \times 10\text{ m/sec} = 200\text{ kg-m/sec}$,
   $p_f = 200\text{ kg-m/sec} = 25\text{ kg} \times v$,
   $v_f = 8\text{ m/sec}$.
   (b) $(E_K)_i = \frac{1}{2} \times 20 \times (10)^2 = 10^3$ joules.
   (c) $(E)_f = 10^3$ joules, by energy conservation.
   (d) $(E_K)_f = \frac{1}{2}(25) \times (8)^2 = 800$ joules.

$$\text{Heat} = (E)_f - (E_K)_f$$
$$= 200\text{ joules} = \frac{200}{4.2} = 48\text{ calories.}$$

2. A hot water heater breaks. A man wishes to heat his bath water from 25°C to 35°C. The tub contains 30 kg of water. How long will it take him to heat the tub if he uses a canoe paddle which he moves through the water at a rate of one meter per second while exerting an average force of 30 newtons? Assume all the work he does goes into heating the water.

$$\text{Work} = 30\text{ nt} \times 1\text{ m/sec} \times T(\text{secs}) = 30\,T\text{ joules.}$$
$$\text{Heat} = (35°\text{C} - 25°\text{C})30 \times 10^3$$
$$= 3 \times 10^5\text{ calories} = 12.6 \times 10^5\text{ joules.}$$
$$12.6 \times 10^5\text{ joules} = 30\,T\text{ joules.}$$
$$T = 4.2 \times 10^4\text{ seconds (about 12 hours).}$$

Electric Charges, Forces, and Fields

CHAPTERS
18-21

These chapters look first qualitatively and then quantitatively at the laws of electric forces, then apply the concept of a force field to the region surrounding electric charge, and consider the motion of electric charge in such a field, using the laws of conservation of energy and conservation of momentum developed in the previous chapters on mechanics. They are concerned from the outset with the atomistic nature of electric forces. The approach is fundamental. While it provides a powerful base for the analysis of electrical devices, electrical technology is not developed as a topic. Even such an old stand-by as Ohm's law is considered in perspective, simply as a useful relation which holds under special circumstances for some special types of circuits, certainly not as *the law* of electricity.

CHAPTER
18 Some Qualitative Facts About Electricity

Chapter 18 is an *introductory* chapter. You may be tempted to neglect it because it contains nothing quantitative. However, it is full of fundamental electrical phenomena, many of which become the basis for quantitative developments later on. Give it reasonable attention through class discussion based on the laboratory experiments, demonstrations, and home reading. Most of the topics in the chapter could be expanded considerably. Indeed, in following chapters, many of them are. Thus, while it is important to ensure that students understand the chapter, it will be well to stay at the level presented and avoid extensions in depth and quantification. If you have time for extensions, it can be used more profitably on the work that follows.

The chapter lays a foundation for a particle view of electricity that is basic to an understanding of the atom. While the chapter begins with electrostatics, it quickly goes beyond classical electrostatics by getting into the movement of charges. Thus, electric currents are introduced early. Current is developed as the motion of electrical particles rather than the "fluid" of Benjamin Franklin's day or even the later ideas of Faraday and Maxwell.

The student should begin to see electricity as a sub-atomic phenomenon—the ions as a kind of atomic fragment, and the electron as one of the building blocks of matter. He should see that electrons are common to all kinds of substances, and that electrons are all alike in that the chemical composition of a substance does not change when electrons are transferred from one substance to another (as in thermionic emission).

Experiment 34 Electrified Objects uses plastic strips rubbed with dry paper to investigate qualitatively the characteristics of static charges. It may be done at home, or if your students have had other courses which familiarized them with the behavior of charged objects, the experiment may be done as a demonstration. Assigning this experiment to be done at home is probably the simplest way to get it done at the optimum time—before getting into Chapter 18.

Experiment 35 Electrostatic Induction familiarizes students with charge separation and conduction resulting from induction. Desirably, this experiment should be scheduled with Section 3.

If your schedule allows time, good demonstrations of electrostatic phenomena are desirable for this chapter. The presentation and discussion of demonstrations on the work of this chapter should focus on the qualitative nature of the effects observed—the existence and movement of charges and

the forces between charges. Quantitative work will come later.

The introductory paragraphs before Section 1 introduce the idea that electric forces are of great magnitude. This will be important later to an understanding of atomic structure. You may not want to emphasize the relatively huge magnitude of the electrical forces at this point, but you should do it sometime during the first two chapters. The first paragraph states that "Sometimes other forces appear which are enormously greater" (than gravitational forces). In our everyday (macroscopic) experience, we only occasionally notice examples where electrical forces are greater than gravitational ones. The reason for this is simply that gravitational mass is the "charge" associated with gravitational force and, here on the earth, we are very near a huge "gravitational charge," the earth itself. Thus the forces between ourselves and the earth are large. However, the gravitational attraction between ordinary-sized objects is trivial compared to the electric forces that can be developed between them. Even further, on a microscopic scale, consider the forces between subatomic particles. If all the positive and negative charges in a pinhead could be shoved to opposite sides of the pinhead, the force of attraction between them would be about 10^{17} tons—truly enormous. The nearly exact equality of positive and negative charges in most matter makes these potentially tremendous forces seem quite small. Quantitative statements must be left for later, but the electrical forces must not be treated as if they were weak and relatively unimportant.

One of the reasons that the movement of both positive and negative charge carriers is introduced in this chapter is to facilitate seeing electric current as not limited to the motion of electrons. This general approach to electric current —the transport of charge in matter, not just in metals—contributes to visualizing and understanding definitions that come later [Chapter 19, Section 3, where electric field is defined as force (and electric potential difference is defined as work done) per unit *positive* charge]. In the world of gases, liquids and semi-conductors, as well as conducting metals, this is the way things are. Both positive and negative charges move in gases and solutions, the negative charges move in conducting metals, and the positive holes move in some semiconductors.

Some student might ask why like charges don't attract and unlike ones repel. The only answer is that they don't. With gravitational forces, however, all masses attract. While no one has ever seen a type of mass that repels, we sometimes speculate about the possibility of its existence. Presumably such "unlike" masses would be repelled by mass as we know it. Another possibility might be a kind of inconsistency, for example, A repels B; A repels C; but B attracts C. It happens that nature is kind in this regard. All this is just to emphasize that it is an *experimental fact* that all charges can be separated into two classes called positive and negative. There is no logic that says things *have* to be that way; they just are.

(You will not want to go into the following in detail, but it may be worth mentioning in passing.) Like most categorical definitions, the classification of all materials into either conductors or nonconductors is an oversimplification. Metals are better conductors than anything else by many orders of magnitude. Some materials like polystyrene are such good insulators that they will hold a charge placed on them for months.

Some materials, such as germanium and silicon (from which transistors are made), and copper oxide are much better conductors than plastics but much poorer than metals. They are often called semiconductors. Salt solutions are

poor conductors, but many orders of magnitude better than plastics. Gases are sometimes nearly perfect insulators, and sometimes nearly as good conductors as metals, depending on the degree of ionization.

	Conductivities in amperes per volt-meter:	
Silver	61.4×10^6	metals
Steel	5×10^6	
Carbon	2.8×10^4	a-case by itself
CuO	10^{-3}	semiconductor
Bakelite	10^{-10}	plastics.
Glass	10^{-12}	
Rubber	10^{-16}	
Polystyrene	10^{-18}	

Having dealt with models in several earlier chapters, students should have no difficulty in understanding what is being attempted in developing a model for electric charge. Quantitative aspects of the model will be developed later; at this point the discussion should be kept qualitative.

In addition to performing Experiment 35, one of the best ways to handle electrostatic induction is to demonstrate various instances of charging by conduction and induction, and ask the students for a step-by-step analysis of the effects in terms of changes in the distribution of charges and the consequent forces.

Comment

The text points out that the attractive force between a charged body and a neutral conductor is somewhat larger than the force between a charged body and a neutral insulator. This is not important enough to warrant full class discussion, but in case it stimulates questions, here is the picture.

The difference is not large, and for certain insulators may be almost undetectable. How this can be when the charges are perfectly free to move in a conductor and completely unable to move any large distance in an insulator may seem paradoxical.

Without excessive mathematical detail, only a qualitative answer to this paradox can be given. Consider a long conductor near a positively charged body as at (a). The positive charge attracts negative charge toward the near end as shown. Even though the charges are perfectly free to move on the conductor, the negative charge that can build up on the end of the conductor nearest the positive charge is limited because the negative charges repel one another and cannot completely bunch up at one end. In fact, the negative charge on the end of the rod can never build up to a magnitude greater than the attracting positive charge. The charge induced on a conductor, then, is limited by other factors than how easily it can move in the conductor.

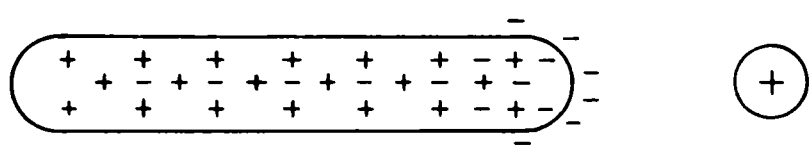


Figure (a)

Consider the insulator at (b), made up of neutral atoms (rather large!). When a positive charge is placed near it as at (c), the negative charges in all the atoms are displaced ever so slightly toward the positive charge.

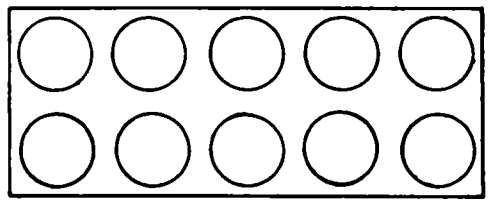


Figure (b)

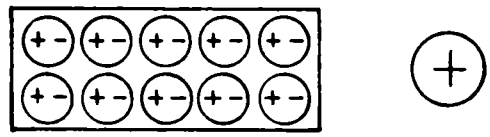


Figure (c)

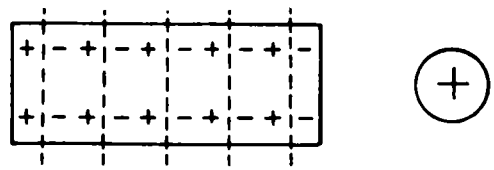


Figure (d)

The net result is a neutral region in the center and a net negative charge at the end closest to the external positive charge. There is a net positive charge at the opposite end. An easy way to see the effective charging of the two ends of the bar is to regroup the adjacent + and − charge pairs, omitting those closest to the ends, as in (d). Now we see that one of the end surfaces is predominantly positive, the other negative. In between, all charges are neutrally paired. Thus the "dielectric polarization" of the entire rod is equivalent to a migration of charges from one end to the other, as occurs in the case of a conductor.

Many electroscopes have a metal case that performs two principal functions which are somewhat related. The first is that of shielding. A charge completely surrounded by a conducting case can have no forces exerted on it by charges outside the case. This shielding is perfect if the case completely surrounds the charge, but it is very good even if rather large openings are left in the case. Even if charges are placed on the case the force on a charge inside remains zero.

The second function of the metal case is to increase the reproducibility of measurements made with the electroscope. As long as charges from rubbed rods and combs are being detected, no great gain is made with a metal case. But if charges are to be placed on the electroscope with a battery, then there must be something to which each end of the battery can be connected. In principle, one battery lead could be connected to the electroscope and the other to

the earth, but then the reading would vary with the distance of the leaves from the earth. The metal case provides a constant reference point to which the other end of the battery can be connected, thus making the readings reproducible.

Since a battery is a very complicated device, it is probably not wise to attempt a detailed explanation of the mechanism by which it works. It should simply be explained as a device which transfers negative charge from its positive to its negative terminal. (Actually positive charges may also be moving within the battery.) The battery continues this transfer of charge until the repulsion between the accumulated charge on the terminals and any additional charge of the same sign, counteracts the chemical forces. Then the process stops. If some process, external to the battery, removes negative charge from one terminal and places it back on the other terminal, the battery will keep on transferring charge until it runs down.

The students should be brought to understand that the battery does not produce charge. It simply moves it. At no time does the battery accumulate a net charge. With batteries, the words "charge" and "discharge" are unfortunate since they seem to imply the storing of charge within a battery. What is stored is chemical energy which can be used to do work, i.e., move charge even against the electric forces tending to recombine positive and negative charges. *After the students have studied the next chapter* it might be well to emphasize this point with the following: A typical 100 ampere-hour automobile battery will transfer $100 \times 60 \times 60 = 360{,}000$ coulombs during one discharge. The force between two 360,000-coulomb charges placed a foot apart would be roughly 10^{18} tons. Surely that net "charge" was never in the battery. You wouldn't dare comb your hair near it, or put two "charged" batteries anywhere near each other!

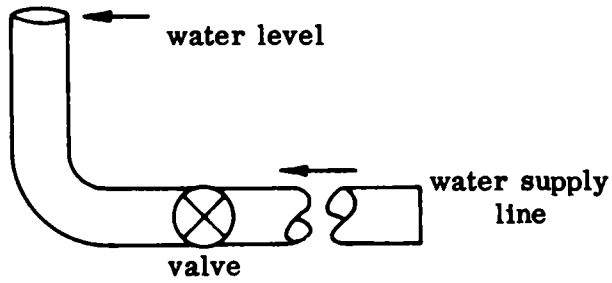
Often students wish to know how fast charges flow. Each case is a complicated problem in itself, but a few order of magnitude statements can be made. The individual charges usually move very slowly; drifting along in a typical electrical circuit at a few centimeters per second or less (see Section 21–4C). On the other hand, patterns of charge rearrange themselves on conductors very rapidly, a good order-of-magnitude estimate being with the speed of light. If a charge were put on one end of a metal rod from, say, a piece of fur, that amount of charge would be distributed more or less evenly along the rod in the time it takes light to travel the length of the rod, sometimes somewhat longer (perhaps 100 times as long), never less. Now the individual charges from the fur do not travel along the metal with this speed. One of the original charges pushes a like charge in the metal which in turn pushes its neighbor so that the *electrical effect* travels with a speed which approximates that of light.

One way of clarifying this point is to present the example of an open standpipe full of water, with a closed valve at its bottom end, as shown.

When the valve is opened, water begins to overflow the top of the pipe. From the rate of overflow and size of pipe, we can calculate the speed of flow in the pipe:

$$\frac{\text{volume}}{\text{second}} = (\text{area of pipe}) \times (\text{speed of flow}).$$

We may now ask the question, "How long a time lapse is there between the instant that the faucet is opened and the instant when water overflows the pipe?" If the standpipe was initially empty, the answer is simple:



$$\text{time lapse} = \frac{\text{length of pipe}}{\text{speed of flow}}.$$

A different answer is obtained if the pipe was initially full. There is a time lag between opening the valve and the beginning of the overflow. The lapsed time is the time required for the pressure wave to travel from the faucet to the open end. This time is determined essentially by the length of the pipe and the speed of sound through water.

In the case of the flow of electrical charges through conductors, the conductors are already "full of charges" (free electrons). Thus when new charges are fed in one end, other charges flow out the other end almost immediately. The lag is determined by the speed of propagation of an electrical impulse through the metal. This is of the order of the speed of light. No single charge moves nearly that fast, a typical "drift speed" being about 0.02 cm/sec.

Sections 18–6, 18–7, and 18–8 discuss briefly the mechanisms of charge transport (electric current) in gases, liquids, and solids and to present briefly the evidence for these mechanisms. The aim in presenting these sections is not to provide an encyclopedic coverage of electrical conduction but to make it clear that there are many ways in which electric charge can move in addition to conduction in metals, that there are various charge carriers and conducting mechanisms, and to further familiarize students with the atomistic picture of electricity.

These sections will stimulate as much discussion as you wish to devote to them, but keep an eye on your time schedule. Limiting your development to the material presented in the text will provide enough depth to prepare students for the material which is to come. Probably the most important material for future reference is the evidence presented in Section 18–8 for charge transport in metals by relatively free electrons.

EXPERIMENT

Electrified Objects 34

This experiment requires little time and equipment and may be done at home. It is best done before the study of Chapter 18. If your students are already familiar with the simple behavior of positively and negatively charged objects, it may be done as a demonstration. In any case, it prepares the student for the three following experiments, which assume a familiarity with simple electrostatics.

The success of Experiments 34 to 36 depends greatly upon the relative humidity in the laboratory. If it is about 50 percent or less, the experiments will work well, but if it is about 80 percent or more, charges will leak off across insulators, making the experiments difficult to do. To reduce leakage in high humidity, store the apparatus in a large box heated by an electric bulb or in a heated room, and remove it just before use. Vinylite, which takes a negative charge when rubbed with a dry piece of paper, and cellulose acetate, which becomes positively charged when treated the same way, develop strong charges and work better than glass, Bakelite, or other plastics when the humidity is high. However, if the surface becomes dirty, it may be difficult to keep charge on a rubbed plastic strip. Charge cannot be easily removed from vinylite and cellulose acetate by rubbing the surface with the hand. Passing the strip quickly over a flame will discharge it.

The two different strips can be hung from a crossbar attached to a ringstand. They should be far apart and far enough from the metal of the ringstand so they will not interact with each other or with the induced charges on the ringstand.

Students may find that a charged plastic strip will always attract an uncharged strip, although they are not told to look for this effect until the end of the experiment. This inductive attraction is not easy to understand until induction has been studied in the text and in Experiment 35. The explanation should probably be deferred until later.

Answers to Questions

A vinylite strip rubbed with a dry piece of paper repels another similarly rubbed vinylite strip, and attracts a cellulose acetate strip that has been rubbed similarly. There appear to be at least two kinds of electric charge.

As a result of bringing acetate rubbed with paper near another similarly charged strip, one can conclude that the like charges we used repel each other and the unlike charges attract.

At this point the student may make the assumption that there are only two kinds of charge, which can be given names such as *a* and *b*, *p* and *q*, or plus and minus. He can then try to explain all further results on the basis of this assumption.

Almost any reasonably good insulator when rubbed will become charged to some extent, and the student can identify its charge as being the same as or opposite to the charges he has found by observing its effect on charged vinylite and cellulose acetate strips.

The general conclusions resulting from the experiment are:

1. There are two kinds of charge.
2. Like charges repel, opposite charges attract.

The names are of no importance. It makes no difference which we call "plus" or which we call "minus" (or "a" or "b").

If a student has observed that both kinds of charged plastic attract an unrubbed object, he is justified in concluding that an unrubbed object has a "charge" that attracts all rubbed objects but does not affect another unrubbed object. He will later find that there is really no new kind of charge involved.

Unless the student is sophisticated in matters electrical, you should not expect him to give an adequate explanation of how uncharged paper or thread can be attracted. The question is given to him simply to make him aware that all is not so simple as it seems.

Students should be encouraged to try charging various substances with different materials. They will conclude that there is no easy way to predict what charge substance x will gain when rubbed with material y. They are right! In fact, depending on the humidity and cleanliness of the surface, some substances can be charged with either sign when rubbed with the same material.

The difference between insulators and conductors can be learned by having the student try charging metal rods. The charge will leak off as fast as it is formed; but if the rubbed metal rod is held by a good insulator, it will become charged like the plastic but the charge will spread all over the rod and will not stay localized as on an insulator.

Apparatus

Dry pieces of paper
2 Cellulose acetate strips
2 Vinylite strips
1 Ringstand, 36″
1 Rod, 24″
1 Clamp holder
Cellulose or masking tape

EXPERIMENT

Electrostatic Induction 35

Electrostatic induction is not easy for students to understand. It may be best to do this experiment at the same time or shortly after the topic is studied in the text. Very able students however, may find it a challenging problem before they have learned about induction. The purpose of the experiment is to familiarize the student with the separation and transfer of charges. In the last part of the experiment he will become familiar with the behavior and use of an electroscope.

If metal rods about 10 cm long and a centimeter in diameter are not available, other similar objects such as spoons, table knives, or pieces of metal tubing may be used. Wires also may be used, but their capacity is so low that they produce very small deflections of the suspended foil. In any case, try to avoid using objects with sharp points or edges since they lead to corona discharge or spark discharges between the objects and the charging plastic strips. If such effects occur, the rods must be discharged and the experiment repeated.

If the humidity is high, to prevent leakage it may be necessary to support the rods on pieces of low-molecular-weight polyethylene.

The sign of the charge on the electroscope can be determined by bringing charged vinylite and cellulose acetate strips close to the instrument.

A leaf electroscope or a "soda straw" electroscope is most sensitive for small deflections. If it is charged to more than about a 45° deflection, it will show little change when another moderately charged object is brought close.

If there is sufficient time, students may try the following variation on the separation of charges by induction: charge two insulated metal rods by induction as in the first part of the experiment, then charge one electroscope by contact with one rod and another electroscope with the other rod. The two

electroscopes will then discharge completely when their terminals are connected. The electroscopes must be quite sensitive to show this clearly.

It might be pointed out to students that when a charged object picks up bits of insulating materials like paper and thread, the attraction is primarily the result of the separation of charge like that in a conductor rather than polarization, the distortion of charges in individual molecules. A perfect insulator would show only the weak attraction resulting from polarization. In ordinary insulators there is sufficient conduction so the greater charge separation resulting from conduction masks the effects of polarization.

Answers to Questions

The tin foil is attracted to the near rod (since they have opposite signs), and is repelled by the far rod (since they have like signs).

The rod may be attracted (by induction) to either rod. (There was the same amount of charge on each rod, but of opposite sign; when the rods touched, the charges neutralized each other.)

The charge on the rod is now opposite to the charge on the plastic.

Apparatus

2 Metal rods, 6″
2 Beakers (200 to 500 ml)
Dry pieces of paper
1 Cellulose acetate (or vinylite) strip
1 Ringstand, 36″
1 Rod, 24″
1 Clamp holder
Thread, aluminum foil, thin cellulose or masking tape
Electroscopes, several

Demonstrations

Charge Transfer by an Oscillating Pith Ball

This demonstration can be done after Experiment 34 to illustrate the transfer of charge by a conductor. Suspend a conducting pith ball or a small square of aluminum foil midway between two aluminum foil plates as shown in Fig. (a). Give the plates opposite charges by using charged plastic strips. Allow the pith ball to touch one of the plates. It will become charged by conduction, be repelled, and fly across to touch the opposite plate. There it will lose its charge, become oppositely charged, and again be repelled. It will oscillate back and forth until the plates are nearly neutral.

A Shielded Electroscope

The effect of shielding may be shown by using two coffee-can electroscopes, a type of electroscope generally available from manufacturers of PSSC Physics equipment. Leave one of them intact and remove the soda straw and its mount from the other, supporting it with a ringstand and burette clamp in a battery jar as shown in Fig. (b).

Pass a charged plastic strip around the battery-jar electroscope, letting the class see that the soda straw moves whenever the strip is brought near any

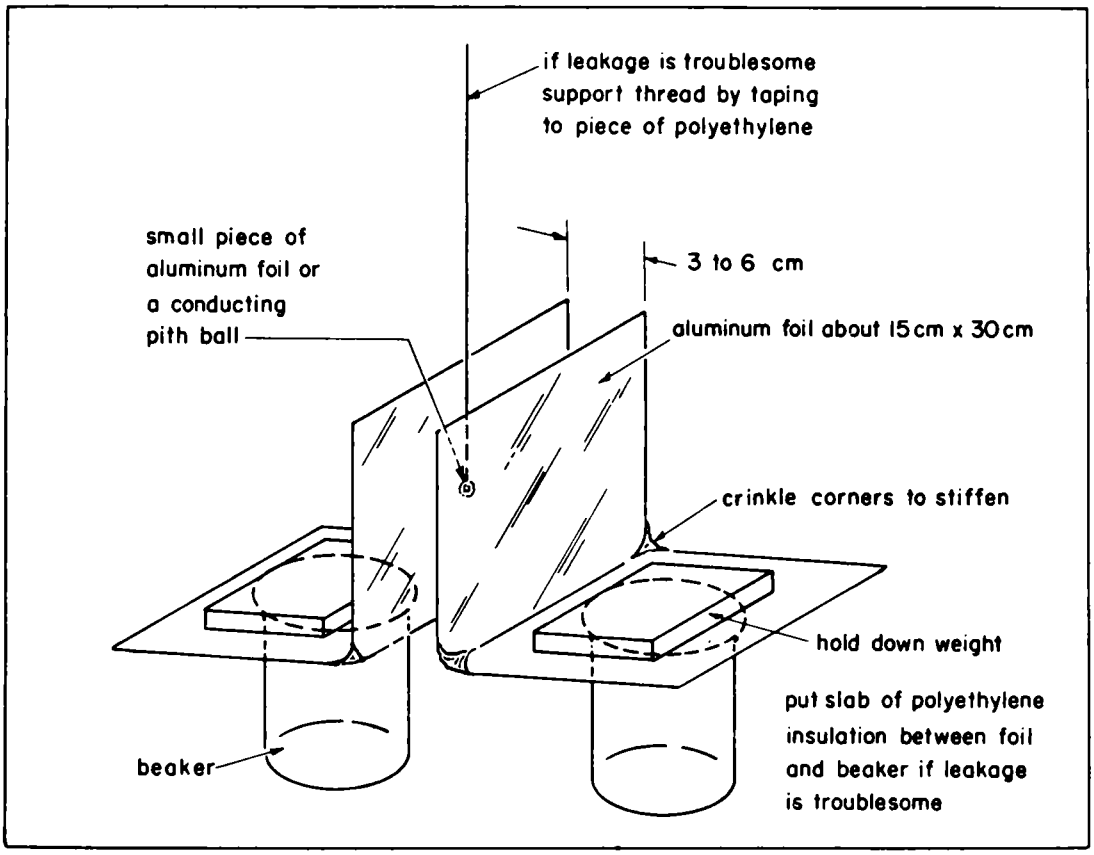


Figure (a)

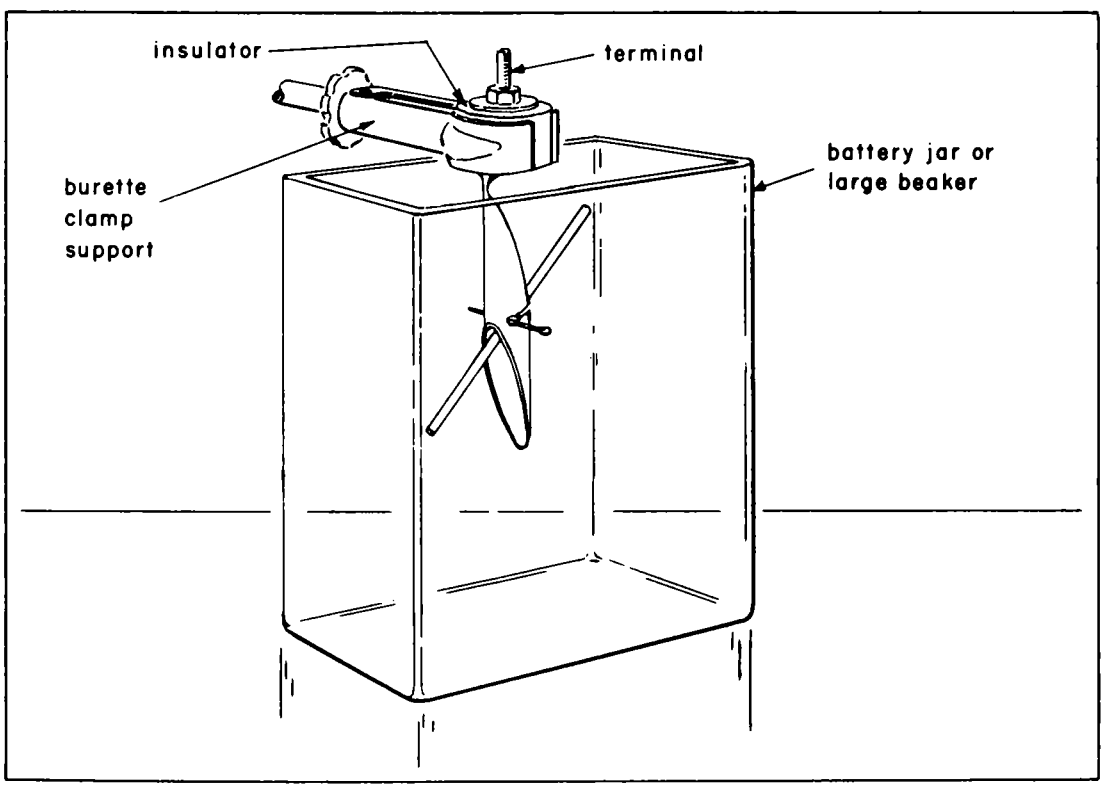


Figure (b)

part of the glass case. Now pass the strip around the coffee-can electroscope to show that the soda straw moves only when the strip is near the terminal or directly in front of the electroscope.

Separation of Charge Using Tape

Two electroscopes, each with a horizontal metal plate attached to the insulated terminal, are needed for this demonstration.

Fasten a 1 to 2-inch piece of vinyl electrical tape to the plate of one electroscope. Hold the electroscope with one hand and peel the tape off without touching the plate. The electroscope will become charged. Without touching the bottom of the tape or the plate of the second electroscope, stick the tape to the plate of the second electroscope to show that the tape is also charged. If the two plates, with the tape still stuck to the second, are then touched together, both electroscopes will discharge.

This demonstration shows that charge separation depends on close contact rather than on rubbing against a force of friction. It also shows that whenever a given amount of charge of one sign is separated from a body, an equal amount of charge of the opposite sign is left on the body.

Discharge of an Electroscope through a High Resistance

Students too often think of electrostatic phenomena as having little relation to the phenomena of current electricity. This demonstration will show that the charge from rubbed plastic can flow in a current similar to the current produced by a battery.

First, show that the rate of discharge from a charged electroscope is negligible. Then, with pinch clothespins, connect a 3 to 10-inch piece of magnetic recording tape between the insulated terminal of the electroscope and the case. When the electroscope is charged, a current will begin to flow through the tape, discharging the electroscope in 5 to 100 seconds, depending upon the initial charge, length of tape, and humidity conditions.

The Transfer of Charge by an Ionized Gas

Hold a burning candle or match near the terminal of a charged electroscope. The positive and negative ions produced by the chemical reactions taking place at high temperature in the flame will discharge the electroscope rapidly.

An intense radioactive source will also ionize the gas and discharge the electroscope. However, sources strong enough to discharge an electroscope rapidly are dangerous to handle. The luminous face of a clock, placed close to the open chamber of an electroscope, will discharge it in about 10 minutes. The glass face should be removed, since it stops alpha particles which produce a large fraction of the total ionization.

Deflecting a Stream of Water

A simple demonstration showing the effect of induced charges can be done by bringing a charged rod near a fine stream of water. An eyedropper tube, mounted in a one-hole rubber stopper and held in a burette clamp supported by a ringstand, can be used as a nozzle to give a fine stream from the faucet. When a charged rod is brought near the stream, the stream will be deflected and drops may tear away from it and circle the rod.

Other, more complex experiments involving the effect of charged objects on streams of water will be found in the book "Soap Bubbles" by C. V. Boys, in the Science Study Series.

The Conductivity of Liquids

This series of demonstrations, designed to accompany Section 18–7, shows that some liquids contain ions and are conductors while others are insulators.

Most of these demonstrations utilize the apparatus shown in Fig. (c) connected to a 110-volt A.C. line. To reduce the hazard of electric shock, the carbon rods can be fitted into a pair of holes drilled in a block of wood resting across the top of the container. The rods will stay in place if tape is wrapped around them close to their tops. If small arc-light carbons are used, connections to them can be made with battery clips. The carbon electrodes from No. 6 dry cells are more convenient, since they have screw terminals on their tops. However, dry-cell carbons should be thoroughly cleaned before use.

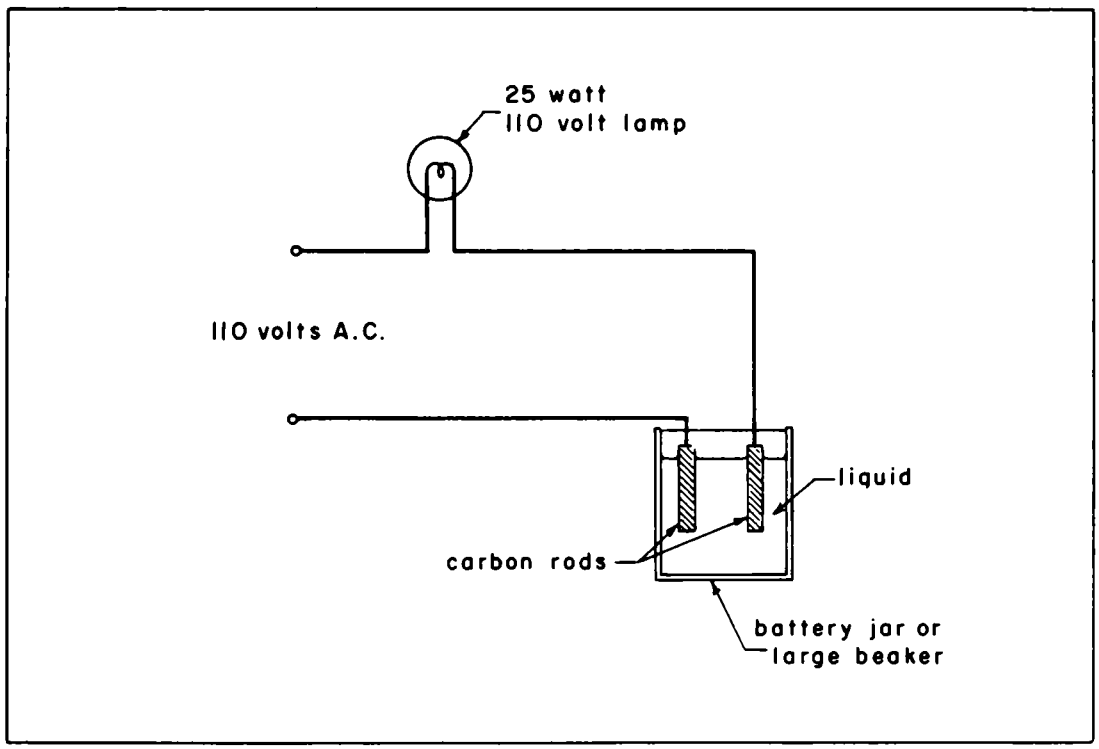


Figure (c)

A clean container, pure water, and clean electrodes must be used for each demonstration. If your tap water contains too high a concentration of dissolved salts, use distilled water.

Fill the cell with water. The lamp will not light. Show that the circuit is in operating order by shorting the cell with an *insulated* screw driver. (It should be pointed out to the students that most water contains some dissolved salts and therefore cannot be trusted as an insulator.) Add some sugar to the water and stir with a glass rod. The lamp will not light, since sugar does not form ions when dissolved in water.

With the electrodes rinsed and in pure water, add drops of concentrated sulfuric acid from a pipette or piece of glass tubing until the lamp lights.

Table salt added to pure water will show that it also forms ions, making a conducting solution. Acetic acid can be used and will show only slight conductivity, barely lighting the bulb because of the small amount of ionization.

If the carbon electrodes are placed in a saturated solution of copper sulfate and supplied with low-voltage D.C., copper will deposit on the cathode. Reversing the current will remove the copper from one electrode and deposit it on the other. This suggests that copper ions are positively charged.

Some students will probably want to electroplate various objects. Results, in general, will be disappointing unless special precautions are taken. Handbooks and other references give details on various plating solutions.

A More Complex Demonstration for Classes That Have Studied Chemistry

Prepare, before class, solutions of barium hydroxide and sulfuric acid as follows: Dissolve 6.4 gm of $Ba(OH)_2 \cdot 8H_2O$ in 200 cm^3 of water to make 0.2N solution. Mix 5.6 cm^3 of concentrated sulfuric acid with 1,000 cm^3 of water to make about 0.2N solution.

Using the same apparatus as before, half-fill the cell with water and, with 110 volts across the apparatus, add 10 cm^3 of the barium hydroxide solution and stir. The lamp will now light.

Next, add (while stirring) about 10 cm^3 of the 0.2N sulfuric acid. Barium sulfate will precipitate, fewer and fewer ions will remain available for conduction, and eventually the lamp will become extinguished.

If you now add more sulfuric acid, the lamp will light up again; this time you can extinguish it by carefully adding barium hydroxide solution. The process can be played back and forth as long as your patience, your supply of acid and base, and your time permit.

Don't attempt to do all the above demonstrations unless you have plenty of time and, in any case, don't try to explain all the details of what happens when salts, bases, and acids ionize and conduct current. At this stage in the course, the only object of these demonstrations is to show that some solutions contain mobile charged particles that make them conductors.

Coulomb's Law and the Elementary Charge

CHAPTER
19

Chapter 19 puts electrostatics on a quantitative footing. Through a discussion of the Coulomb experiment leading to Coulomb's law and a discussion of charge sharing and the forces between charges, the idea of charge is made quantitative. The similarity between Coulomb's law and the law of gravitation suggests introducing the concepts of electric field and electric potential. The existence of a natural unit of electric charge is established through analysis of a Millikan "oil drop" experiment. A large-scale version of this experiment then enables us to determine the electric force between two elementary charges. This is equivalent to determining k in Coulomb's law. This makes it possible to measure ordinary sized charges in terms of the number of elementary electric charges. Finally, the chapter discusses experiments leading to the determination of the number of elementary electric charges on particles such as the electron, alpha particle, and proton.

Experiment 36 The Force Between Two Charged Spheres is a high-priority experiment and should be done as an introduction to Chapter 19. The experiment uses charged pith balls to investigate the relation between electric force and separation, leading to the conclusion that there is an inverse-square relation. Incidentally, the experiment employs an elegant measurement technique —measuring the displacements of shadows—for a situation in which the charge would be lost if a measuring instrument were put directly on the objects.

Understanding charge sharing is important in order to follow the development of Coulomb's law in the text. Students' understanding of charge sharing can be checked by asking how to divide a charge into thirds. An arrangement like is satisfactory. Such an arrangement as is not. Recognizing the symmetry in a situation is important in physics and particularly in electricity. In the second situation above, not all three balls are equivalent. In fact, the students should know that the two outer balls will receive the greatest charge. Dividing a charge into fourths can be accomplished by halving, then halving the "halves." An arrangement like , and a tetrahedron will also work; but not .

To say that a particular "charge" can be divided into two equal parts by charge sharing, presupposes the idea of conservation of charge which is mentioned explicitly in Section 19–9. If we touch an uncharged metal sphere to an identical charged one, symmetry tells us only that the final charges on the two spheres are equal but not that their sum is equal to the original charge. The text has already developed the idea of electrical *particles* quite thoroughly,

and from this point of view conservation of charge reduces almost to conservation of electrical particles, which should be accepted quite readily. Do not worry your class with this unless they raise questions.

Classwork on Coulomb's law can be handled by working through such problems as:

Two charged metal balls A and B repel each other with a force of F newtons.

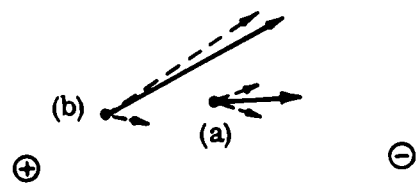
(a) Suppose we cut the charge on *one* ball in half. Will it make any difference which ball we choose to divide the charge? (No.) What will be the force between the balls if the charge on one ball is cut in half? ($\frac{1}{2}F$)

(b) What will be the force between the balls if the charge on *both* balls is cut to half its original value? ($\frac{1}{4}F$)

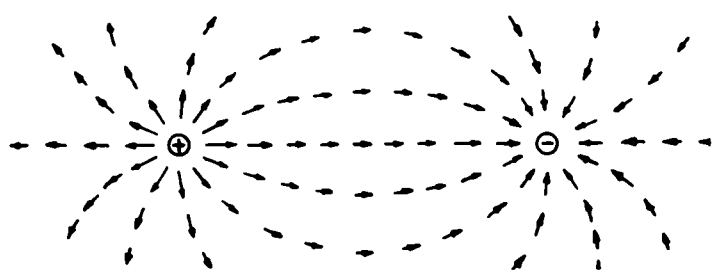
(c) With the charge on each ball reduced to half the original value, how much closer will the balls have to be moved to restore the original force? ($\frac{1}{2}$ the distance)

The concept of a *field* was introduced in Chapter 12 to describe the gravitational properties of the region in the neighborhood of the earth. Here the idea is applied to the description of the "electrical condition" of some region in space. It would be appropriate to reintroduce the concept of field with a specific example:

Suppose we have equal positive and negative charges as shown below, and we ask what the force would be on a small positive charge placed at (a).



Even without performing a detailed vector addition one can see that the positive charge pushes up and to the right, and the negative one pulls down and right. The ups and downs cancel, leaving a force to the right as shown. At (b) the force up and right due to the positive charge is so much stronger that it produces the net force shown. Continue drawing "vectors" for a few more points until you can draw the force pattern on the small positive charge *anywhere* as shown below.

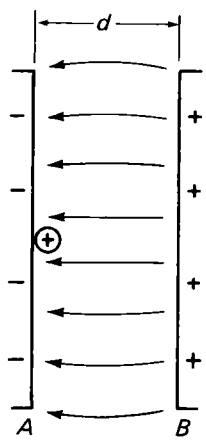


This *pattern* of forces is called the electric force field or even just the electric field in the region of the positive and negative charge.

Only two electric field configurations will be used extensively during the remainder of the course: (1) the field due to a point charge (this should be well understood from Sections 1 and 2); and (2) the field between two parallel conducting plates which are oppositely charged. The uniformity of the field between charged plates can be inferred from the observed behavior of the plastic spheres in the Millikan experiment, as well as observed in the photograph of Fig. 19–6c.

There should be little trouble with the quantitative expressions for electric field and potential energy if the corresponding expressions for the gravitational field have been understood. However, the concept of electric potential is new and should be stressed; the definition is fundamental, and grasping it now will head off difficulties that arise later when potential difference is encountered in the motion of charges. For many students, the subject of field and potential will remain vague until they can use realistic situations. In the meantime, be sure to assign Problems 12 and 13 and try to devise some examples to make the students use the terminology, such as the following:

Two flat plates, A and B, are charged, so that there is an electric field of E nt/unit charge in the uniform region between them. A small body of mass m carries a charge $+q$, and is situated adjacent to plate A. The plates are d meters apart.



What is the force on the small body? ($q\vec{E}$ newtons.)

How much work is done if the body is moved from plate A to plate B? (qEd joules.)

Suppose the potential energy of the body is zero at plate A; what is the potential energy at plate B? (qEd joules.)

What is the difference of potential between plate A and plate B? (Ed joules/unit charge.)

If the body is released at plate B, with what speed will it strike plate A? $\left(\sqrt{\frac{2qEd}{m}} \text{ meters/second.}\right)$

Now, as a preliminary to the Milliken experiment, the idea is developed that a small sphere driven in air by a constant force has a constant velocity which is directly proportional to the driving force. Since the electric part of the driving force is proportional to the electric charge, we can compare electric charges by observing the velocities with which a standard sphere is driven. Knowing the weight of the sphere and observing its velocity under the driving force of gravity, we can calibrate the apparatus in familiar force units.

The process of calibration also experimentally confirms that the electric force on a charged object between two charged plates is the same at all points, and is directly proportional to the number of batteries (later voltage) across the plates.

Experiment 37 Driving Force and Terminal Velocity should be done in connection with Section 19–5 and is a prerequisite to the major experiment, 38. Here the students establish the linearity between terminal velocity and driving force which forms the basis for the Millikan Experiment measurements. The experiment can carry most of the teaching of this section and will provide the background necessary for the following sections. If the Millikan apparatus is not available, the film "The Millikan Experiment" provides an excellent coverage.

Basically the electric force is not vital to the material of the first part of the section on measuring small electric forces. The whole point is to prove that the velocity of a small sphere moving through air is proportional to the driving force acting on it. The physics involved is really hydrodynamics rather than electricity. The electrical force is simply used as a convenient way of doubling and tripling the force. Of course the use of the electric force prepares the way for Section 19–6, where it will be all-important. Your class might enjoy knowing that Millikan knew that velocity was proportional to driving force before he began his experiment. The viscous drag of the air on a sphere moving through it is know to be $6\pi krv$ where r is the radius of the sphere, v is its velocity, and k is the viscosity of the air. It is this drag force which balances the gravitational and electrical forces so as to produce uniform velocity.

A word of warning with regard to the dynamics involved in the motion of the plastic spheres: Be careful in your choice of the words for describing the forces that act on a sphere; the *net* force on the sphere is zero when it is moving at constant velocity. Consider a sphere moving upward with constant speed, v_0. The gravitational force acts downward and has a magnitude F_g. The electrical force acts upward and has a magnitude F_{el}. It is then tempting to say that there is a net force acting on the sphere, the net force being given by $(F_{el})_{up} - (F_g)_{down}$. *This is not true. The net force acting on any object which is moving with constant speed is zero.* To calculate the true net force, one must take into account, besides the gravitational and electrical forces, the retarding force of the air resistance: $[(F_{el})_{up} - (F_g)_{down}] - (F_{air})_{down} = 0$. It may then be useful to use the term "driving force" to indicate the resultant of gravitational and electrical forces. The net force will be zero.

The ideas in this section are simple. They are understood when students can answer the following kinds of questions:

A steel marble will drop through syrup at a constant speed (why constant?) of 10 cm per second. How fast will a glass marble fall if it is of the same size but half the weight? Ans.: Half as fast—though bright students should probably see that the buoyant force of the syrup should be taken into account, just as the text mentions that the buoyant force of air must be considered for extreme accuracy in the plastic sphere case.

If the steel marble is pulled upward, by a string, with a force equal to its weight, how fast will it move? Ans.: Zero speed. The driving force is zero; if the ball had been moving, the viscous force would immediately stop it.

If the string pulls up with a force twice as great as the weight, how fast does the marble go? Ans.: The driving force is an upward force as large as the weight; consequently the marble moves up at 10 cm per second.

The Laboratory Experiment 28 The Millikan Experiment should be used to carry the teaching load of the section on the elementary charge. Direct student experience with the equipment and the measurements indicating the existence of an elementary unit of charge cannot be surpassed by any other technique of teaching. The experiment, using plastic spheres, is now simple enough that good measurements can be obtained in one laboratory period, but it must be preceded by Experiment 37. If the equipment is not available to you, the film "The Millikan Experiment" should be used instead. If you do perform the experiment, this film makes an excellent summary and ties down any "loose ends" that may still exist.

Students should not be allowed to get so involved in the descriptive material of the experiment that they lose sight of what it shows. Some of your students may have heard the Millikan oil drop experiment described as an experiment to measure the charge on the electron. That is the way it is usually described. It is *not* to be thought of in that way here. This experiment shows that in a particular electric field, a particular plastic ball shows changes in velocity and hence, changes in the force on it. These changes in force come in small integral multiples. Since the force is proportional to the charge on the ball, the conclusion is that the charge cannot have just *any* value, but must be subject only to discrete changes which are integral multiples of a fundamental charge unit. In our development we have not set up the coulomb, so we need not measure our elementary charge in terms of any existing unit. Instead, we adopt this elementary charge as our fundamental unit charge.

Millikan, who had no plastic spheres, was faced with a much more formidable task than the authors of the text. He used small oil droplets whose size was unknown. If a drop is made of oil of density d and has a radius a, it has a mass $\frac{4}{3}\pi a^3 d$ and gravity exerts a force on it of $\frac{4}{3}\pi a^3\, dg$. If it falls with a velocity v, the retarding force on it is, from hydrodynamic considerations, $6\pi kav$, where k is the viscosity of the air. Then if the drop falls with a constant velocity v

$$\tfrac{4}{3}\pi a^3\, dg = 6\pi kav.$$

Measuring the velocity of fall allows a calculation of a, the radius of the drop. The experiment proceeds from this point on in a way very like that described in the text.

A fine firsthand account of Millikan's experiments is given in his book, listed under "Further Reading" on page 443. Some of your more enterprising students might be interested.

The discussion of the large electric balance extends two ideas on electric forces between parallel plates: how they change with the area of the plates (they don't), and how they change with separation of the plates (inversely as the separation for the same battery applied). It might be well to ask students to summarize what they know about electric forces on charged objects between metal plates. They should come up with something like the following:

The electric force on a charged object placed between two parallel metal plates is:

(1) independent of the position of the object between the plates, i.e., the same everywhere (as long as the object is not near the edge of the plates).

(2) independent of the area of the plates as long as the plates are large compared to their separation.

(3) inversely proportional to the separation between the plates for the same battery applied.

(4) directly proportional to the number of batteries connected in series between the plates (later this will be defined as proportional to the voltage across the plates).

(5) directly proportional to the charge on the object.

Students should not be left with the idea that any of this is proved deductively in this course. The best we can do is state the facts and verify them by observing charges between plates. You can tell your students that all these statements can be proved, though the development is too long for the present text and properly involves the use of calculus or differential equations. At this point it will be well to leave the proportionality of force between the plates and the number of batteries as simply experimentally demonstrable, and not go ahead to the addition of voltages.

All the statements made about forces on charges between parallel metal plates apply only to the case where the battery is kept connected. Consider the case given in the text. A battery is connected between two metal plates, resulting in a force being exerted on a test charge placed between them. The distance between the plates is doubled, the battery remaining connected. The force on the test charge is halved. The charge which is now on the *plates* has decreased to half its former value. As the plates were moved apart, charge actually flowed back through the battery.

On the other hand, if the battery is disconnected and the plates are then moved apart, the force on the test charge remains unchanged. In this case the charges on the plates had no place to go and hence remained the same.

You might introduce the discussion of the measurement of k in Coulomb's law by asking your class to calculate the force between two elementary charges placed 0.1 mm apart. When they find they lack knowledge of the constant, ask them how to measure it. The problem always comes down to how to count or measure the number of excess elementary charges on an object which is large enough to permit making a force measurement. In order to discourage a proposal of a direct measurement you may have to give the class the value of $k = 2.3 \times 10^{-28}$ newton meter^2/elem charge^2. As soon as the class realizes that the measurement is not easy, you can discuss the experiment in the text.

You will probably wish to summarize the logic of the measurement something like this:

(1) We know the force per elementary charge between two standard plates (from the Millikan experiment).

(2) We scale up the experiment, multiplying both the plate separation and the number of batteries by 100, so that the *force per elementary charge is the same* as before (and hence known).

(3) We place one of two large identically charged spheres between the charged plates and use a spring balance to measure the electrical force exerted. This force divided by the force per elementary charge gives us the number of elementary charges on the sphere.

(4) We measure the force of repulsion between the two identically charged

spheres at a given distance. We can then solve for k in

$$F = k\frac{q_1 q_2}{r^2}.$$

Students should see that k, 2.3×10^{-28}, is the measure in newtons of the force between two elementary charges at a distance of one meter. It will be important to place this force in scale, which you may do after the next chapter when you can compare the magnitude of this force with the gravitational force between two protons at the same distance.

Notice the difference in development used in this text and that usually encountered. Most books *define* an electrostatic unit of charge to be that charge, q, which, when placed 1 cm from an identical charge, exerts a repulsive force of 1 dyne (10^{-5} newton). That is, Coulomb's law is written $F = q_1 q_2 / r^2$. (The constant k is thus set equal to one.) Then the charge on the electron, the elementary charge, must be measured as some fraction of an electrostatic unit (4.8×10^{-10}). Other books define a unit of charge called a coulomb in terms of magnetic forces on currents. Since the unit of charge is found in some other way, the constant k must be measured. Also the charge on the electron must be measured (1.602×10^{-19} coulomb).

Our book adopts the idea that nature supplies a natural unit of charge and that we must measure the factor k thus defined by Coulomb's law. In this scheme there is no such thing as "measuring the charge on the electron." *This section describes a means of measuring k.* For the purposes of atomic physics, this scheme is superior both conceptually and practically. For the use in electrical engineering the charge unit is inconveniently small.

It is probably best to tell students that the scheme for measuring k as described in this section is not one that is actually used. As the film "Coulomb's Force Constant" shows, this experiment is possible, but much more accurate methods exist. The actual methods involve a whole series of very indirect measurements which would be impossible to describe at this level. This is in contrast to the Millikan experiment which, at least when it was first performed, was the best method for determining the charge on the electron in terms of the previously defined electrostatic units of charge.

The basic notion of a conservation law is probably very easily accepted by the majority of high school students. They have an intuitive feeling for the conservation of objects which can be exploited here. The nomenclature "plus" and "minus" makes the idea of pair creation less distasteful.

The final section confirms what most students will have already suspected, that the electron carries a single elementary charge unit. As a matter of fact all elementary particles we know today carry no charge or a single elementary charge. Physicists know of no reason why this is so. It is simply a fundamental, empirical fact.

The material of this section can easily involve you in rather deep discussion of pair creation and other related subjects. Don't get into that unless you are competent to get out!

EXPERIMENT

The Force Between Two Charged Spheres 36

This is an important experiment and should not be omitted. If it is done before Chapter 19 is studied, the student will discover for himself the inverse-square nature of electric forces.

The experiment is delicate and difficult to perform, and the results will probably have to be analyzed at home. A few practice runs, taking data but not analyzing it, are advisable so a quick final run can be made before leakage becomes excessive. Excessive leakage can be prevented by making sure that all insulating parts (including the charging strips) are kept clean.

Under very humid conditions you can set up the pith-ball arrangement and apply heat to the inside of the box for about three minutes with an infra-red drying lamp (reflector type) before doing the experiment. If the effect of this drying wears off before the students can get enough data, you can apply the lamp again.

The pith balls are coated with a colloidal suspension of graphite in alcohol (commercially known as "Dag 154") or with India ink. (Aluminum paint is *not* a good conductor.) A single nylon fiber is used to support the hanging pith ball because it is a good insulator and weighs much less than the pith ball. If the supporting threads are too heavy, they will hang in a curve (a catenary) when the pith ball is deflected. As the deflection increases, the curve will become more and more pronounced and the horizontal deflection will not be proportional to the force exerted; large errors will occur.

If the nylon fiber is not straight, this can be remedied by stretching it with a firm pull. The pith ball is then attached by threading two fibers through a fine needle hole poked in the ball. Apply a bit of glue to the fibers and pull the glue inside the ball. After the glue has dried, the short ends of the fibers may be cut off. Both fibers should come out of the same side of the ball.

Both spheres are given charges of the same sign by conduction from the electrophorus. If the spheres are given opposite charges and attract each other, they may swing together, touch and discharge, making the experiment more difficult to perform and yielding results which are not very satisfactory. The two spheres should be at the same level above the base, and when taking readings they should not be close to the walls of the box, the hand, or other objects since induced charges will cause errors.

The student may find that if he brings the spheres very close together, the inverse-square relation seems to break down. This is because the inverse-square law holds for point charges. The spheres behave like point charges only when separated by distances several times their diameters. When very close, the balls cease to act like point charges because of the separation of charge on the spheres by induction. [See Fig. (a).]

Since the distance between corresponding sides of the spheres equals the separation of the centers of the spheres, the readings of the spheres' positions are taken from these sides because they can be read more accurately than the centers.

Answers to Questions

At the end of the run, the sphere on the base can be moved to the position where the first reading was made. If there has been any leakage, the deflection of the hanging ball will be less than it was originally.

The graph of the forces as a function of separation will not, of course, tell the student at a glance the algebraic form of the function. However, the force at any distance r should be close to one-quarter the force at $\frac{1}{2}r$ and close to $(\frac{2}{3})^2 = \frac{4}{9}$ the force at $\frac{2}{3}r$. This suggests an inverse-square dependence, and a graph of F as a function of $(1/r^2)$ should give a straight line.

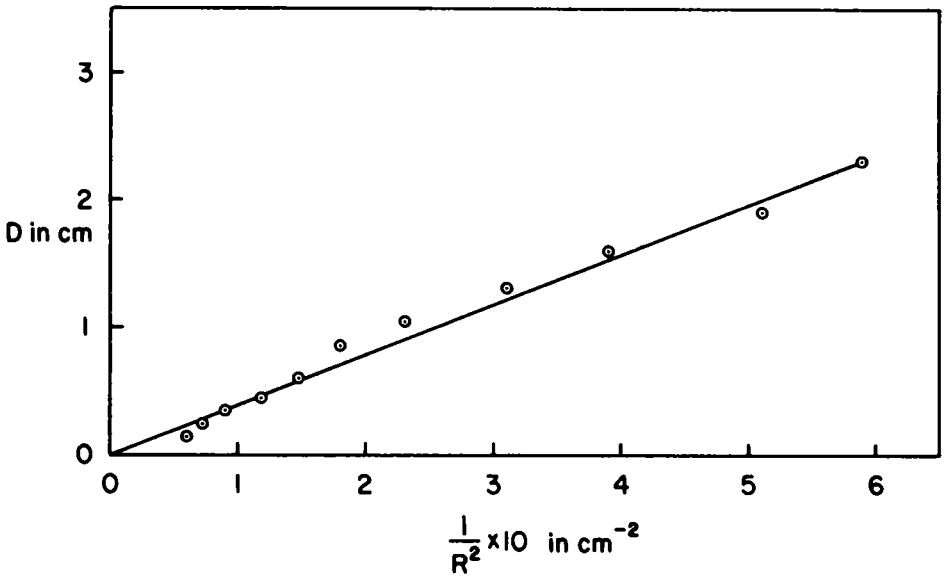


Figure (a)

Apparatus

1 Coulomb's Law Apparatus Kit, consisting of
Box with mirror, scale, base, and lid
Styrofoam spheres, 0.5 cm diameter
Nylon fibers
Plastic strip with charging cloth or paper
Electrophorus disc with handle
Block with sphere on insulating rod
Dag 154 suspension of graphite

EXPERIMENT

Driving Force and Terminal Velocity 37

This experiment is designed to familiarize the student with the Millikan apparatus, to help him feel at home with this kind of measurement, and to show evidence for the proportionality of driving force and terminal velocity. The experiment first establishes that the spheres reach a terminal velocity very quickly and thereafter move at constant velocity. This constant terminal velocity is then seen to be directly proportional to the driving force.

Data can be taken most efficiently if one student watches a sphere and also starts and stops the stopwatch as the sphere passes two references marks. A second student should note down the watch reading, the direction of motion, and whether the plates are charged. Note that the microscope inverts as well as magnifies: therefore, spheres will appear to "fall up" under the action of gravity.

The spheres are supplied as a concentrated (10 percent solids) water suspension that should be diluted in the ratio of one part suspension to two parts water. One teaspoonful or less of this mixture is enough to put in the atomizer jar for best results. The diluted mixture can be poured into a clean container

and saved for future use. The jar and particularly the atomizer nozzle should be cleaned frequently. The jar is cleaned by rinsing it several times in clean water, and the nozzle by blowing water through it until the spray is clear.

The plates should be charged from a well-filtered D.C. power supply or battery giving about 500 volts. The source impedance should be as high as possible for safety reasons; the current drain is small, because the only load is a 10-megohm bleeder resistance across the plates. Four 90-volt "B" batteries will serve equally well. If you use batteries, be sure the connectors are well insulated and that a 1-megohm safety resistor is connected in series with the output to increase the source impedance.

It is most satisfactory to perform the experiment in a darkened room. The contrast between spheres and background can be controlled by rotating the light source around its pivot.

Answers to Questions

The spheres move upward in the field of view.
All move in the same direction.
No acceleration is apparent; they move at constant velocity.

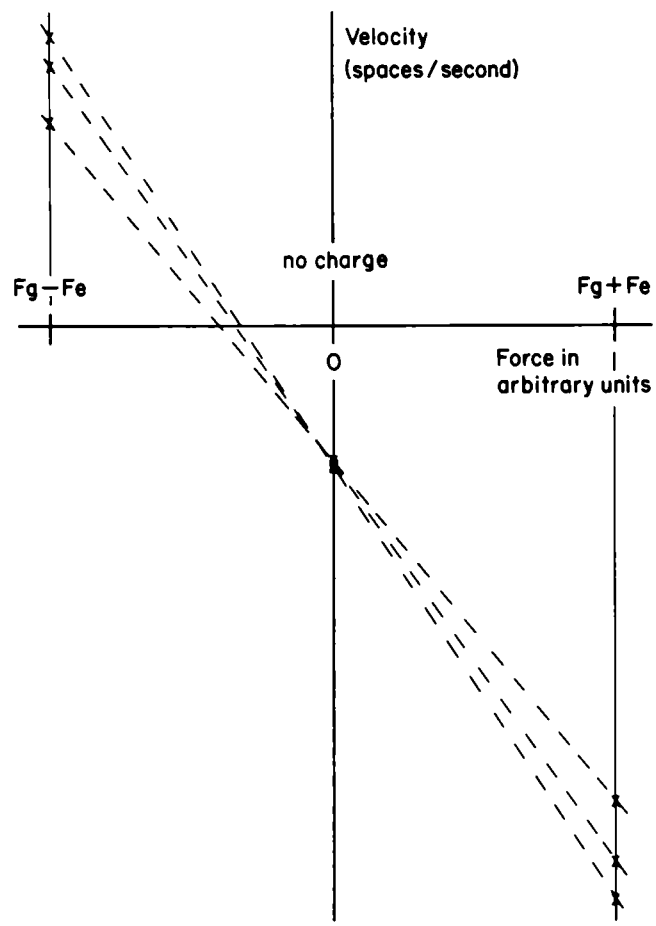


Figure (a)

With the charging switch centered, gravity and the air resistance are the only forces on the spheres. They are equal and opposite.

Putting charge on the plates makes some spheres move faster, some slower, and some reverse their direction of motion.

Reversing the direction of the electric field will reverse the direction of most spheres.

The value V^+ is greater than the terminal velocity under gravity alone, while V^- is less. Often V^- is in the opposite direction.

Each graph is a straight line. They differ in their slope because there are different charges on the spheres shown in Figure (a).

The speed is directly proportional to driving force.*

Apparatus

1 Millikan apparatus
1 Stopwatch
1 Power supply (500-volt D.C.; 6.3-volt A.C. for lamp

EXPERIMENT

The Millikan Experiment 38

In this experiment the student makes measurements of the speeds of a number of small plastic spheres in an electric field. The distribution of velocities is seen to cluster around multiples of a definite unit velocity, which suggest that the charges also come only in multiples of a unit charge.

The apparatus is the same as in the previous experiment, and the same words of advice and caution apply. Again, a team of two is essential. Note that there is no need to make measurements of speeds in free fall. It is advisable, however, to make several (say, three) pairs of measurements on each sphere and average them.

Remind your students first to let the electric field clear away the spheres that are moving too fast for accurate timing and carry too many charges to offer good evidence for the discreteness of charge. After a while only a few spheres will be visible. It is best to track the slowest ones. Having chosen a sphere, the student puts it through its paces by reversing the field after each timed run. The student who is observing the spheres should operate the stopwatch, while his partner reads it and records the results. The observer must keep the sphere in view while the watch is being read.

The time for a single sphere to fall 10 divisions is approximately 35 seconds. By using only spheres with this falling time, F_g will be the same for all of them and a direct analysis can be made from the measured velocities.

It is recommended that 370 volts be used across the plates. With this voltage the times for calculating V^- and V^+ will be more spread out than if a higher voltage is used. The approximate times for spheres with 1, 2, 3, and 4 elementary charges to move 10 divisions against F_e will be, respectively, 140, 25, 13, and 9 seconds.

Since $V^+ + V^- \propto 2F_g$, students may check their data to see that they did have single spheres by comparing $V^+ + V^-$ with $2\ V_{\text{free fall}}$. However, this is not necessary.

* Some of the spheres are fragments or clumps of spheres and will have different terminal velocities because of their different masses. Clumped spheres will fall faster than single spheres.

Answers to Questions

The difference $V^+ - V^-$ is proportional to $(F_g + F_e) - (F_g - F_e) = 2F_e$. Since F_e depends only on the charge, $V^+ - V^-$ can be used as a measure for the charge.

Figure (a) shows the results of a set of readings. The clustering of the readings around multiples of a smallest speed is good evidence for the existence of a natural unit of charge.

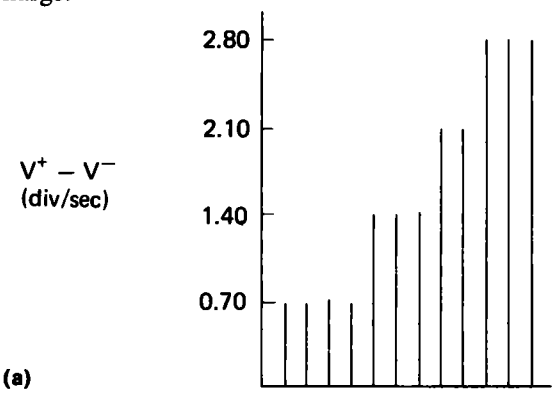


Figure (a)

In this set of data the smallest charge consisted of one elementary charge. If single charges were absent from the sample, the ratio of the two lowest speeds would be 3:2, and not 2:1.

The speeds corresponding to one elementary charge may vary from group to group because of slightly different plate spacings and potential differences. Therefore, to make it possible to compile the readings of the whole class, each group has to normalize its data and express them in terms of the speed corresponding to one elementary charge.

Apparatus

1 Millikan apparatus, 1 Stopwatch
1 Power supply (500-volt D.C.; 6.3-volt A.C. for lamp)

Demonstration: Electrostatic Fields

This demonstration is described briefly in Section 3. A 4–15 kilovolt neon sign transformer, suitably protected, is the source of the electric fields. The output voltage can be varied by means of a *Variac* autotransformer connected as shown in Fig. (a); for most patterns an output of 4 kilovolts is adequate.

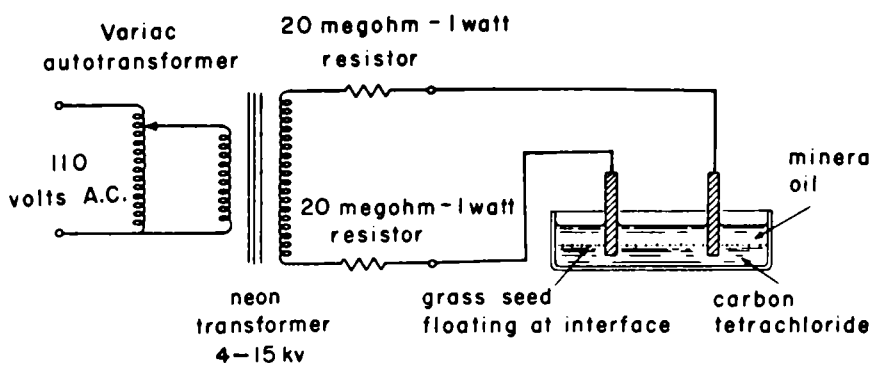


Figure (a)

The 20-megohm, 1-watt resistor in each high-voltage lead is connected directly to the high-voltage terminals of the transformer and is wrapped with several layers of vinyl electrical tape so the transformer terminals cannot be touched. Do not use resistors smaller than 1 watt; they may be so short that current will flash over and short-circuit them. It may be advisable to enclose the transformer and protective resistors in a box, bringing the insulated high-voltage leads out through holes in the box.

The grass seeds must be small enough to be easily aligned by the electric field (length about $\frac{1}{8}$ in.). A type called "Colonial Bent" works well.

Pour carbon tetrachloride to a depth of about 5 to 15 mm in a Pyrex glass pie plate or similar insulating container. Sprinkle about $\frac{3}{4}$ of a teaspoon of grass seed over the liquid surface (about 1 level teaspoon for each 500 cm^2 of surface). The seed will gather in clumps, but this is normal.

Since carbon tetrachloride evaporates rapidly and its vapor is toxic, it is necessary to cover the carbon tetrachloride with a layer of another liquid. Mineral oil, which is less dense than grass seed and carbon tetrachloride, is used for this purpose. Pour mineral oil slowly down a stirring rod held in the liquid so the oil flows evenly over the carbon tetrachloride, forming a layer 5 to 15 mm deep. The grass seeds will be pushed out to the edges of the container. Gentle stirring will distribute the seeds evenly over the entire area.

Place the electrodes in the container so that they extend below the interface of the two liquids. The electrodes can support themselves in the container; can be supported by a wooden frame from above; or can be held in place by stiff wires which also connect them to the transformer.

When using two parallel plate electrodes or concentric cylindrical electrodes, you should place identically shaped pieces directly beneath the container and connected to the corresponding voltages. This prevents the fringing field from the bottom of the electrodes in the dish from distorting the predicted field pattern. One of the electrodes must be grounded (the outer one when concentric cylindrical electrodes are used).

High temperatures from high-power illumination may cause boiling; it is better to use fluorescent lamps for direct observation with a black background and side lighting. The patterns can be projected as shadow patterns on the ceiling, but the electrodes may not show up clearly, especially if auxiliary electrodes beneath the container are used.

Films

"Coulomb's Law," by Eric M. Rogers of Princeton University. This film shows the inverse-square dependence of electrical forces, and demonstrates the consequences of this law. Use with Sections 1–3. Time: 28 minutes.

"The Millikan Experiment," by Alfred Redfield of IBM and Francis Friedman of Massachusetts Institute of Technology. This film presents a direct photographic record, with commentary, of the experiment described in Sections 19–5 and 19–6 of the text. All pertinent quantities are measured. This film can be used to great effect as a summary after the two related student experiments, 37 and 38. Running time: 30 minutes.

"Coulomb's Force Constant," by Eric M. Rogers of Princeton University. This film shows how macroscopic experiments are interpretable in terms of Millikan's experiment, and derives the value for the constant k in Coulomb's law. This film parallels Sections 19–7 and 19–8. Running time: 34 minutes.

CHAPTER

20 Energy and Motion of Charges in Electric Fields

Chapter 20 considers the kinetic and potential energies of individual charged particles in electric fields, and introduces the concepts of current, EMF, and potential difference. Throughout, the major concern is with the motion of elementary charges.

Batteries in this chapter are essentially "black boxes" which furnish a constant amount of energy per unit charge. Our atomistic approach tells us something about what must be happening in the way of charge-exchange reactions within the cell, but we are not specifically concerned with the chemistry of the cells or with the design of practical batteries.

At this initial stage, in order to avoid obscuring the central points, several assumptions which underlie the development given in Section 1 are not stated explicitly. For clarity at this early stage they should not be taken up with the class. Some of them come up a bit later in the course. We mention them here so that you may be forearmed if questions arise.

(1) We assume that the electric force acting on a charged particle which is moving between two flat metal plates is constant, *independent of the speed of the particle.* This fact is not obvious, but can be verified by experiment. One definitive test is described in Section 4, where the increase in thermal energy of a barrier struck by moving charges is shown to be equal to the predicted kinetic energy of the charges. This is also shown in the film "Elementary Charges and the Transfer of Kinetic Energy."

(2) We assume that energy conservation holds on a microscopic scale, so that the work done on the charged particle does in fact only go into an increase in the particle's kinetic energy. This is justified on the basis of its consistency with *all* the known facts.

(3) We assume that the kinetic energy of the particle is given by the expression $\frac{1}{2}mv^2$. This holds exactly only for particles moving with speeds small compared to the speed of light. When the accelerating potential becomes high enough, the motion of the particle can be described accurately only with relativistic dynamics.

The time-of-flight experiment to measure the masses of elementary particles is completely analogous to the methods used in Chapters 15–17, where we were concerned with the energies of macroscopic objects. If we know the force acting on a body and the distance the body moves in the direction of the force, we know the work done on the body. If energy is conserved, this work must

produce an equal change in the energy of the body—in this case, in the body's kinetic energy, since there is no other place for it to go. The situation is exactly the same as that for a Dry Ice puck or any other object accelerated by a constant force. If we know the force, the distance, and the mass, we can deduce the final speed; if we know the force, the distance, and the final speed, we can deduce the mass.

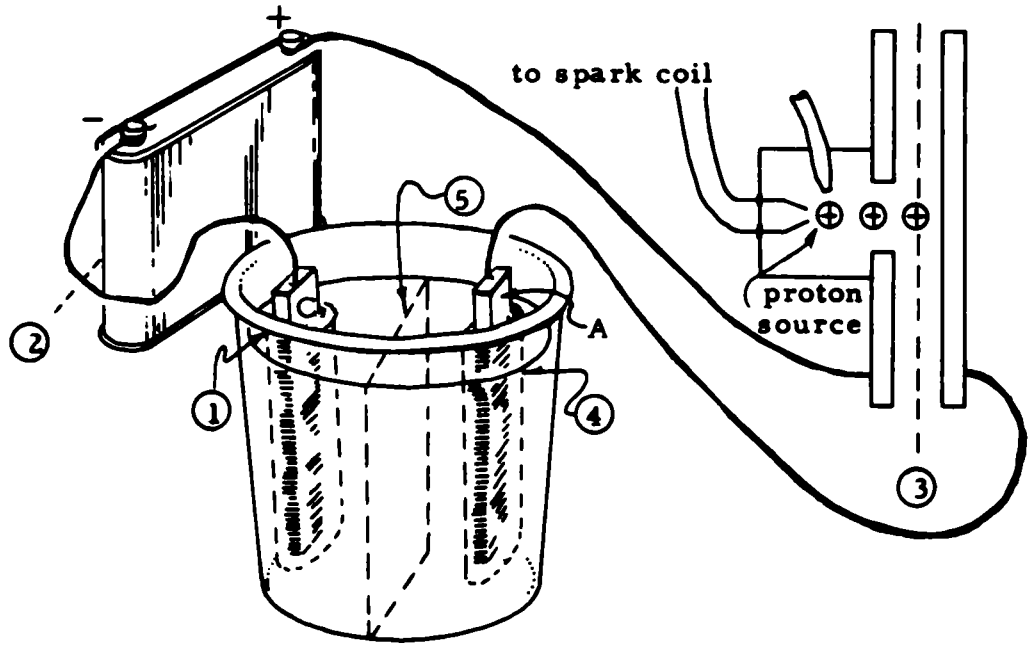
The potential difference used to accelerate the ions or electrons determines the kinetic energy per elementary charge; it follows that what we measure is the mass per elementary charge. In fact, this is all that is determined in any experiment in which the motion of charged particles in an electric field (or in a magnetic field or both) is observed. All such experiments effectively measure the acceleration in a situation where we know the force per elementary charge but not the force on the particle. To find the mass itself we need to know the charge from an entirely different kind of measurement. For particles as abundant as electrons and protons this can be done directly by counting the particles and measuring the charge transported by a beam of particles (Chapter 19). For rarer particles, or for individual particles, one must resort to less direct methods. For instance, the densities of the bubbles in a bubble chamber track and of developed grains in the track in a photographic emulsion depend on the charge of the particle which makes the track (the density depends also on the velocity). Indirect arguments such as charge conservation can also be used. For example, if we observe a hydrogen atom to break up into an electron and a proton, we infer that the electron and proton have equal and opposite charge.

The results of the mass measurement on the electron, the proton, and the hydrogen atom suggest that a hydrogen atom is made up of a proton and an electron. The analogy to a sun and a much lighter planet is obvious, particularly since there is a similar, central attractive force. This model should be easily grasped and requires no discussion at this point.

The measurement of the velocity of an atomic or subatomic particle by a time-of-flight method was preceded historically by methods involving the effect of a magnetic field. This is discussed in Chapter 22 and should not be discussed now. It may be interesting to know that time-of-flight measurements are used frequently in nuclear physics. In the discovery of negative protons, for instance, these particles were distinguished from other negative particles, and their mass determined, by observing the time of flight over a flight path of about 30 feet. This determination of speed is identical to the one we have used. Another measurement (this happened to be deflection in a magnetic field) had to be made in order to determine the mass per unit charge, and it was necessary to establish that the particles in question carried one and only one elementary charge.

Students should grasp the concept of an electric current as comprised of moving elementary charges. However, unless this course is integrated with one in chemistry, the details of the electrolytic reactions need not be stressed.

It probably will be useful to go through a blackboard discussion of the relation between the motion of charges and the current in various parts of a circuit. In doing this it will be wise to use a circuit in which the current is carried by different charges in different parts of the circuit. For example, one can insert the apparatus of Fig. 20–1 into the circuit of Fig. 20–6, as illustrated in the sketch. The polarity of the battery in our sketch is reversed from that in Fig. 20–6. (Note that no charge is injected into the circuit by the spark coil.) Assuming that the ions in the electrolytic cells are all singly charged, and as-



suming that a current of, say, 10^6 elementary charges per second flows around the circuit clockwise, at what rate do various charged particles cross surfaces 1, 2, 3, 4, and 5? The answers are:

(1) 10^6 positive ions, each carrying one elementary charge, cross from the liquid to the cathode each second. The current flows from right to left.

(2) 10^6 electrons per second cross from left to right, opposite to the direction of the current.

(3) 10^6 protons per second cross from the left to the right, in the same direction as the current.

(4) 10^6 negative ions cross this surface per second from the liquid to the anode, opposite to the direction of the current.

(5) Negative ions cross the surface from left to right and positive ions from right to left. All that can be said is that the rate at which negative ions cross and the rate at which positive ions cross *add* to 10^6 per second. This could be, for example, 400,000 negative ions and 600,000 positive ions per second or 700,000 and 300,000 or any other combination. The direction of the current coincides with the direction of motion of the positive ions. (In the above, we have assumed that the ions present are singly charged.)

Someone usually raises the question of the sign convention in currents. For better or worse, a long time ago it was decided to call the charge of a proton positive and that of an electron negative. The only reasonable convention regarding currents is one that a current flowing into a region shall be defined as positive if it increases the net positive charge (or decreases the net negative charge). This implies that a current carried by negative charges (such as electrons) is opposite to the motion of these charges.

Nature has apparently provided an equal number of positive and negative electric charges in the universe. Under some conditions (e.g., ionic conduction in solid salts), electric currents are carried predominantly by positive charges. Under other conditions (e.g., electronic conduction in metals), the current is

carried by negative charges. In some cases (e.g., in p-type semiconductors), the current is actually carried by negative charges, but it is more convenient to picture the conduction process in terms of positive charge carriers ("holes"). In any event, with the spectrum of electrical phenomena which must be described, there is nothing to be gained by reversing our present arbitrary definitions of + and −, although there have been frequent proposals to do just this (usually from those who are concerned principally with electronic conduction in metals and vacuum tubes).

The quantitative study of electrolytic reactions such as that in Fig. 20–10 enables us to determine the charge on ions in solution, and to show that all charges are integral multiples of the same elementary charge (see Problem 18). This is the main point to be made in discussing electrolysis. It is not important in this course that students know, for example, that silver exists in solution as Ag^+ and copper as Cu^{++}. They should know that, by reference to a standard electrode, the charge carried by any ion can be determined. If this course is integrated with one in chemistry, you will undoubtedly wish to expand considerably on this material, correlating the results of such experiments with known chemical valences.

Experimental checks on energy transfer, electric forces, and elementary charges are discussed in Section 20–4, for it is important to see that the assumptions made previously (e.g., that all elementary charges are the same, that the force on a moving charge is independent of its speed) can be verified by directly measuring the kinetic energy carried by a beam of moving charges. However, extensive class discussion should not be necessary. This section can be discussed in conjunction with Problem 18 in HDL. It is only by a quantitative comparison of the results of electrolytic measurements and measurements on accelerated charges that we can assert that all elementary charges are in fact indistinguishable except for sign.

Be sure to consider the similarity between this heating experiment and that in Section 5.

As some students do not understand the meaning of EMF, it would be well to emphasize the sequence of ideas arising in conjunction with batteries, as follows:

(a) A battery is basically a device for converting stored chemical energy into electrical energy.

(b) A battery supplies a fixed amount of energy per elementary charge to a circuit; for example, a 270-volt battery supplies 4.34×10^{-17} joule/elementary charge.

(c) The energy supplied by a battery per elementary charge (defined as the EMF of the battery) in an ion beam accelerated from one parallel metal plate to another, is independent of the distance between the plates, the area of the plates, and the amount of current flowing.

(d) If batteries are connected in series, the total EMF available to the circuit is the sum of the EMF's of the individual batteries.

(e) A battery of EMF $\mathcal{E}$, through which a charge q flows, supplies a total energy $q\mathcal{E}$; if the current (charge per second) flowing is I, the battery supplies power (energy per second) at a rate $I\mathcal{E}$. If I is measured in elem chgs/sec, and $\mathcal{E}$ in joules/elem chg, $I\mathcal{E}$ is in joules/sec (watts).

Unless this course is integrated with chemistry, it will be wise not to discuss the detailed chemical reactions which occur in actual batteries. It is sufficient for our present purposes to consider batteries simply as "black boxes" which supply a fixed EMF.

It will be helpful to point out the similarity between the basic experiment of Section 20–4 and that shown in Fig. 20–11 of this section. In the first, electrons are allowed to gain all the kinetic energy supplied them by a battery hooked between two plates, and then allowed to smash into a metal plate, and the energy delivered is measured by the temperature rise in the plate. In Fig. 20–11 the battery again supplies energy to electrons, but this time instead of accelerating across a vacuum and hitting a plate, they move through a metal resistor, transferring any energy gained to the metal atoms in collisions (see Section 21–4*c*). This leads to a temperature rise which is measured. In both cases the battery was the ultimate energy source; the energy was given to the electrons; they were slowed down by a material which warmed up. The difference was only in the behavior of the electrons—and the logical use to which the results were put.

Extensive use is made of "practical" electrical units; it is easy for students to lose sight of their nature, and it would be well in class to tabulate the essential ideas:

(a) In batteries, chemical energy is transformed into electrical potential energy; a cell gives a fixed energy per elementary charge, as long as its chemical composition and temperature remains constant.

(b) The EMF of batteries is conveniently measured in volts; 1 volt = 1.6×10^{-19} joule/elem chg.

(c) Charges may be measured in coulombs; 1 coulomb = 6.25×10^{18} elem chg.

(d) Current is conveniently measured in amperes; 1 ampere = 1 coulomb/sec.

(e) EMF (volts) × charge (coulombs) = Energy (joules): EMF (volts) × current (amperes) = Power (watts).

(f) The electron-volt (1.6×10^{-19} joule) is a convenient measure of energy on an atomic scale.

The final section of the chapter raises some questions, especially about the nature of the forces between charged particles when both are moving, and sets the stage for what follows; however, it may be treated as a reading assignment.

(No student laboratory experiments are provided for this chapter.)

Films

"Counting Electrical Charges in Motion," by James S. Strickland of Education Development Center, shows how an electrolysis experiment enables one to compute the number of elementary charges passing through a circuit in a given time. An ammeter in the circuit is calibrated. The random nature of the motion of elementary charges is demonstrated using very weak currents of only a few charges flowing per second. In contrast to the electrolysis experiment, charges are counted directly one by one as they flow through a circuit. Running time: 23 minutes.

"Elementary Charges and Transfer of Kinetic Energy," by Francis L. Friedman of M.I.T. Electrons are accelerated in a vacuum through a known distance in a known electric field between two charged plates. The separation of the plates and the strength of the electric field are identical to the plate separation and the electric field used in the film, "Millikan Experiment." The experiment checks the predicted energy acquired by the electrons as they traverse the distance between the plates. This is done by measuring the rise in temperature of the anode as a result of a known current flowing for a specific length of time. This predicted total dissipation of energy, when produced by a mechanical

source, gives the same temperature rise in an identical anode. The elementary charges in an electrolysis experiment are shown to be identical to those in a Millikan experiment. Running time: 33½ minutes.

"E M F," by Nathaniel Frank of M.I.T., shows that the energy transfers demonstrated in the preceding film are independent of the geometry of the electrodes in the diode. The energy per elementary charge delivered by a battery depends only on its chemical constitution. Running time: 20 minutes.

Demonstration: A Time-of-Flight Measurement

A demonstration similar to this described in Section 20–1. The cathode-ray oscilloscope is set at approximately 1 sweep per second. (On a 5-inch tube the spot will move about 10 cm per second.) A steel ball is then rolled down an inclined trough made of two meter sticks and passes over three pairs of aluminum-foil strips, making electric contact between the two halves of each pair [Fig. (a)]. When the first gap is closed, the oscilloscope sweep is triggered.

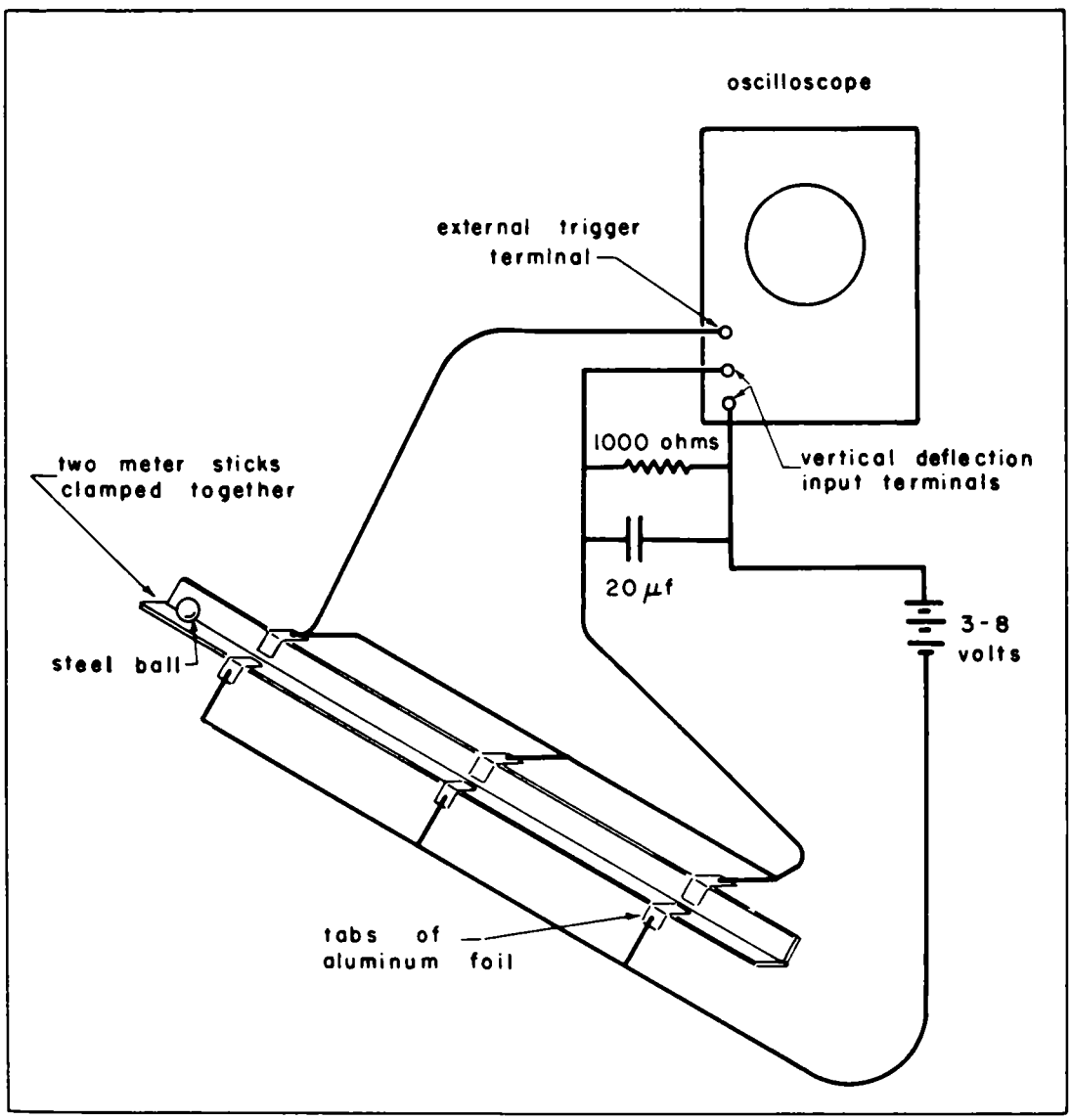


Figure (a)

Shortly after the sweep has started, the rolling ball closes a second gap which momentarily connects a voltage to the vertical deflection plates of the oscilloscope, giving a sudden vertical deflection of the spot as it sweeps across the scope. This marks the instant of passage of the ball across the second gap. The ball closes a third gap at a later instant, again deflecting the spot.

From a knowledge of the distance between the second and third gaps, and the speed of the spot, the average speed of the ball can be computed from the distance between the vertical deflections or spikes on the screen.

The capacitor-resistor combination across the oscilloscope deflection circuit input gives the vertical spike a steep side on the left and a sloping side on the right. When the ball initially closes the gap, the capacitor charges almost instantly from the battery, and the oscilloscope spot shoots upward very steeply. As the ball leaves the gap, the charging voltage to the capacitor ceases, whereupon the capacitor discharges through the resistor, causing the oscilloscope spot to move downward in a sloping curve to the base line, rather than dropping instantly as it would if there were no capacitor in the circuit.

The time constant for the capacitor-resistor combination is about 20 milliseconds. This will make the base of the spike appear to be about 1 centimeter long since, after about 5 time constants, the voltage falls nearly to zero. The spike can be made sharper by reducing the time constant. Use either a smaller capacitor, a smaller resistor, or both. If the spike is made too sharp, the spot will rise and fall so rapidly that it will leave a very faint trace which is not visible to the class.

CHAPTER

21

Electric Circuits

Chapter 21 turns to a more detailed consideration of energy flow in electric circuits, both ohmic and non-ohmic. Throughout, the major concern is with the motion of elementary charges rather than with circuits per se. While the text provides a sturdy base for the analysis of electrical devices, a detailed treatment of Ohm's law and associated problems is aside from the main purpose of the course.

As noted in the text, Ohm's law describes current flow only in a very special (but technologically very important) case—that of electronic conduction in metals at constant temperature. It would be a serious error to consider the development of this chapter as "leading up" to Ohm's law as a climax. Instead, Ohm's law is properly considered as a by-product, having first thoroughly treated the effect of electric fields on individual elementary charges.

We are not concerned primarily with an analysis of what we ordinarily call "D.C. circuits." We are interested in currents as a manifestation of the fact that elementary charges move in a predictable manner under the action of an electric field. We shall need this concept of currents and moving charges to discuss magnetic interactions in the next chapter. Otherwise, this subject is not particularly important to the continuity of the volume.

Laboratory

Experiment 39 The Charge on a Capacitor is intended to follow Section 20–1. It deals with the relationship of the potential difference between the plates of a capacitor to the quantity of charge on the plates.

The ideas contained in Section 21–1 are centrally important to advanced work in electricity, but are not crucial to the continuity in later parts of this course. If this section is treated, it should be done well—one or two days on this section alone—but it may be treated as reading assignment only.

If you cover this section in class, it will be wise to stress the various conclusions in terms of the motion (or lack of it) of individual elementary charges.

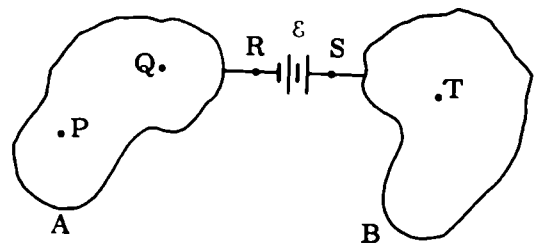
(1) That a single conductor forms an equipotential should be easy to show the class. Simply be sure the logic is definite and clear:

(a) If the situation is a static one—settled down—no charges are moving.

(b) A conductor contains charges which can move if a force is applied. If the charges are not moving, no force is being applied to them and the electric field is zero everywhere inside the conductor (and perpendicular to its surface just outside the conductor).

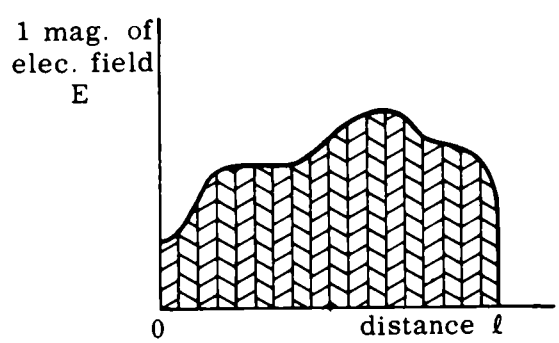
(c) A single elementary charge can therefore be moved from place to place on a conductor without doing work on it. Therefore every point on the conductor is at the same potential.

Sometimes students will ask how good a conductor we are talking about. The question is not how good a conductor, but how long we are willing to wait for the charges to "settle down." Consider some magnets resting on corks which are floating in water. They will quickly arrange themselves into a proper pattern. If we use syrup instead of water, the arrangement will take longer, but they will eventually arrive at the same arrangement. If we wait long enough, even a rubbed piece of plastic will eventually become an equipotential, though it may take months or years.



You may wish to draw two odd-shaped conductors A and B attached to a battery of EMF $\mathcal{E}$. Consider how much work is done in moving an elementary charge from P to Q (none) Q to R (none). Why? Because this is a single conductor—an equipotential. How much work is required to get a charge through the battery, i.e., from R to S? Answer: $\mathcal{E}$ (or $-\mathcal{E}$ depending on the sign), because that is what an EMF means. S to T? (none). What is the potential difference between P and T? ($\mathcal{E}$); R and S? ($\mathcal{E}$); Q and S? ($\mathcal{E}$); etc. The idea to develop is that a battery simply maintains, in an open circuit, a constant potential difference $\mathcal{E}$ between conductors attached to its two terminals.

(2) If students understand the situation when current does not flow, then they should readily understand what happens when current does flow. If no current flows when there is no electric field inside the conductor, then when current does flow, there must be an electric field inside the conductor. The work done in moving a unit charge a distance $\Delta\ell$ under the action of an electric field $\vec{\mathbf{E}}$, in the direction of $\vec{\mathbf{E}}$, is just $E\Delta\ell$. Thus we can easily find the total work done on a unit charge if we know how $\vec{\mathbf{E}}$ varies with distance. We simply add



all the little $E\Delta\ell$'s—this gives us the total work done per unit charge, or the energy per unit charge in moving a distance ℓ. This is just the potential difference between 0 and ℓ; so $V_0 - V\ell$ = area under graph of E vs. distance from 0 to ℓ. We can break any circuit up into an arbitrary number of series intervals, and in this way find the energy per unit charge in each interval. Finally in our circuit we must come to a battery, which supplies an EMF $\mathcal{E}$. The EMF is the energy per elementary charge supplied to the whole circuit. Thus, it must just equal the sum of the potential differences across the various series intervals in the circuit. So

$\mathcal{E}$ = sum of V's around a circuit

= area under graph of E vs. distance for circuit between one battery terminal and the other.

What we have stated, of course, is simply a somewhat formal declaration of the conservation of energy. Thus these considerations must hold for any circuit, no matter how complicated.

The section on the measurement of potential difference is not central with reference to later work and need not be stressed in class discussion. It can be handled as a reading assignment. It is easier to discuss voltmeters after Ohm's law has been covered. See Problem 23 of the HDL, for example.

The change in kinetic energy of a charged body moving in an electric field is equal to the potential difference through which it moves times the number of elementary charges it carries. A knowledge of the actual electric field along the path is not required. This point is very important, but should be obvious from previous considerations. It results in a great simplification in the treatment of many problems. If you have the time to treat the material on electricity thoroughly, the illustrations provided here are very worthwhile. If you are pressed for time, the ideas presented here will not be required in later material.

The reasoning follows from conservation of energy and is therefore not new. It is probably appropriate to develop it by discussion and suggestions by the students. They should be able to arrive at it independently, perhaps through a mechanical analogy. A frictionless roller coaster is a suitable simple mechanical analog.

A roller coaster of mass m starts from rest at A. What is its kinetic energy as it passes B (and beyond)? To find the details of the motion we would need to know the resultant force at each point along the path, but to predict the kinetic energy at B we need to apply only conservation of energy. Therefore we need only to know the change in gravitational potential energy. Conservation of energy tells us that, starting from rest at A, the roller coaster will have, at B, an amount of kinetic energy equal to the change (i.e., decrease) in gravitational potential energy between A and B:

$$\text{Kinetic Energy at } B = (\text{Potential Energy at } A) - (\text{Potential Energy at } B) = mgh_A - mgh_B.$$

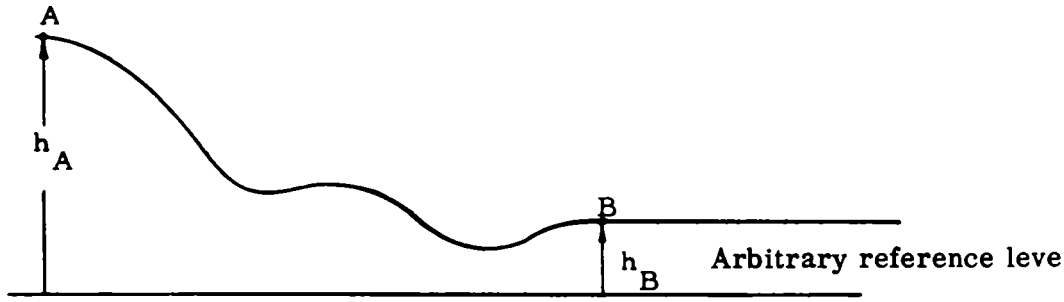
We note that as long as the roller coaster does get from A to B we need not know anything about the potential energy or the force at intervening points.

We note also that it does not matter what we take as the zero of potential energy; only the difference counts.

We can make the analogy even more complete by talking about "gravitational potential difference," i.e., change in potential energy per unit mass.

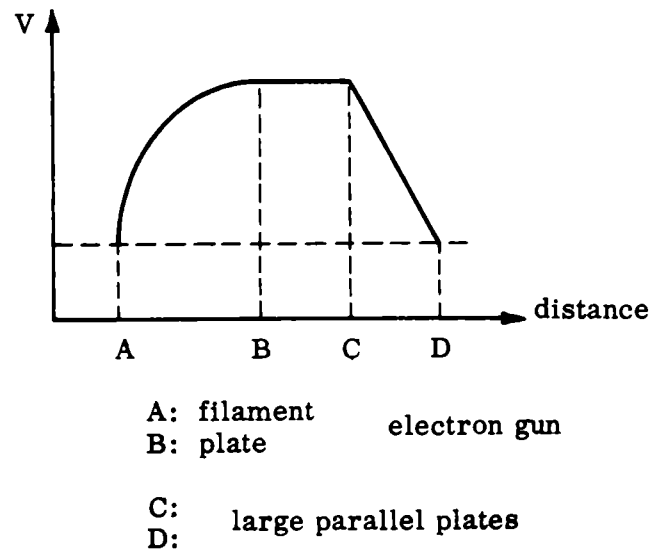
The gravitational potential difference in our example is $gh_A - gh_B$, and we have:

(Kinetic energy at B) = mass × (potential difference) = $m(gh_A - gh_B)$.



It may be interesting to let the student sketch, or help you sketch, the potential along the path of the electrons in Fig. 21–6, with the uniform retarding field adjusted to just stop the electron current. The sketch should look something like the one below.

Why is this "roller coaster" upside down? (The electrons have negative charge, the roller coaster has positive inertial mass.)



The qualitative discussion of the relationship between current and potential difference in ionized gases, in a vacuum diode, and in a metal is valuable because it is one of the few treatments of this subject which puts Ohm's law in proper perspective.

Many students will be interested in reading the material presented. There is, however, little to be gained by extensive class discussion, since a deeper understanding would require a background in atomic physics which is lacking at this stage. The following brief comments may help you in answering questions by interested students.

The device shown in Fig. 21–7 (minus the X-ray tube) is an ionization chamber. It can be used to detect ionizing radiation such as is produced, for instance, by radioactive substances. The passage of a fast charged particle through the gas leaves a trail of electron-ion pairs. The chamber can be used with a current measuring device to measure the average rate at which such charged particles pass through the chamber, or the change in potential difference following the passage of a single particle can be amplified and used to detect the passage of individual particles. With a sufficiently high potential difference, the electrons produce secondary ion pairs and this can result in breakdown of the gas and an appreciable pulse of current. This is a Geiger tube.

The region where the current is limited by space charge is just the beginning of the curves in Fig. 21–10, where the current vs. potential difference curve is concave upward. Crystal diodes exhibit curves which are in some respects different from those for a vacuum diode. Since crystal diodes operate at an absolute temperature lower by a factor of $\frac{1}{3}$ to $\frac{1}{8}$ than vacuum tube diodes, the current falls off much more abruptly when a reversed potential difference is applied. This is because the electrons in the crystal diodes have thermal energies smaller by a factor of $\frac{1}{3}$ to $\frac{1}{8}$ and therefore can be stopped by a retarding potential smaller by approximately this amount. Crystal diodes also do not exhibit saturation, since there is an enormous supply of electrons available.

A quantitative statement of the derivation of Ohm's law from the simple model discussed is not difficult to obtain. We suppose electrons to be making collisions with the positive ions and to start off with a random velocity after each collision. Suppose the time interval between collisions is t. Between collisions each electron will gain an additional velocity equal to the impulse (force $\times$ time $= qEt$) divided by the mass. The average added velocity due to the electric field is then $\frac{1}{2}(qEt/m)$ which is clearly proportional to the electric field.*

* Actually it is not the collisions with positive ions of the lattice which cause the electrical resistance. In a perfect solid all atoms are equally spaced from one another (at the absolute zero of temperature). The collisions are then specular (not diffuse), and do not cause any resistance. Resistance results from imperfections in the lattice. At normal temperatures, these arise mostly from the thermal vibrations of the lattice atoms. When the atoms vibrate, they are not exactly equally spaced, and there is some diffuse scattering. Thus the resistance generally increases with increasing temperature. At very low temperatures, the resistance arises mostly from other lattice imperfections, particularly impurity atoms. The actual treatment of electrical resistance is quite complicated, since only *some* of the electrons in a metal are "free" in the sense that they can move under an electric field. In simple metals, such as copper, about one electron per atom is "free." In some metals, such as germanium, only a very tiny fraction of the electrons are free, and this number decreases sharply with decreasing temperature. This effect far over-rides the effect of temperature on the thermal scattering, so that the resistance of germanium increases sharply with decreasing temperature, and is infinite at absolute zero.

Because of these possible complications, it is best to specify Ohm's law at constant temperature, i.e., that the current is directly proportional to the potential difference *if the temperature of the metal is kept constant.* This law is obeyed over a wide range of voltage and currents for a large number of conductors.

From reading or class discussion of the section on an overall view of a circuit, students should understand the following points:

(a) A circuit consists of a battery which causes charge to move through a number of devices or elements.

(b) All of the energy supplied by the battery to the circuit must be accountable in some form, by heat, by mechanical work, etc.

(c) The EMF of the battery is divided into potential differences across each of the elements connected in series in the circuit. The actual fraction of $\mathcal{E}$ across a particular element depends on the physical nature of the element and (generally) on the current.

(d) If two circuit elements are connected in parallel, the potential difference across both must be the same; if they are connected in series, the current flowing through both must be the same.

(e) The law of conservation of energy shows that the sum of the potential differences around the various series elements of a circuit must exactly equal the applied EMF for any circuit, no matter how complex.

One approach to a discussion of Section 21–5 is through a quantitative example such as Problem 14. The approach in terms of energy exchanges is quite useful even in more complicated circuits, for instance in circuits in which nonsteady currents flow, and energy can be stored in capacitors and inductors. Laboratory Experiments 39 and 40 will be of great help in getting across the essential points of this section.

The very general description of a circuit presented in the text is highly preferable to one which may be simpler conceptually, but is of limited applicability, such as a circuit containing only a battery and a number of resistors. Note that the text's description encompasses all electrical circuits, as long as we are careful to include all sources and sinks of energy as parts of the circuit. For example, if our circuit contains an electric motor, we must also include as a part of the circuit the mechanical system on which the motor is doing work and thereby storing or dissipating energy. If our circuit contains a radio transmitter, we must also include in the circuit all energy radiated into space which is not dissipated in a receiver. Thus we see that our "circuit" is essentially only a closed system (possibly as big as the whole universe in the case of a radio transmitter) in which the total energy is conserved.

Experiment 40 Energy Transferred by an Electric Motor is intended to be used at the start of Section 5. It gives a quantitative view of energy transfer in an electric circuit.

EXPERIMENT

39 The Charge on a Capacitor

Measuring electric charge directly is rather hard. Therefore, an indirect method is used. It consists of measuring an average current and multiplying it by the time it flows.

In this experiment the amount of current passing through the milliammeter for one discharge of a capacitor is very small, and the meter movement has too much inertia to give a reading. However, if many discharges occur in a short time the meter will measure the average current and the charge placed on the capacitor each time can be calculated.

For this method to work, the switch must make contact long enough for the capacitor to charge and discharge completely each time. Later, resistors are

placed in the circuit to slow down the charging rate of the capacitor so that the student can think about the transient time during charge and discharge.

It is important to have the timer motor running smoothly and steadily. The shaft should rub smoothly on the metal clips during the rotating motion. A steady reading on the discharge milliammeter indicates proper adjustment. If the rate of rotation changes, the current reading will change; therefore, readings can be compared directly only at the same rate of rotation. You may want to include a rheostat to allow the students to adjust the potential difference applied to the timer motor.

An extension is included concerning capacitors connected in series and parallel.

Answers to Questions

The milliammeter will jump only slightly on a single discharge of the capacitor, because the momentary flow of current through the meter gives only a small impulse to the meter movement.

It can be assumed that the capacitor collects its maximum charge during each charging.

During a full rotation of the motor, the capacitor will charge and discharge.

The inertia of the meter movement averages out the individual pulses of current.

The charge and discharge rate will be the same as the rotating rate of the timer.

The number of coulombs placed on the capacitor in each charge cycle depends on the potential difference applied to the capacitor. See data.

The ratio of the charge to the potential difference applied remains constant.

The 100 and 200-ohm resistors have no effect on the current. The contact time is long enough to completely charge and discharge the capacitor. The presence of 10^3 and 10^4-ohm resistors in the charging circuit results in less and less current passing through the meter, because the capacitor does not become fully charged during the contact time. See data.

When two capacitors are connected in parallel, their capacities are added. When they are in series, the total capacity is less than that of either alone. In this case $\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2}$ or $C = \frac{C_1 C_2}{C_1 + C_2}$.

Sample Data

NO. OF CELLS TO MOTOR	CHARGING VOLTAGE (volts)	MOTOR FREQUENCY (cycles/sec)	CURRENT (10^{-4} amp)	CHARGE (10^{-6} coul)	CAPACITY (μfd)
2	1.5	92	1.3	1.4	0.93
2	3.0	92	2.6	2.8	0.93
2	4.5	92	3.8	4.1	0.91
1	1.5	48	0.7	1.5	1.00
1	3.0	48	1.3	2.7	0.90
1	4.5	48	1.9	4.0	0.89

With Resistor in Circuit

NO. OF CELLS TO MOTOR	RESISTOR (ohms)	CHARGING VOLTAGE (volts)	MOTOR FREQUENCY (cycles/sec)	CURRENT (10^{-4} amp)	CHARGE (10^{-6} coul)
1	100	4.5	48	1.9	4.0
1	200	4.5	48	1.9	4.0
1	10^3	4.5	48	1.8	3.8
1	10^4	4.5	48	0.7	1.5
2	100	4.5	92	4.0	4.4
2	200	4.5	92	4.0	4.3
2	10^3	4.5	92	3.3	3.6
2	10^4	4.5	92	0.6	0.7

A 1.0 μfd and a 0.25 μfd were connected in parallel and then in series

CONNECTION	VOLTS	CURRENT 10^{-4} amp	FREQUENCY cycle/sec	MEASURED cap'y in μfd	CALCULATED cap'y in μfd
Parallel	6.0	5.1	60	1.4	1.25
Series	6.0	.8	60	.22	.20

Apparatus

1 Timer with electric switch modification
1 Milliammeter 0 – 1 ma. Full scale
4 1.5-volt dry cells
1 1.5-volt No. 6 dry cell
1 Capacitor paper 1.0 μfd
1 Capacitor paper 0.5 μfd
4 Resistors, $\frac{1}{4}$-watt carbon, 100-ohm, 200-ohm, 10^3-ohm, 10^4-ohm

EXPERIMENT

40 Energy Transferred by an Electric Motor

Mechanics and electricity are too often kept apart in the mind of the student. This experiment offers an opportunity to relate the two by concentrating on the energy transfer rather than on the internal working of the motor.

The motors used vary considerably in their characteristics and the data your students obtain may differ quite a bit from the sample data given in this Guide.

Answers to Questions

The EMF of the cell is 1.5 volts. This will not change very much under load if the cell is fresh.

From the sample data, at a speed of 0.42 m/sec it takes 3.6 second to lift 6 washers 1.5 m.

The energy supplied by the battery is $\mathcal{E}It$ = 1.5 volts × 0.81 amp × 3.6 sec = 4.4 joules.

The change in potential energy of the washers is 1.5 m $\times$ 0.02 kg $\times$ 9.8 m/sec^2 = 0.29 joule.

The energy supplied by the battery is about 15 times the change in potential energy of the washers.

The current is nearly constant for different speeds and the same load. (See sample data.)

A graph of current as a function of speed is shown in Fig. (a) and of efficiency as a function of speed in Fig. (b).

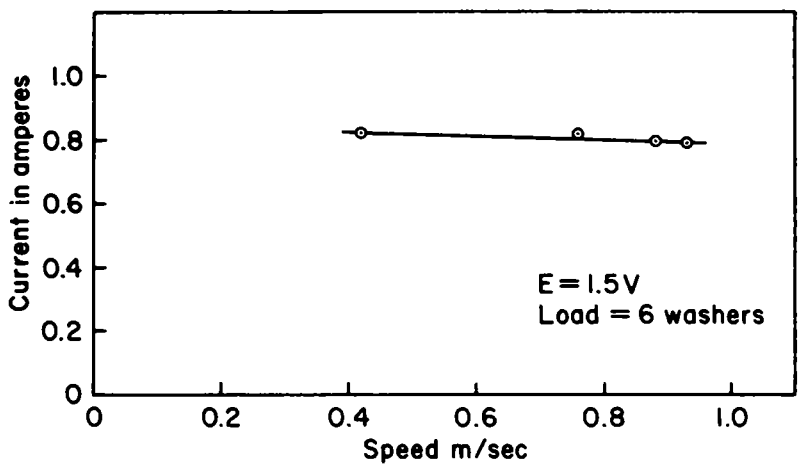


Figure (a)

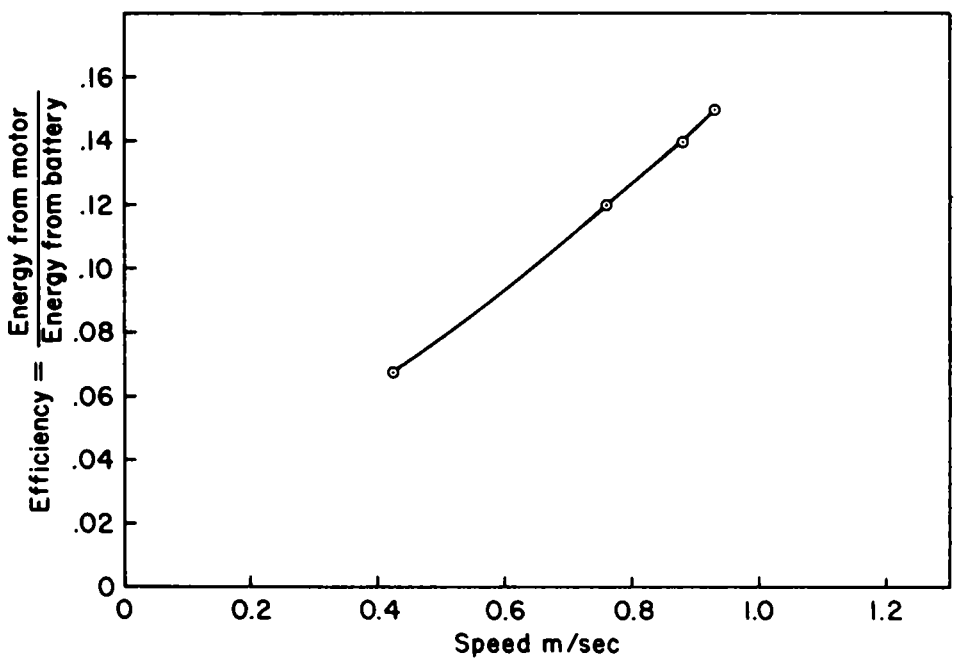


Figure (b)

A graph of efficiency as a function of the number of washers is shown in Fig. (c). The efficiency is nearly constant up to about 4 washers and then drops rapidly.

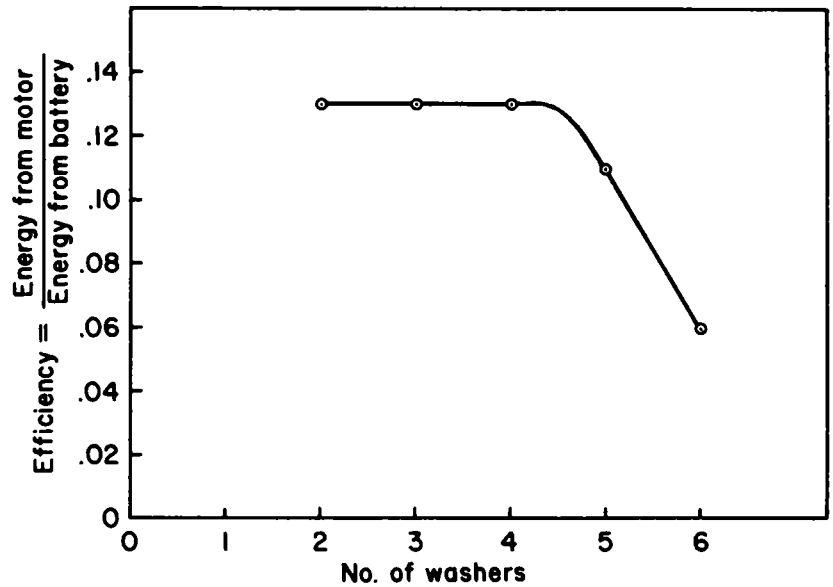


Figure (c)

The energy that is not converted into potential energy in lifting the washers appears as thermal energy in the resistance and motor.

Sample Data

Mass = 6 washers = 0.020 kg
Distance = 1.77 m

RESISTANCE SETTING	CURRENT in amp	TIME in sec	AVERAGE SPEED in m/sec
1	0.81	4.3	0.42
	0.81	4.4	
	0.81	4.0	
2	0.82	2.2	0.76
	0.82	2.4	
	0.82	2.6	
	0.82	2.1	
3	0.80	2.0	0.88
	0.80	2.0	
	0.80	2.0	
4	0.80	1.9	0.93
	0.80	1.9	
	0.80	1.9	

TRIAL	WASHERS	MASS in kg	CURRENT in amps	TIME in sec	AVERAGE SPEED in m/sec
1	6	.020	.82	4.6	0.39
2	5	.017	.70	2.5	0.71
3	5	.017	.70	2.5	
4	4	.013	.60	1.8	0.93
5	4	.013	.60	2.0	
6	4	.013	.60	1.9	
7	3	.010	.55	1.7	1.1
8	3	.010	.54	1.6	
9	2	.007	.45	1.4	1.3
10	2	.007	.45	1.4	

Apparatus

1 Energy transfer kit
1 Ringstand, 24″
1 Ringstand clamp as shown
1 No. 6 dry cell
1 Ammeter, 0–3 amps
1 Stopwatch

Magnetic Fields, Induction, Electromagnetic Waves

CHAPTERS
22-23

With these two chapters, the treatment of classical electricity and magnetism is completed. A quantitative study of magnetic fields is followed by a quantitative study of induced electric and magnetic fields. It is then possible to show that energy can be propagated by electromagnetic waves. At this point, the course has begun to bind together optics, dynamics, electricity, and waves in an integrated picture.

CHAPTER

22 The Magnetic Field

In contrast to the sequence of electrical chapters, this chapter undertakes all at once to introduce magnetic phenomena, build a mathematical theory for them, and give that theory content through applications. The first part, running through Section 22–5, establishes a law for the forces on currents in a magnetic field and from it sets up a unit of magnetic field strength. This unit is known variously as the *weber/meter*^{2} and as the *tesla,* but your students will have less to memorize if you stick with *newton/ampere-meter.* In this part, the application is to meters and motors. The second part is concerned with the production of magnetic fields by currents, and with introducing a concept, the circulation of the magnetic field, which will be useful for the study of electromagnetic radiation. It is important for students to recognize the equivalence of a current and a stream of charged particles as far as deflection in a magnetic field is concerned. Not only does this equivalence have practical applications, but its recognition gives an orderliness and a consistent aspect to the scheme.

There is a lot of physics in the proton example at the end of Section 22–6 and its further development in Section 22–7. Concepts of energy, centripetal force, and the conditions for circular motion all enter. If you have time it will be worthwhile to consider this in detail.

You can count on home reading to cover a good fraction of the material, particularly when it is reinforced by Experiments 41 to 45. Use the HDL's to increase the students' familiarity with the subject, and reserve class discussion for the puzzling detail.

It is common to refer to the "north pole" and the "south pole" of a magnet as though there were equal and opposite magnetic charges at these points. As the text remarks, however, no one has been able yet to separate north and south magnetic charges although a number of experimental attempts have been made, some very recently. The text therefore does not introduce magnetic poles, but develops the picture of magnets and magnetic fields as created by electric currents. The current loop approach is quite different from the "pole" approach, even though both lead to the same answers to the questions dealt with in high school courses. Insofar as we know, no poles exist as such, only current loops.

Mnemonic devices like the right-hand rule are dangerous, and the student should realize that he still must remember what goes along the thumb and what along the fingers. He is not ready for vector product notation, but ordering the factors can be helpful. Memorize the order IB. The first goes along the thumb and the second along the fingers. The student who is familiar with right-handed coordinate systems may prefer to think of I along x, $B_\perp$ along y, and F along z.

The teacher must decide whether or not to bring in the progression of a right-handed screw as I is twisted into B.

Experiment 41 The Magnetic Field of a Current uses a magnetic compass to explore the magnitude and direction of the magnetic field at the center of a loop of wire in which current is flowing. The experiment shows that magnetic fields add vectorially and are proportional to the current that produces them. This experiment is designed as an introduction to Section 3. The experiment should not take a whole period, so it can be combined with discussion.

Experiment 42 The Measurement of a Magnetic Field in Fundamental Units uses a simple current balance to measure magnetic forces and establishes a unit of magnetic field strength. This high-priority experiment is intended to follow study and discussion of Section 4.

Experiment 43 The Mass of the Electron uses a common "tuning-eye" radio tube and the coils used for Experiment 42 to determine the order of magnitude of the mass of an electron. This is a high-priority experiment. It should be preceded by Experiment 42, and can be done after Section 7 has been studied and discussed.

Experiment 44 The Magnetic Field Near a Long, Straight Wire uses the methods of Experiment 41 to investigate the magnetic field near a current-carrying, straight wire. Magnitude as a function of distance from the wire is considered.

Experiment 45 Magnetic Circulation is an involved experiment which requires considerable care and thought on the student's part. It ties together the concepts of magnetic circulation and is best done after a thorough discussion of the text material in Section 22–10.

EXPERIMENT

The Magnetic Field of a Current 41

A square rather than a circular loop of wire is used because it is easier to make. The entire 30 feet of wire should be used throughout the experiment. This will insure a constant current in the wire provided the total battery drain is not excessive. The small (No. 41) flashlight bulb limits the current to about 0.3 ampere with a single No. 6 dry cell.

As an indicator for the magnetic field, one can use a small magnetic compass of about 3 cm diameter.

The angles can be read directly from a small compass if it has a scale divided in degrees. The compass should be tapped lightly before taking each reading to prevent errors due to friction. If the pivot is not centered with respect to the scale, the deflections can be read at both ends of the needle and the average of the two readings recorded. The experiment can be done using only graphical methods in the taking of data and in the analysis, or by reading angles in degrees and using simple trigonometry to find the field strength.

The direction of the compass needle can be indicated by putting dots on the paper at the ends of the needle.

The data can be analyzed as shown in Fig. (a). The direction lines of the compass needle for different numbers of turns should mark off equal segments on a line drawn perpendicular to the zero current line. Each of these segments corresponds to the field added as another turn of wire is added to the coil.

If the deflection angles of the compass are measured in degrees, the data can be analyzed [Fig. (b)]; the constant horizontal component of the earth's field B_e, the field due to the current B_c, and the angle which the needle makes with

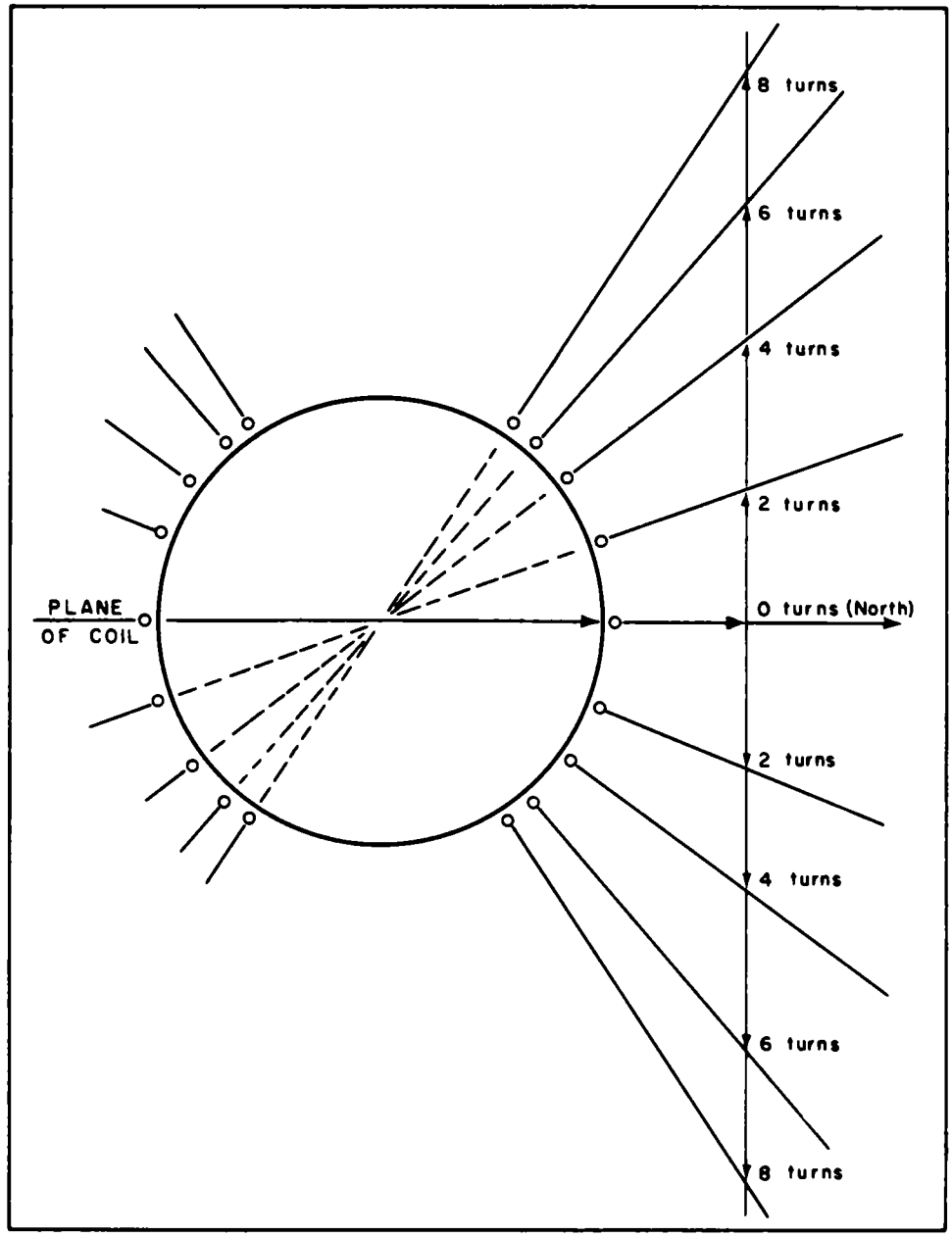


Figure (a)

its zero position are related by $B_c = B_e \tan \theta$. Plotting tan θ as a function of the number of turns of wire will give a straight line.

Some students may object to the statement in the Laboratory Guide that the current in the coil can be increased by increasing the number of turns. The important point is that, even though the current in the wire remains constant, the current flowing around the coil in three wires close together is the same as three times the current in one wire. This method of increasing the current around the loop in measurable amounts eliminates the need for an ammeter.

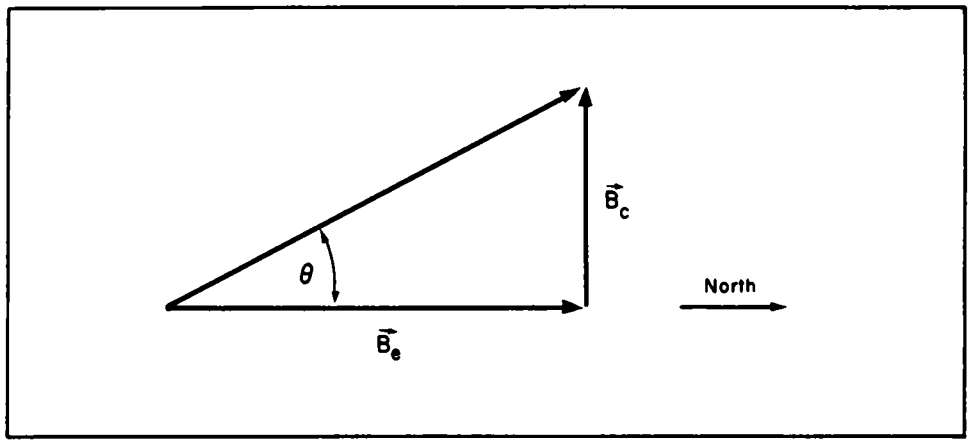


Figure (b)

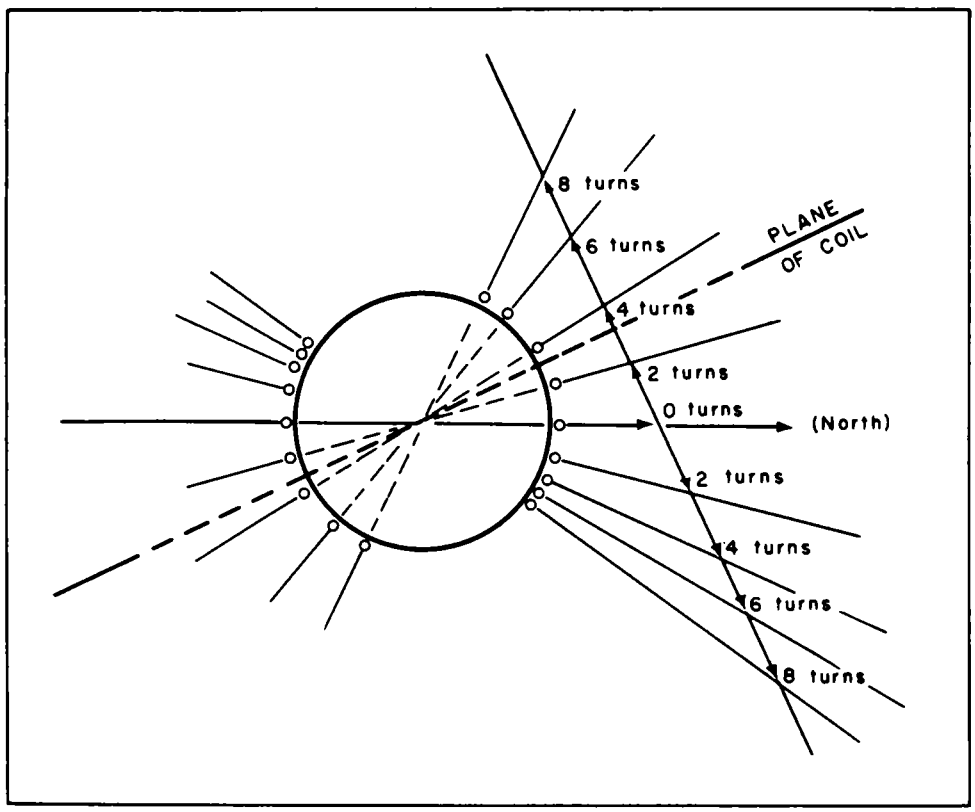


Figure (c)

Answers to Questions

The field in the center of the coil due to the current is perpendicular to the plane of the coil. Reversing the current through the coil reverses the direction of the field.

The magnitude of the magnetic field is proportional to the total current flowing around the coil.

Winding turns in opposite directions will produce fields in opposite directions. The net field will be due to the excess number of turns in one direction.

It is possible to measure the field even if the compass needle is not parallel to the plane of the coil, but reversing the current will no longer give equal deflections [Fig. (c)]. In this case, the graphical method is much easier to do.

The strength of the magnetic field of the compass needle does not influence the results of the experiment.

Apparatus

1 Tangent galvonometer kit which includes:
  Insulated wire, 30′
  1 Magnetic compass
1 No. 6 Dry cell, 1½-volt
1 Flashlight bulb, No. 41, and socket
1 Sheet polar or rectilinear graph paper

Alternate for light bulb:
1 Variable resistor, 10 ohm, 4 watt
1 Ammeter, 0–3 amp

EXPERIMENT

42 The Measurement of a Magnetic Field in Fundamental Units

The coil calibration determined in this experiment will be needed in Experiment 43. The coils should therefore be numbered if their characteristics are different, so they can be identified when used again.

In Fig. 2 of the Laboratory Guide, the loop and coil are shown connected in parallel to the source of current, each having a variable resistor and an ammeter in its circuit to permit independent control of their currents. This arrangement will be most easily understood by students, but if you are short of rheostats and ammeters, the apparatus can be connected in series as shown in Fig. (a). If this circuit is used, it will not be possible to make several determinations of a given field in the coil by varying the current in the loop.

For the accuracy of the experiment, the wire or string can be assumed to be uniform in cross section, and the mass of the short length of string can be calculated from the mass of several meters of string.

The length L of the straight end of the loop (A in Fig. 1) should be measured as the distance between the centers of the strips forming the sides of the loop.

Notice that a systematic error can occur if the ammeter's calibration is off.

The ratio of field strength in the center of the coil to the current is about 3.6×10^{-3} newton/meter $(\text{amp})^2 \pm 10\%$.

Answers to Questions

The ratio of the field strength of the coil to the current in the coil is constant.

Although the current-carrying loop can be balanced in the magnetic field near a small magnet, the measured value of the field strength will be hard to interpret since the field about the magnet is far from uniform over distances

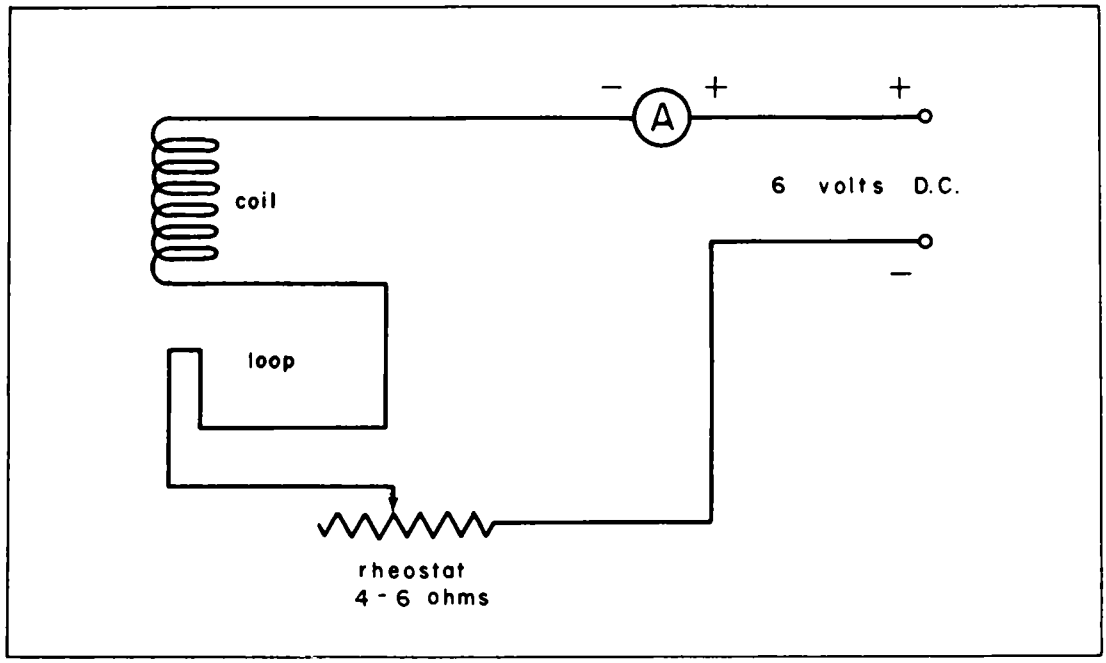


Figure (a)

comparable to the dimensions of the loop. We would have no idea at what point near the magnet the field would have the strength we calculated from the balancing force on the loop.

In principle, the balance could be used to measure the field of the earth, which is uniform over large distances, but the earth's field is so weak that, without using excessive currents through the balance, the force is too small to measure. Students might try this, however, to see if any motion of the balance can be observed. If more than 5.0 amperes are used in the loop, the contacts will become hot and corroded. A piece of fine sandpaper or emery cloth will clean the contacts.

If iron wire is used in the loop, it becomes magnetized and adds an unknown field to the field being measured.

Apparatus

1 Air-core solenoid
1 Current balance and contacts
1 DC current source, 0–10 amp (6 to 12 volts) this can be used simultaneously for several setups
2 Ammeters, 0–10 amp
2 Variable resistors (to regulate the current above)
8 Clip leads, 2′ or 3′

Pan balance (sensitive to 0.1 gram)
String
Ruler or meter stick

EXPERIMENT

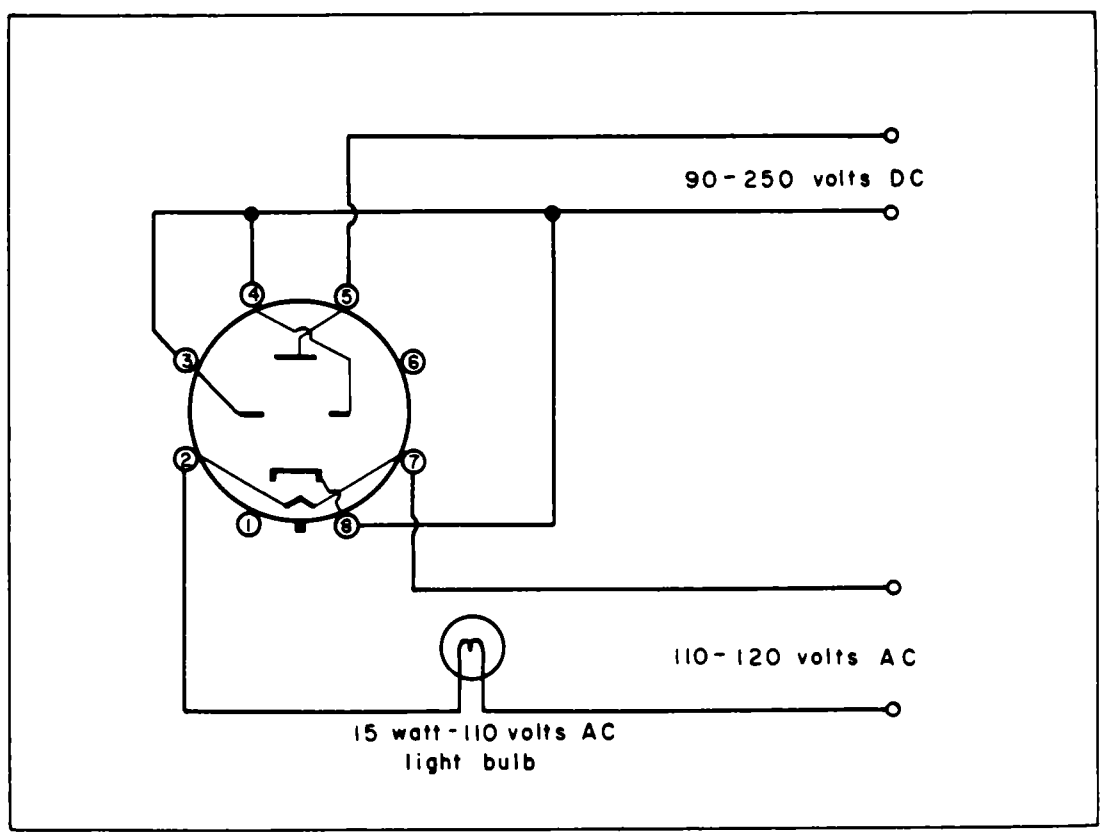
43 The Mass of the Electron

The conditions inside the 6AF6G tube are more complicated than as described in the Laboratory Guide. The tube elements and their supports are slightly magnetic, introducing a small amount of distortion in the magnetic field. In addition, since the accelerating electric field extends from the cathode to the anode, electrons in the beam are not moving at constant speed while being deflected by the magnetic field. However, owing to the "geometry" (the size, shape, and spacing of the elements) of the tube, most of the increase in speed takes place before the electrons move out from under the center cap. Both the magnetic distortion and the change in speed during deflection result in a mass determination in error by a factor of 2 or 3, but the mass of the electron is determined to the right order of magnitude, 10^{-30} kg. This is a very important accomplishment in measuring such a small mass.

Note that the pattern we see is not the path of the electrons, but the *end point* of the paths of many different electrons as they hit the conical target.

It may be asked why the helical grid surrounding the cathode cannot be connected to the anode so the electrons will gain all their kinetic energy between the cathode and this grid and move at constant speed from it to the anode. As a careful examination of Fig. 1(b) shows, this grid is internally connected to the cathode.

Figure (a)



The filament supply for the 6AF6G tube is 6.3 volts at 0.15 amp. Batteries or a filament transformer can be used, or the filament can be operated from the 110-volt A.C. line, using a 15-watt, 110-volt incandescent lamp as a potential-dropping resistor as shown in Fig. (a).

It is also possible to use 110 volts A.C. for the anode voltage. One still sees a sharp shadow with the rapidly changing anode voltage owing to the persistence of vision and the increase in brightness of the beam as the anode potential increases. The peak voltage should be used as the potential difference in the calculation. These points are difficult to explain to the student at this time, and therefore D.C. is to be preferred for the anode voltage.

When wiring the circuit, remember that the pin numbers on the tube socket shown in Fig. 4(a) of the Laboratory Guide apply when looking at the *bottom* of the socket, not when looking down at the top.

The voltages used in this experiment are dangerous. Be sure precautions are taken to avoid shocks.

The solenoid used here is the same as that used in Experiment 42, and currents up to 5 amperes will be needed. A low-voltage D.C. supply operating from the 110-volt A.C. line can be used for the solenoid current, but these supplies are sometimes poorly filtered. In Experiment 42 this was not noticeable because of the inertia of the wire loop, but electrons will follow the rapid fluctuations in the magnetic field and the edge of the shadow will be fuzzy. Hence, it may be necessary to use a 6-volt storage battery. Do not use dry cells; they won't last long with the current used.

Consistent units must be used in the analysis. To obtain the mass m in kilograms, measure R in meters, q in coulombs, B in newtons/amp-meter and V in volts. The following data were taken in an experiment:

Coil constant,

$$\begin{aligned} B/I &= 3.6 \times 10^{-3} \text{ newton/amp}^2\text{-meter} \\ I &= 3 \text{ amperes} \\ B &= 10.8 \times 10^{-3} \text{ newton/amp-meter} \\ V &= 135 \text{ volts} \\ R &= 5 \times 10^{-3} \text{ meter} \end{aligned}$$

Substituting in the equation for m in the Laboratory Guide, we find a value for m of

$$\begin{aligned} m &= \frac{B^2qR^2}{2V} \\ &= \frac{1.2 \times 10^{-4} \times 1.6 \times 10^{-19} \times 25 \times 10^{-6}}{270} \\ m &= 1.8 \times 10^{-30} \text{ kg.} \end{aligned}$$

Answers to Questions

It is possible to use the earth's magnetic field to deflect an electron beam. With a vertical component of the earth's field of about 5×10^{-5} newton/amp-meter and an accelerating voltage of 100 volts, the radius of the circle in which an electron beam will be bent is about 67 cm. Consequently, we would need a tube at least one meter in diameter if we wanted to use the same method. However, such a deflection can easily be measured in a straight tube.

Using the free fall of an electron to determine its mass is theoretically sound but impractical. An electron gains a very high velocity for even very small accelerating potentials, and it would travel a very long horizontal distance before "falling" a measurable distance. We would have to determine its "initial" velocity from the distance it had fallen and compute the mass $mv^2/2 = qV$. An electron accelerated through a potential difference of only 1 volt will acquire a velocity of nearly 6×10^5 meter/sec. During $\frac{1}{100}$ sec it will travel a horizontal distance of 6000 meters and will fall under gravity a distance of 4.9×10^{-4} meter, or about 0.5 mm. Obviously, this will make it impossible to perform any experiments of this nature.

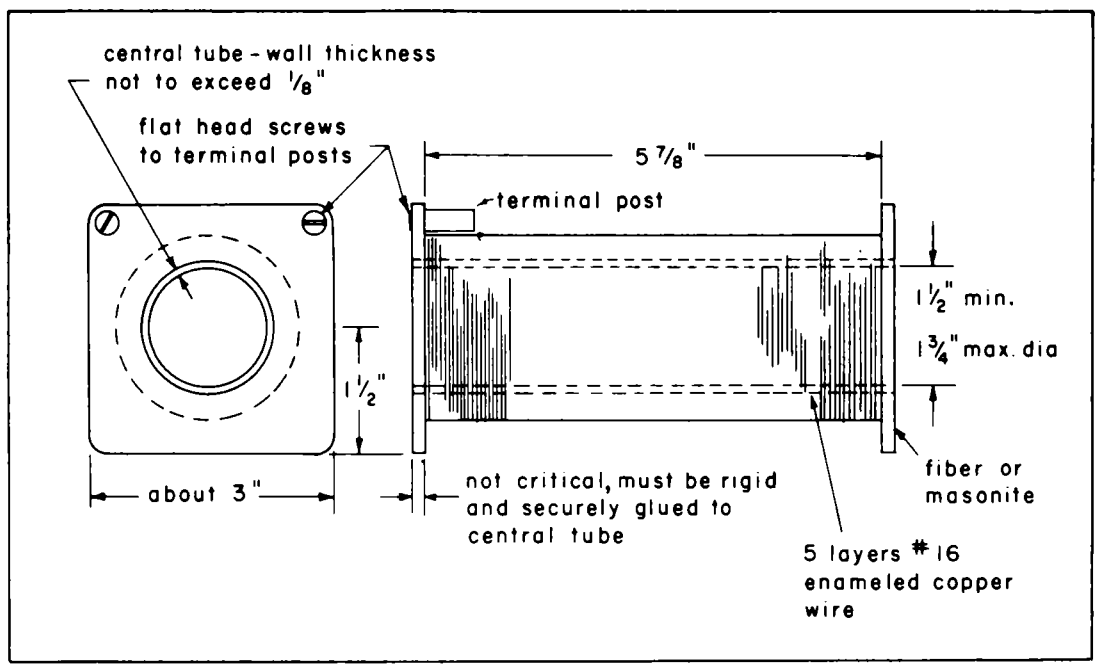
Apparatus

1 Air-core solenoid
1 Mass-of-the-electron apparatus
4 45-volt batteries (dry cells) or power supply
1 D.C. source; 0–5 amp with at least a 6-volt tap for filament supply; this may be used simultaneously for several setups
1 Ammeter, 0–10 amp
1 Variable resistor (to regulate the current above)
4 Clip leads, 2′
1 Ruler or meter stick
1 Set of wood dowels of various diameters

Design of Solenoid

You can make your own coil as in Fig. (b). The ratio of the field strength in the center of the coil to the current is about 3.6×10^{-3} newton/meter-(amp)^2 (plus or minus 10 percent). Computing this ratio from the geometry

Figure (b)


central tube - wall thickness not to exceed 1/8"
flat head screws to terminal posts
5 7/8"
terminal post
1 1/2" min.
1 3/4" max. dia
1 1/2"
about 3"
not critical, must be rigid and securely glued to central tube
fiber or masonite
5 layers # 16 enameled copper wire

of the coil and assuming an infinitely long coil gives 4.0×10^{-3} newton/-meter(amp)^2. If corrections are made for the finite length of the coil, this computed value will come closer to 3.6×10^{-3} newton/meter(amp)^2.

EXPERIMENT

The Magnetic Field Near a Long, Straight Wire 44

This is the only experiment in which the dependence of magnetic forces on distance is investigated. Furthermore, it is easy and gives accurate results.

The vertical section of wire must be long, extending, if possible, as far above the table as the table is above the floor. The effect of iron objects in the vicinity can be checked by moving the compass along the graph paper with no current flowing in the wire. If the compass shows no change in the direction of the earth's field as it is moved out 20 cm along the paper from the wire, it can be assumed that there will be no error due to neighboring iron. If there is much iron in the table, it may help to use the nonmagnetic coil frame used in Experiment 41.

The compass should be handled as in the preceding experiment. If time permits and greater accuracy is desired, the current through the wire can be reversed at each position of the compass to give deflections in opposite directions.

If a 5-ampere D.C. source with a self-contained rheostat and ammeter is not available, batteries or other power supplies can be used. Students may have to be shown how to connect a rheostat and ammeter in series with the vertical wire and power supply. Dry cells are not recommended for this experiment, since they are expensive and deteriorate rapidly when supplying large continuous currents.

Answers to Questions

The strength of the field in terms of the horizontal component of the magnetic field of the earth is inversely proportional to the distance from the wire. This can be found by determining the field strength at different distances and then drawing a graph of field strength as a function of the reciprocal of the distance from the wire. If data is taken with the compass too close to the wire, the curve will deviate from a straight line at short distances as shown in Fig. (a). The shorter the compass needle, the less the error will be.

Because iron becomes magnetized even by the earth's field alone, close proximity will produce an additional unknown field. Similarly, the close proximity of the rest of the wire in the circuit would add unwanted fields to the field being measured.

If the wire were only 20 cm long, the field would drop off faster than inversely with the distance. Students should be encouraged to check this experimentally if time allows. Fig. (b) illustrates the situation. The field at the point M is primarily due to the current in the wire in the vicinity of O, because the distances MB and MC, for example, are already several times larger than MO, and therefore the current at B and C will contribute very little to the field at M. Whether the wire extends to A and D will have little effect on the magnetic field at M. On the other hand, the distances AN, ON, and DN are about the same; therefore the sections AB and DC of the wire will contribute significantly to the magnetic field at N. Shortening the wire will decrease the field at

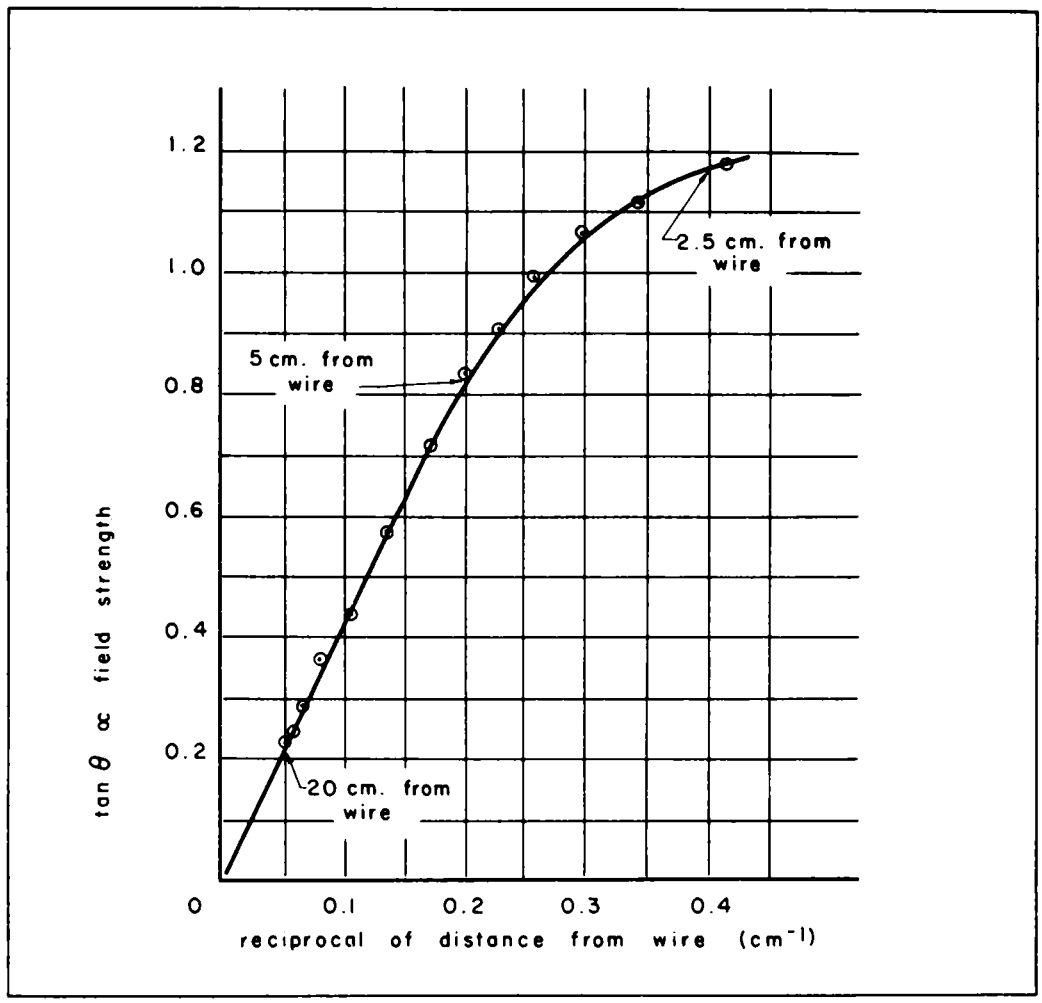


Figure (a)

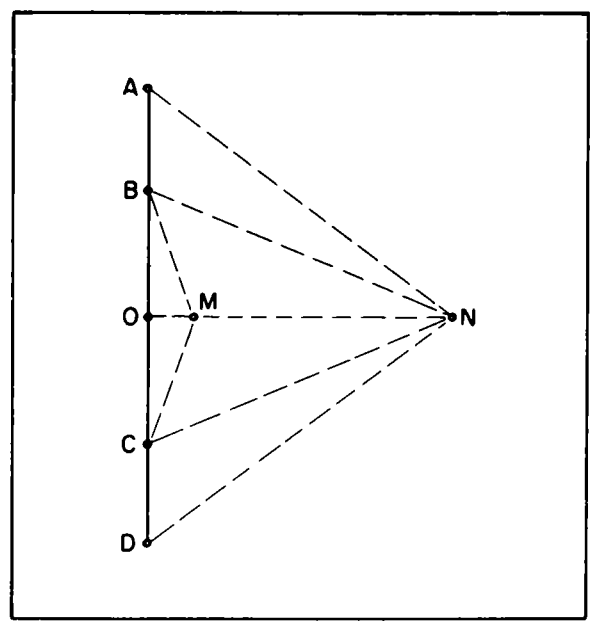


Figure (b)

N. Hence, for the short wire the field will drop off faster than inversely with the distance.

A current one hundred times as large would make the field about the wire very large and give compass deflections so close to 90° that the small changes in angle as the compass is moved out to 20 cm from the wire could not be measured accurately. If the current were one hundred times smaller, the maximum deflection would be only a fraction of a degree and the changes in deflection could not be measured accurately.

If the laboratory tables cannot be moved to make their edges parallel to the earth's field, it will be necessary, in the last part of the experiment, to clamp a properly oriented board to the table edge as shown in Fig. (c). Computed graphs of the field strength as a function of distance are shown in Fig. (d). This part of the experiment is intended only for those students who wish to find out more about the fields surrounding wires. Some students will be able to predict the results by the superposition of the two fields before they do the experiment, as was done in drawing Fig. (d).

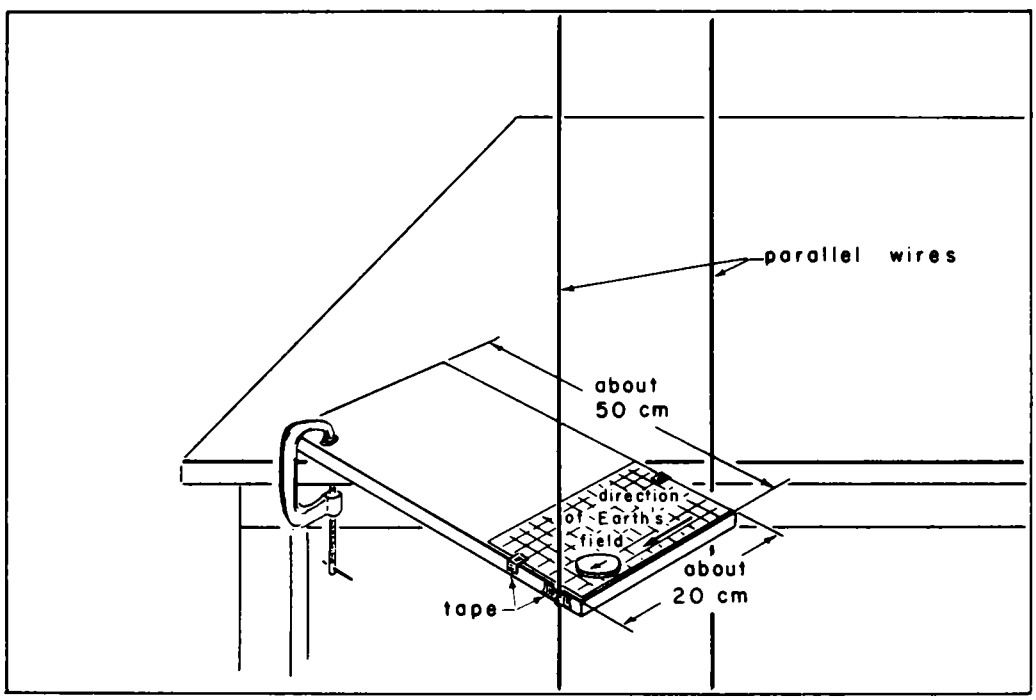


Figure (c)

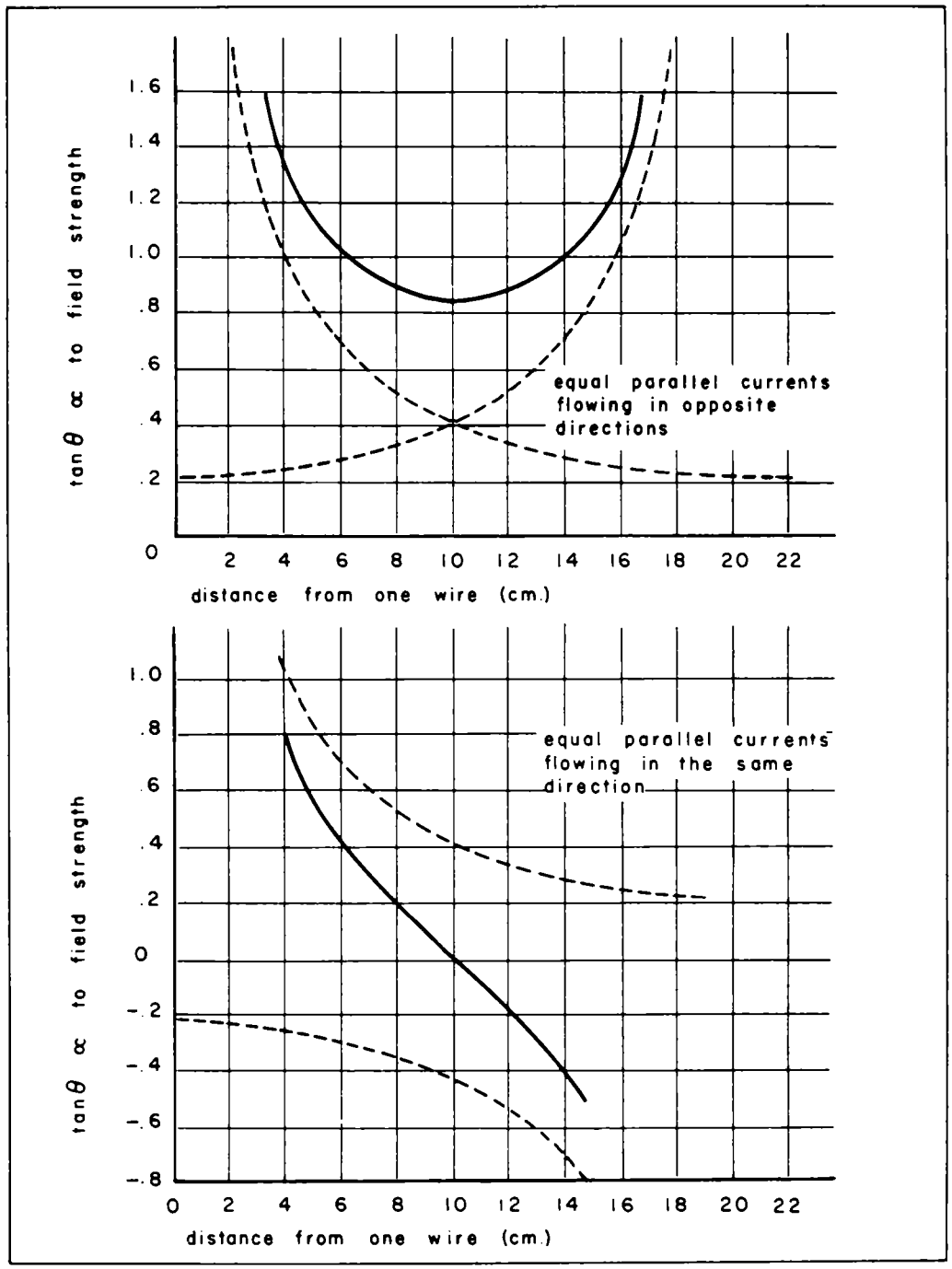

tan θ ∝ to field strength
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
2 4 6 8 10 12 14 16 18 20 22
distance from one wire (cm.)
equal parallel currents flowing in opposite directions
tan θ ∝ to field strength
1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
0 2 4 6 8 10 12 14 16 18 20 22
distance from one wire (cm.)
equal parallel currents flowing in the same direction

Figure (d)

Apparatus

1 Magnetic compass
Insulated wire, 30′ No. 18 stranded
2 Sheets rectilinear graph paper
1 Ringstand, 36″
1 Clamp holder
1 Meter stick
1 DC source, 5 amp (6 to 12 volt); this source may be used simultaneously for several setups
1 Ammeter, 0–10 amp
1 Variable resistor (to regulate the current above)
2 Bricks

Cellulose or masking tape

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1 C clamp
1 Board $\frac{1}{2}'' \times 8'' \times 12''$

EXPERIMENT

Magnetic Circulation 45

Encourage your students to use fairly simple polygons and to make careful readings of the current at each position of the probe. Good results are obtained with seven turns of wire on the square coil and a current of 3 amp. More turns or a larger current may result in a few probe-coil readings over 1 amp, which generally occur in the center of the square coil.

If all groups use seven turns and 3 amp of current, the $\Sigma\Delta SI_p$ for all polygons can be compared directly; otherwise, you can compare $\Sigma\Delta SI_p$ per amp-turn by dividing the summation Σ by the number of amp-turns used in the square coil.

The probe coils should be calibrated at different distances out from the long straight wire carrying a current of 5 amp. The product $2\pi rI_p$ should be constant, regardless of the radius used. This is the circulation of the current in the long straight wire.

Answers to Questions

The current in the square coil must remain constant, because the circulation being measured depends upon this current.

A reversed current reading in the probe coil indicates that the magnetic-field vector $\vec{\mathbf{B}}$ is pointing in a direction opposite to the direction of the field in the previous segment. The reading ΔSI_p for this segment is therefore negative, if the previous one was marked positive.

The circulation around the square coil is proportional to the amp-turns in the coil. The circulation around the straight wire is proportional to the current in the wire. The ratio of the circulations is the same as the ratio of the amp-turns.

For the same number of amp-turns, the circulations measured for different polygons may vary by 10 percent. Comparing class data should give fairly close agreement between amp-turns and circulation.

The circulation is directly proportional to the product of amp × number of turns. The earth's magnetic field has an effect on the readings, but in making a closed loop the effect is canceled out.

The circulation should be zero. Class data should be compared here. The I in the expression $2\pi KI$ is the current threading the loop. It is zero in Fig. 6. In Fig. 7 the current threads the loop only once, so $2\pi KI$ will give the value for the circulation.

The constant of proportionality between $\Sigma\Delta SI_p$ and $2\pi KI$ depends upon the construction of the probe coil. Five turns on a coil of 2π-cm diameter or six turns on the 7.7-cm diameter coil will make the number of values nearly equal, disregarding the power of 10.

Apparatus

1 Seven-turn square coil
1 Plotting board
1 Small circular probe coil
1 Magnetic compass
1 1.5-volt source (dry cell)
1 6-volt D.C. source (3 amp)
2 Variable resistors to regulate current
1 Ammeter (0–1 amp)
1 Ammeter (to read 3 amp)
Cellulose or masking tape
1 Meter stick
1 Set of apparatus for Experiment 44 (to be shared by class)

Films

"Electrons in a Uniform Magnetic Field" by Professor Dorothy Montgomery, Hollins College. A spherical cathode-ray tube (containing gas to show the path of the electron beam) is used to give a fairly precise measurement for determining the mass of an electron. The electron beam is deflected by Helmholtz coils. Running time: 10 minutes.

"Mass of the Electron" by Professor Eric M. Rogers, Princeton University. A current-carrying loop of wire is used to deflect the electron beam in a cathode-ray tube. From circuit measurements and measurement of the deflection, the mass of the electron is calculated. Running time: 18 minutes.

These two films cover similar material. One or the other, but in most cases probably not both, may be shown to advantage in connection with Sections 22–6 to 22–8.

Demonstrations

1. Oersted's experiment can be done easily; a single wire, practically any battery, and a compass needle are all that are needed. A current of 5 amperes will cause a large deflection of the compass needle.

2. Perhaps the best way of getting the ideas across in Section 22–2 would be to let the students play with bar magnets and compasses, perhaps at home. For instance, anyone with a long bar magnet and compass can see that the compass always lines up like the filings in Fig. 22–5a. The fields around bar

and horseshoe magnets can easily be explored with iron filings. Rather large currents (an automobile battery is probably necessary) are needed, to produce such patterns as those shown in Fig. 22–5. Some students might like to make permanent their iron filing patterns on blue print or photographic printing paper.

3. The only hard part of Section 22–11 is the evaluation of the circulation over the special path around the solenoid. After this is convincingly established, it might be wise to set up the current balance again and allow some of the students to try the experiment mentioned here to evaluate the circulation constant.

CHAPTER
23 Electromagnetic Induction and Electromagnetic Waves

The ideas of the chapter are really the culmination of classical physics where electricity and light become one subject. While this material is abstract and difficult, many of the ideas in this chapter can be adequately comprehended if the students get a qualitative understanding of what happens. Demonstrations can get these ideas across more effectively and faster than lecturing. Of course, if you have time, the quantitative aspects of this chapter can be handled as in previous work. There are, however, many ideas presented in a condensed way in the text, and complete, quantitative comprehension can be expected only from good students and plenty of time.

Before starting this chapter it is essential that you know exactly which parts of the chapter you want your students to understand in detail and which parts you wish to treat qualitatively.

The first three sections form the basis for understanding an electrical generator. They are straightforward extensions of Chapter 22 and should be handled readily by most students. A thorough treatment can be made in one class period.

Sections 23–4 and 23–5 extend the relatively straightforward material of the first three sections to consideration of the induced EMF in the circuit. If Section 23–3 is well understood, this material should go easily. All students will find the simplicity of Lenz's law appealing. Section 6 deals with electric fields created by changing magnetic fields. Some students will find this material more abstract and difficult than the previous two sections. Probably two class periods will be required as a minimum to get these basic ideas adequately planted.

All the phenomena described in Sections 23–1 through 23–6 can be easily demonstrated to small groups with equipment usually available. For demonstration to a large class more elaborate setups may be necessary. Such demonstrations might well bear the major load of teaching this material. You may have your own favorites, but if not, suggestions are made under Demonstrations.

Sections 7 and 8 will be rather hard going for the average class, but even a partial understanding of this material can be very rewarding to most students. Here is the real, surprise "pay-off" of the chapter when we find that electromagnetic radiation is predicted to travel at a unique velocity, the velocity of light. Section 9 can be handled as a reading assignment supplemented by

a film if desired. Minimum time on these sections is about 2 days, but three or four days would be more desirable.

Section 23–5 is the all-important section which gives a quantitative result for all of the induced effects we have been observing. It is a particularly important example of the use of conservation of energy, but students can appreciate it without much detailed work. Most students will realize that electrical generators require mechanical forces. It is not necessary to solve quantitative problems using this principle. On the other hand, if you have enough time to treat the chapter thoroughly, you can use this interesting material to give the students worthwhile practice.

Demonstrations 6 through 9, which use the same apparatus as that used for the preceding sections, with inexpensive additions, illustrate in a convincing way the two points which are made in the text; (1) there must be a force *on the coil* when it is moved in the magnetic field; and (2) the force depends only on the relative motion. The demonstrations are so simple and persuasive that they should be given *before* a detailed discussion of the text. The text material can then be regarded almost as a series of generalizations of these simple experiments.

Section 23–6 is important if you plan to discuss the mechanism of electromagnetic radiation in detail. However, you need not take much class time because the existence of the electric field can be inferred from the operation of a transformer.

Many people find it hard to believe that a changing magnetic field produces an electric field at points in space where there is no magnetic field. There is no way of proving this except experimentally. The betatron is probably the best example, but it is an unfamiliar one to students. Be sure to remind your class that the operation of a transformer is very much like the circular electromagnet described in the discussion of the betatron. The battery causes a current in the electromagnet which results in a magnetic field in the magnet. If the gap is plugged with iron as shown here almost no magnetic field will exist outside the ring. Yet a voltage is induced in a coil wound around the ring when the current is cut off. There must have been an electric field induced even where there was no magnetic field.

At this time be sure to point out how far we have come from the original ideas of Section 23–1. The force on a moving charge led directly to the idea of an induced EMF in a wire which moves in a magnetic field. Step by step, further experiments led to abandoning the necessity of motion of a wire in favor of any change of flux through the loop of wire. Finally in this section, the wire is abandoned, and we are led to the very abstract idea of a changing magnetic field producing an electric field.

A qualitative understanding of this material is necessary to an understanding of the nature of electromagnetic radiation discussed in the next section.

Even the best students have trouble with a quantitative treatment of induced magnetic fields. You will probably not have time to explain this quantitative material in Section 23–7 to average students.

The mechanism of electromagnetic waves is the climax of the sections on induced electric and induced magnetic fields. However, a qualitative treatment may have to be sufficient. Class time probably can be spent more profitably in going over the figures in the text.

Section 23–9 is an important section and even if you have been rushed and have omitted considerable material on electricity and magnetism, some class time should be given to discussing electromagnetic waves and the electromag-

netic spectrum. Above all, for material which follows, students should know that accelerating charges produce electromagnetic waves even if they have only a vague idea of what an electromagnetic wave is.

Furthermore, at some point, you should stress that the wave model of light involves electromagnetic waves. The speed of light, the effects caused by light, and the origin of light can all be understood in terms of electromagnetic effects.

To show that accelerating charges are responsible for electromagnetic radiation, it may be wise to discuss examples which cover a very broad wavelength (or frequency) interval.

At the long-wavelength, low-frequency end of the spectrum, the frequency is low enough to make the mechanism apparent. In radio antennas, an alternating electric field accelerates the charges, first in one direction, and then in the other. The emitted radio waves have a frequency that is the same as the number of cycles of acceleration. This mechanism is commonly used to give frequencies from about 10^4 to 3×10^{11} per second, corresponding to wavelengths from 3×10^4 meters to 10^{-3} meter.

At the higher frequencies associated with infrared, visible, and ultraviolet light, we would predict on the basis of the physics we have thus far studied that accelerated charges in atoms might be the source. However, we are not yet ready to examine atomic processes, which we will do in later chapters. These electromagnetic radiations are also produced by high-energy electrons undergoing centripetal acceleration in a modern particle accelerator. The energy loss (due to radiated electromagnetic energy) is a crucial factor in designing electron accelerators with energies above about 10^8 electron volts.

Still higher frequencies, up to the highest that can be produced, are generated when charged particles are decelerated. This deceleration mechanism produces not only the standard X rays (with frequencies above 3×10^{16}/sec) but also the highest frequency or highest energy gamma rays. Several accelerators now produce electrons of 10^9 ev and these, in stopping, radiate gamma rays with frequencies of about 2×10^{23}/sec. Cosmic-ray particles generate even higher frequency γ rays.

Be sure to emphasize in class that light is an electromagnetic wave. Explain that if time were available you could return to the characteristics of light and analyze them in electromagnetic terms. For example, electric fields cause motion of charges in metal. When a light wave hits a metal the charges near the surface accelerate and send back a reflected wave. When light interacts with matter, the charges within matter are basically what respond.

Point out too that knowing the electromagnetic character of light allows us to use light, (and ultraviolet, and X rays, and gamma rays) to explore the position of, and the forces on, charged particles in matter. In this way, electromagnetic waves become probes of the structure of matter.

Note that many phenomena do not vary smoothly with wavelength or frequency. For example, consider the penetration of electromagnetic waves through a piece of paper. At low frequencies, transmission is high; radio and radar waves go through paper unimpeded. At higher frequencies, such as infrared, there is a strong absorption; paper will burn when sunlight is focused on it. At still higher frequencies, in the visible range, there is selective reflection and absorption depending on the exact frequency (which with visible light is associated with color). In the ultraviolet, absorption again takes over. At still higher frequencies, the far ultraviolet and X rays, there is some slight absorption but most of the radiation is transmitted.

Note that at this stage of the course (i.e., before you discuss photons and

photon energies $E = h\nu$) the principal clue to what a wave will do is its wavelength. As you discuss different phenomena, be sure to write the relevant wavelengths on the blackboard.

Of course, the wavelength varies smoothly from one end of the spectrum to the other. The overall pattern of electromagnetic interaction also varies smoothly although there are many detailed features in some parts of the spectrum. With long wavelengths, at frequencies below infrared, electromagnetic interactions depend on free charges. Any metal behaves as other metals do; the key to electromagnetic effects in this range is "how free are the charges."

When the wavelengths become smaller, the detailed structure of matter plays a more important role. Infrared radiation affects molecules easily. Visible and ultraviolet radiation move charges relative to one another within an atom, especially at certain natural frequencies. These wavelengths are still large compared with the atom (by about a factor of 10^2 or 10^3), so that the crude picture of wavelength determining the size of the object with which interaction occurs is not exact. (It may miss by several orders of magnitude. But even several orders of magnitude are not much in the entire electromagnetic spectrum.) As the wavelength becomes still smaller, the interaction is with the innermost atomic electrons (X rays). A further decrease in wavelength involves either interaction with individual electrons (the Compton effect) or interaction including the atomic nucleus and the closest electron (the atomic photoelectric effect). Finally, at still shorter wavelengths the main interaction is nuclear. For X rays and gamma rays the interaction is governed by the charge of the nucleus and not by the arrangement of atoms (i.e., in molecules, in crystals, in metals, etc.).

It may be worth while to describe the action of a simple antenna. You cannot at this level prove anything, but the class may get a feel for some aspects of electromagnetic radiation.

At (a) we have shown a simple, very familiar antenna which consists of two rods which are connected near the center to a voltage source—for the moment

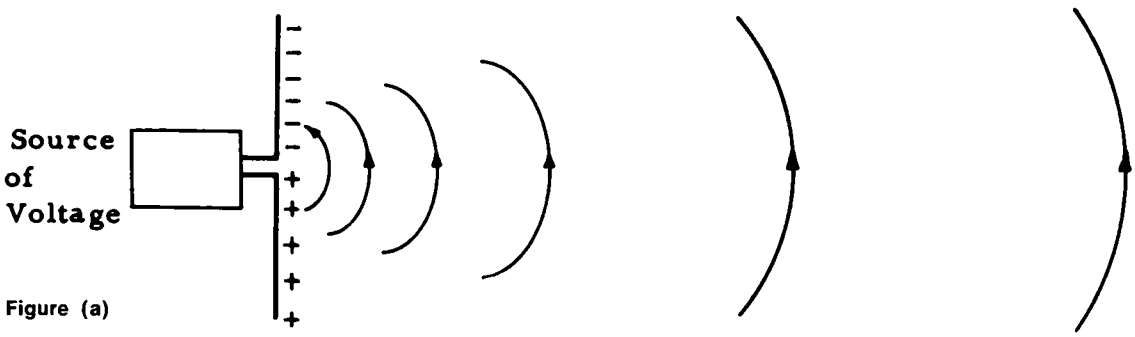


Figure (a)

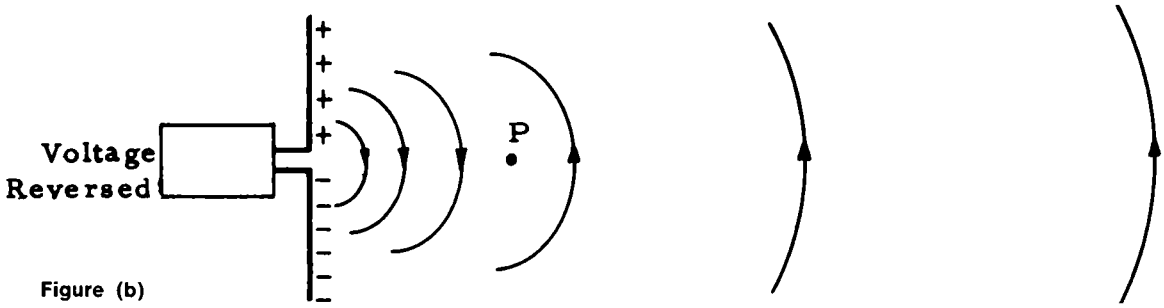


Figure (b)

just a battery. The voltage charges the upper rod negatively, the lower one positively. An electric field extending through all space is set up by these charges. We have shown it only to the right, but of course it extends in all directions. Now let us suddenly switch the battery to the other polarity so that the top rod becomes positively charged and the lower one negatively. We then have the situation shown at (b).

The field beyond P is shown just as it was before, while only the field near the antenna has changed. The point is that the regions beyond P have not yet had time to "learn about" the change of the polarity of the antenna. Points distant from the antenna can "learn about" the change in polarity of the antenna only through a changing electric and perpendicular magnetic field which can travel only at the speed of light as discussed in the last section.

It only remains to add that the antenna itself must produce not only an electric field but also a magnetic field; for if the bottom was originally positive and later on the top was, positive charge must have run up the antenna or negative charge down. In either case a current ran up the antenna. A current going up the antenna would produce the necessary magnetic field. Thus we see that in every case a changing electric or a changing magnetic field implies the other; the two are irrevocably coupled together.

To get back to our antenna and battery, we see that if now the battery is switched back and forth, changing electric and magnetic fields sweep out across space alternating in direction. They try, in a sense, to carry the message as rapidly as they can, i.e., with the speed of an electromagnetic wave, as to which way the charge is on the antenna at a given instant. The result is an oscillating electromagnetic wave.

Film

"Electromagnetic Waves" by George Wolga of M.I.T. shows why we believe in the unity of the electromagnetic spectrum. Experiments are performed to show that radiation arises from accelerated charges, that this radiation consists of transverse waves that can be polarized, and that interference can be produced in four different regions of the spectrum: X ray, visible light, microwave, and radiowave. Running time: 33 minutes.

Demonstrations

1. *Qualitative Experiment with a Loop of Wire.*

Equipment: Magnet and Microammeter (or sensitive D.C. oscilloscope).

A strong magnet is needed. Maximum fields of 0.1 nt/amp-m to 0.2 nt/amp-m (i.e., 1000 gauss to 2000 gauss) are useful.

If you have a choice of microammeters, the smallest internal resistance will be most useful. An oscilloscope which responds to a fraction of a millivolt would be preferable.

Procedure. Make a closed circuit containing a long wire and the microammeter. If you use an oscilloscope, connect a large resistance in place of the meter and connect the oscilloscope across the resistor.

Move the wire quickly through the region between the magnet pole faces. The meter (or oscilloscope) should respond.

Typical Values and Possible Difficulties. Suppose your magnet has a field of 0.2 nt/amp-m and a pole area of 10 cm^2 (or 10^{-3} m^2). The maximum flux through the circuit formed by the wire would be

$$BA = 0.2 \times 10^{-3} = 2 \times 10^{-4} \text{ nt-m/amp}$$
$$= 2 \times 10^{-4} \text{ nt-m sec/coulomb}$$
$$= 2 \times 10^{-4} \text{ joule sec/coulomb}$$
$$= 2 \times 10^{-4} \text{ volt sec.}$$

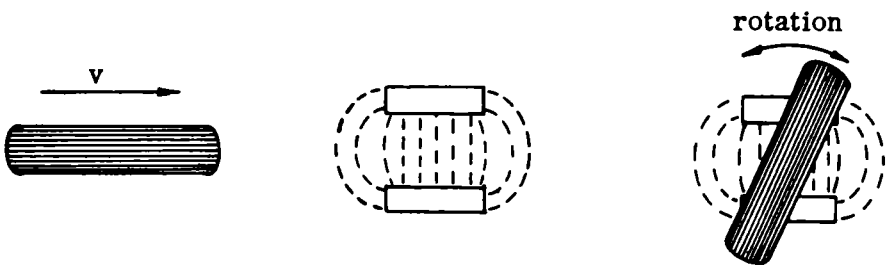
If you move the wire so that this flux passes (or stops passing) through the area enclosed by the complete electrical circuit in, say, 0.5 second, an EMF = $\frac{2 \times 10^{-4} \text{ volt sec}}{0.5 \text{ sec}} = 4 \times 10^{-4}$ volt will be produced. If your circuit uses a resistor with an oscilloscope across it, this voltage will be displayed on the oscilloscope. With a resistor-oscilloscope arrangement, you can show (qualitatively) the dependence on the speed, v.

If you use a microammeter, it may be much harder to show the effects of changing the speed. Imagine an idealized microammeter with an internal resistance of 100 ohms. The 4×10^{-4} volt induced EMF would produce

$$I = \frac{V}{R} = \frac{4 \times 10^{-4}}{100} = 4 \times 10^{-6} \text{ amp.}$$

Before using such a demonstration in class, it will be wise to determine whether your microammeter responds when you change v.

With the microammeter you have available, you may be able to get much better results by using a coil consisting of many turns of wire. With 50 turns, the EMF will be 50 times larger or $50 \times 4 \times 10^{-4} = 2 \times 10^{-2}$ volt.



The demonstrations can be extended by rotating the coil or loop about a diameter as shown in the figure above. This part of the demonstration goes well with Section 3. If you use a resistor and oscilloscope (rather than a microammeter) the output voltage of the coil can be seen to reverse sign. Even with a microammeter, the reversal of the induced current is visible because the needle deflection will reverse. A microammeter with a center zero-point will show the symmetry between positive and negative voltages. For this type of demonstration, it is necessary to stress that $B_\perp$ rather than B is the important parameter.

2. *Relative Motion.*

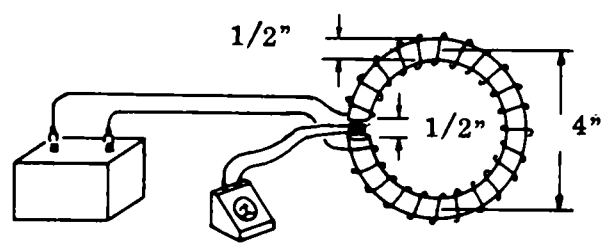
Connect the terminals of one of the solenoids used in Experiments 42 and 43 to a microammeter or a sensitive galvanometer. Move a bar magnet in and out of the coil and, while holding the magnet stationary, move the coil to show that the presence of an induced voltage in the coil depends only on the existence of relative motion between the magnetic field and the coil. Several magnets may be tied in a bundle to give a stronger field.

3. *Induced Current Produced by a Changing Magnetic Field.*

(a) With a stationary coil and a magnet in motion, show that the induced current increases with the rate of change of the magnetic field through the coil. Use a weaker magnet which requires a higher speed of motion to give the same effect, thereby showing it is the rate of change of the flux, not the speed of the magnet, that is important. Demonstrate the difference in strength of the magnets by the quantity of nails each can lift from a pile.

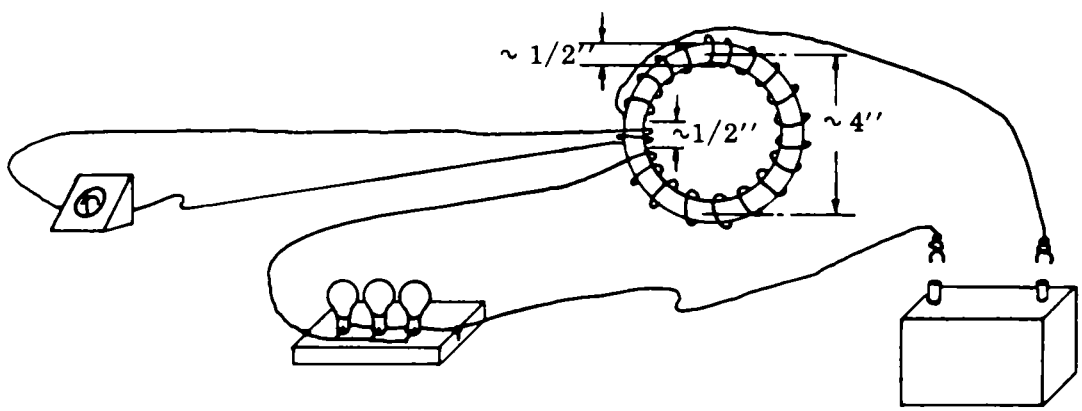
(b) A simple electromagnet experiment can be done using the solenoid of Experiment 42. Wind several turns of heavy insulated wire around the solenoid and connect this wire in series with a knife switch and a dry cell and show each of the following: (1) When the switch is closed, an induced current flows while the magnetic field in the coils is building up (the flux in the coil is increasing); (2) An induced current flows when the switch is opened and the flux is decreasing; (3) There is no induced current while the switch stays closed and the flux produced by the loop of wire is constant.

(c) Effects more like that shown in Fig. 23–11 can be demonstrated if you have an electromagnet. If you do not, a simple version can be constructed quite easily. The illustration of Fig. 23–11 can be duplicated effectively with a soft iron ring from which a piece has been cut. Lacking anything better, you can make a ring by bending a piece of concrete-reinforcing rod. With a ring of diameter 10 cm and cross-sectional area about $2cm^2$, you can obtain a sufficiently large magnetic field by winding the ring closely with one layer of 18 gauge *insulated* copper wire. About 100 to 300 turns should be sufficient.



Using a 6-volt car battery as a current-source, connect the electromagnet as shown above.

For connections to the battery, use clip-on cables so that they may be attached and detached easily. With a few hundred turns of wire the current



drawn from the battery should be reasonable—less than 10 amperes. A current of 6 amperes gives a field of about 0.30 nt/amp-m in the gap of the iron ring. If your coil draws too large a current, it can be cut down with a resistance of a few ohms. About eight 150-watt light bulbs (or two 500-watt heating elements) in parallel will be sufficient. Ten amperes may burn up No. 18 wire if left on continuously.

A small coil of about ten turns of wire can now be connected to the microammeter, and inserted in the gap, as shown in the figure. Upon *reversing* the current in the electromagnet, with the ten-turn coil absolutely still, a deflection will be seen on the microammeter.

Alter the orientation of the ten-turn coil so that its plane is *along* the field. Repeat the experiment now, and no deflection will be observed. The reason is that although B changes, ϕ does not.

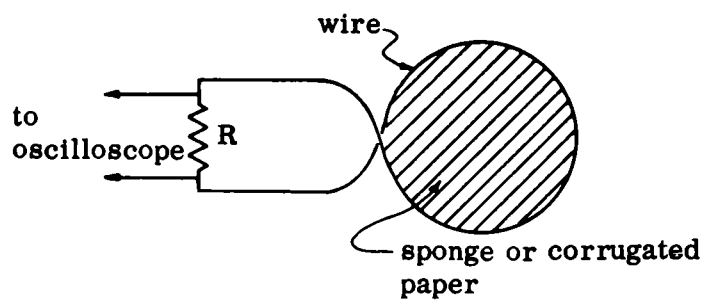
(d) Transformer.

If you have a transformer, you can show your students how it works. Use an alternating current supply and an oscilloscope (or an A.C. meter). Point out that the ratio of secondary to primary voltage is essentially the same as the ratio of secondary turns to primary turns.

If you have an extra transformer, unwrap it so that the students can see the windings. Point out the similarity to the iron ring electromagnet (the primary) and the coil hooked to the microammeter (the secondary). If there is room on the core, wind some turns on and show the students that an EMF is induced on this third winding if either of the others has an energizing alternating current.

4. *Currents Induced by Area Changes.*

(a) Wire on Flexible Form (a variation of Fig. 23–11).



The original loop area should be a bit smaller than the area of a strong magnetic field. If you pull the ends of the wire, the loop area will decrease, and the flux will decrease proportionately. This decrease in flux will produce a clockwise induced current if B is into the page. If your meter is too insensitive or your magnet not strong enough to show a visible deflection with a single turn, wrap several turns on a sponge and squash it with your hand. A sensitive oscilloscope will be better than a meter as a detector.

Rotate the plane of the loop by 90° and show that there is no induced voltage if B is in the plane of the loop, i.e., $B_\perp = 0$ even though B and A are large.

(b) Simple Flexible Wire.

Using a simple flexible wire, you can easily change the area by twisting the wire. The main problem will be the small size of the signal you can get from the single turn of wire.

5. *Currents Induced by Changing the Orientation of the Circuit Relative to the Magnetic Field.*

A small coil can be wound and taped to the end of a dowel or stick. The wires from the coil can be twisted together and brought out along the stick to an ammeter or to a resistor with a sensitive oscilloscope across it. Slow rotation of the coil will reproduce the effects shown in Fig. 23–12.

6. *Lenz's Law and Conservation of Energy.*

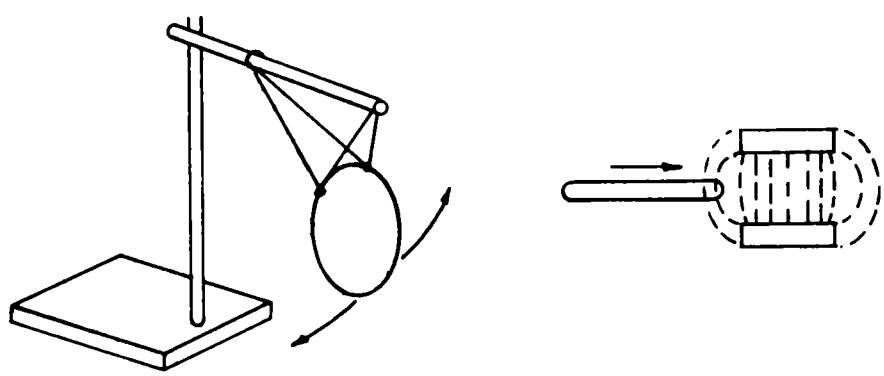
From the direction of the induced current as shown by the meter and the direction in which the coil is wound, find the direction of the magnetic field produced by induced current. Show that this field reacts with the field of the magnet to give a force on the magnet opposite in direction to the magnet's motion and that therefore the direction of the induced current is consistent with the conservation of energy.

A very convincing demonstration of the transfer of mechanical energy can be done as follows: Suspend a strong bar magnet by one end from the spring used in Experiment 32 so that it can oscillate up and down inside the solenoid used in Experiments 42 and 43. Connect the solenoid in series with a knife switch and an ammeter (about 0–3 amp), using heavy wire. With the switch open, set the magnet to vibrating, noting that it loses little energy and rises to very nearly the same height at the end of each oscillation. With the magnet still vibrating, close the switch and note that the vibration is quickly damped and the magnet loses mechanical energy, transferring it to the coil circuit where it appears as heat loss due to the flow of current in the wire. A low-resistance meter and heavy wire must be used to get a large enough rate of transfer of energy to make the damping of the vibrations apparent.

7. *Loop Swinging Freely as Pendulum.*

Make a circular loop of copper tubing (about $\frac{1}{4}$ inch is a convenient size) or of very thick copper wire. (If you have a multiturn coil of the proper loop area, you may use it.) Make a sturdy, electrically low-resistance clamp which can be used to connect the ends of the tubing which forms the loop. If you cannot arrange to close and to open the electrical circuit (loop) easily, make two identical loops, but solder the joint of only one while leaving the other electrically disconnected.

Suspend the loop so that it will swing freely as a pendulum and so that the area of the loop can swing between the magnetic poles, cutting the maximum flux.



When the electrical circuit of the coil is closed, the pendulum motion will be slowed noticeably as it passes through the magnetic field. The stronger the field, the more noticeable the effect (it goes as the *square* of the field strength) so a large magnet is much better than a small one. If the electrical circuit is open (or if the magnet is removed) the pendulum will swing freely. Note that if the area of the loop is smaller or bigger than the magnetic pole area, the coil can make small oscillations with no change in the flux. Consequently no currents are induced as the oscillations become small; in this case, small oscillations may persist.

8. *A Solid Metal Slab as a Circuit.*

The loop may be replaced by a flat piece of metal without changing the basic effect. If this metal swings between the poles of the magnet, electrical currents (called "eddy currents") are set up and the forces on these currents oppose the motion. A flat metal plate, provided it is thick, experiences a big retarding force if the motion is slow. Although the effect is large, it may not be so instructive for the students because they cannot visualize the current so easily. Another similar metal slab with many fine air gaps (produced by a saw blade) will swing much more freely.

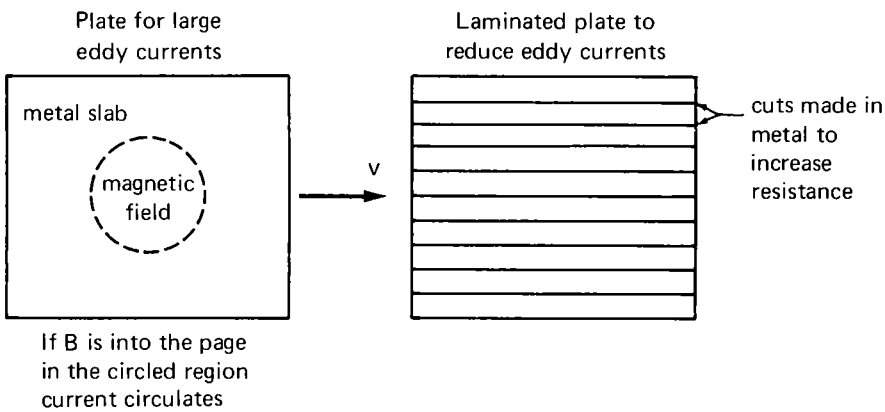
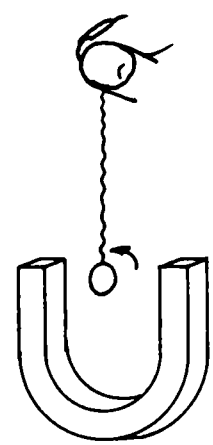


Plate for large eddy currents
Laminated plate to reduce eddy currents
metal slab
magnetic field
v
cuts made in metal to increase resistance
If B is into the page in the circled region current circulates

Note. This effect is often used to damp the motion of sensitive chemical balances. Magnetic damping has the important advantage that the retarding force opposing the motion is large when the velocity is great but reduces to zero when the motion is stopped or almost stopped.



9. *Retarding Force Opposing a Twisting Motion.*

Make a loop of wire using thick wire (large cross-sectional area) but make the area of the loop small enough so that it can be rotated between the jaws of the magnet. Suspend this loop on a fine string which twists easily.

When the loop is not near the magnet, twisting the top of the string will easily rotate the suspended loop. However, when the loop is between the jaws of the magnet, the loop moves more slowly.

Furthermore, the retarding force varies during the course of rotation. The maximum retarding force exists when the loop is moving and the plane of the loop is along the magnetic field. (This same effect comes up in Problem 1.) Students must realize the importance of $B_\perp$ to appreciate this.

Questions

1. A loop of copper wire is pulled through a magnetic field at a certain velocity. If the strength of the field is doubled, what happens to the force required to maintain the original velocity? (It is four times larger. The current goes up by a factor of 2 because the field is doubled. The force is proportional to I and B. Since both double, the force quadruples.) What happens to the power needed to maintain the motion? (Since the force is 4 times bigger, and v is the same, the power needed is multiplied by 4.) What happens to the power going into electrical form in the loop? How does it compare with the old value? (The induced EMF is doubled and the current is doubled. Therefore $I\mathcal{E}$ is 4 times its old value, in agreement with the four-fold power fed in.)
2. A 5-turn coil encircles a rectangular area, 2 cm $\times$ 3 cm, through which there is a uniform (perpendicular) magnetic field of 4×10^{-2} nt/amp-m. If the magnetic field is reduced to zero in 10^{-3} second, what will be the EMF induced in the coil?

   Answer:

$$\begin{aligned}
\text{Flux} = B_\perp A &= 4 \times 10^{-2} \times 2 \times 10^{-2} \times 3 \times 10^{-2} \\
&= 24 \times 10^{-6} \frac{\text{nt-m}}{\text{amp}} \\
&= 2.4 \times 10^{-5} \text{ volt sec.} \\
\text{EMF in one turn} = \frac{\Delta\phi}{\Delta t} &= \frac{2.4 \times 10^{-5}}{10^{-3}} \\
&= 2.4 \times 10^{-2} \text{ volt.} \\
\text{EMF in 5 turns} &= 1.2 \times 10^{-1} \text{ volt.}
\end{aligned}$$

This can be made easier by leading students through the steps: ask for (a) the flux, (b) the EMF per turn, and (c) the EMF in the coil.

The problem can be made more diffiult by (a) stating a flux change which differs from simply the initial flux going to zero, or (b) having the coil area make an angle with the magnetic field.

Atoms, Photons, and Matter Waves

CHAPTERS
24-27

In these chapters, information from scattering experiments leads to ideas about the structure of atoms, and information from photoelectric experiments leads to additional ideas about the nature of light. With these additional ideas, we are ready to apply the wave kinematics and the particle dynamics studied in earlier chapters to arrive at the modern wave-mechanical description of matter which is required for a detailed understanding of the world of the atom. The course ends with a detailed description of the hydrogen atom.

CHAPTER

24 The Rutherford Atom

Starting with the classical laws of physics (Newton's law and Coulomb's law) Rutherford attempted to explore the structure of individual atoms. The attempt was both a success and a failure. Rutherford's experiments led to a "planetary model" of the atom, in which a massive positive nucleus is encircled by one or more electrons which are held to the nucleus by the Coulomb force, much as planets are held by the gravitational attraction of the sun. This model gave an understanding not only of the results of the Rutherford scattering experiments, but also, at least qualitatively, of some of the other properties of atoms. The attempt was a "failure" only in that it led to a model which demonstrated rather conclusively the inability of classical physics to account for phenomena on the atomic scale; according to classical physics such an atom as Rutherford visualized would be unstable and should collapse in an extremely short time by emission of electromagnetic radiation. This is discussed in Section 26–1.

Most of the material in this chapter can be understood in a qualitative way by any interested person, with very little previous knowledge of physics. It is important that the obvious qualitative features not be obscured by an excessively detailed treatment. On the other hand many students will want to try hard to apply all that they have learned about mechanics and electricity. For each class a balance must be found between the extreme of a purely qualitative treatment and the other extreme of getting lost in quantitative detail. How you handle this problem depends on the ability of your class and the time you have to spend on this chapter. The point to be made here is the existence of a Coulomb field inside an atom, and the kind of experimental evidence which assures us that this is so.

In 1911 Rutherford proposed an atomic model. Rutherford saw that the validity of his ideas could be tested by observing the deflections of fast-moving charged particles as they traversed thin foils of various substances. This qualitative model is sufficient for discussing the scattering experiments. Be sure students realize that in Rutherford's time it was already known (as they know) that the negative particles of atoms were all of the same kind and were the light particles in atoms—that the positive parts were heavy.

You should also be sure they understand that a collision of an alpha particle with an electron cannot deflect the alpha particle appreciably since the mass of an alpha particle is about 7400 times greater than the mass of an electron. This is something like a cannon ball hitting a ball bearing. To be deflected appreciably the alpha particle must collide with something having at least a comparable mass.

Students may wonder what happens if an alpha particle passes outside a neutral atom. Unless the alpha particle passes inside the electron orbits, no appreciable deflection is expected. Since the atom as a whole is neutral, there is no field outside. It is only when the alpha particle gets inside the electron orbits that it is acted on by an electric force.

In trying to learn about the structure of atoms it is useful to study the dynamics of a collision between an alpha particle and a single atom. It would be difficult to draw simple conclusions if the observed deflections were due to the cumulative effect of many collisions. For this reason very thin foils are used in scattering experiments. Whether the foils are thin enough to make multiple scattering unimportant can be checked by observing how the fraction of alpha particles scattered through a given angle depends on the foil thickness. It will increase in direct proportion to the thickness only if the deflections are due to collisions with single atoms.

The fact that the majority of alpha particles can traverse a foil many atoms thick without deflection disposes of the idea that atoms are solid chunks of matter, but does not, by itself, tell us that an atom is mostly empty space. The mass of the atom could be distributed uniformly over its entire volume like a thin jelly. However, with such a model we could not explain the observation that in a few cases, very large deflections occur. It is only from a thorough study of the angular distribution of deflected alpha particles, as discussed in the following sections, that we can draw sound conclusions about the structure of the atoms.

Section 24–2 turns to the *orbits* of alpha particles scattered by the nucleus according to the Rutherford atomic model.

This section is qualitative and easy to grasp, but quite important. It should be clear to your class that if one knows the force law, the orbits can be calculated by a straightforward application of Newton's laws of motion. It turns out that if the force is given by Coulomb's law one can calculate the orbits, but the mathematics that is required is somewhat more advanced than the usual secondary school level. It should be clear, however, that in principle one could always calculate the orbits by applying Newton's law repeatedly, on successive small portions of the orbit, using the correct force in each interval to calculate the change in velocity produced by the force.

Before you introduce the potential energy arguments given in the text it might be helpful to question the class about the nature of the path of an alpha particle past a heavy nucleus. Start with a head-on case. Students should see that the alpha particle will simply slow up, stop, and retrace its path. Next take a path with such a large aiming error that the force is always negligible. Such a path will be a straight line. Now take an intermediate case. The direction of the force alone will show the students that the particle is deflected to the side.

Students may wonder why the atomic nucleus does not move when struck. It does, of course. In fact its change in momentum, $m\Delta\vec{\mathbf{v}}$, is as great as for the alpha particle. However, since the gold nucleus has a mass much greater than that of an alpha particle, its change in velocity is correspondingly less. We may usually neglect its motion. However, if the struck nucleus is light, we must consider its recoil. See Fig. 24–2.

The mechanical model shown in Figs. 24–5 and 24–6 should be very helpful in providing an understanding of the dependence of the scattering angle θ on the "aiming error" b. To avoid possible disappointments, however, it must be pointed out that it is a difficult task to build such a model with sufficiently high precision and with sufficiently low friction to yield anything but crude results.

It is possible to play the game on the figures themselves. (See under Demonstrations.)

At this stage students should know:

(1) For an alpha particle of given energy—or marble on a hill—a particular scattering angle corresponds to each aiming error.

(2) Large b's, little scattering; small b's, large scattering.

Section 24–3 describes the actual alpha-particle scattering experiments and shows that they are quantitatively explained by Rutherford's planetary atomic model.

We can calculate the orbits corresponding to different aiming errors. This provides a relationship between aiming error and scattering angle. The relationship depends on the assumed force law. To make the matter concrete, it should be helpful to carry out the computation for one or two simple cases. Problem 16 and Fig. 24–14 refer to the case of an abrupt force such as would arise in the elastic collision of two billiard balls or Dry Ice pucks. It will be worthwhile to discuss this simple case in detail.

The calculation for the more pertinent case of the Coulomb force is rather more complicated and cannot be taken up here. The result itself is quite simple and you may wish to use it to make the subsequent discussion more concrete. The Rutherford result (see Fig. 24–9), relating the "aiming error" b to the scattering angle θ is $2 \tan \frac{\theta}{2} = \frac{r_0}{b}$, with $r_0 = \frac{kQ_\alpha Q_N}{E_\alpha}$, where r_0 is the distance of closest approach for a head-on collision; r_0 is the distance at which the electric potential energy $\frac{kQ_\alpha Q_N}{r_0}$ equals the original kinetic energy E_α. For small angles of scattering θ, the relation is $\theta = \frac{kQ_\alpha Q_N}{bE_\alpha}$.

The relationship between "target area" and scattering probability is simple enough, but will probably require a thorough discussion. A simple example such as the following may help.

We have an array of identical targets distributed uniformly (or randomly) over a certain area, e.g., 15 targets per square meter. If we spray bullets uniformly (or at random) over the target area, what is the probability of hitting within a circle of radius 5 cm on any target? What is the probability of hitting within a circle of radius 2 cm on any target? In the first case, each target has an effective area of $\pi \times (0.05)^2 = 0.008 \text{m}^2$. There are 15 targets in a square meter so that the effective target area in one square meter is $15 \times 0.008 = 0.12 \text{ m}^2$. The fraction of shots that will hit the targets (within the 5-cm circles) is just the ratio of the effective target area to the total area, or in our example:

$$\frac{0.12 \text{ m}^2}{1 \text{ m}^2} = 0.12.$$

If we fire 10,000 shots, we expect $0.12 \times 10{,}000 = 1200$ hits. If we wish to be more choosy and only count hits within the smaller circle of 2 cm radius, the number of counted hits will be smaller. In this case the effective area of each target is $\pi b^2 = \pi(0.02)^2 = 0.001 \text{ m}^2$. We still have $N = 15$ targets in 1 m^2, so that the total effective target area in 1 m^2 is:

$$N_t \times (\pi b^2) = 0.015 \text{ m}^2 \text{ per square meter.}$$

The probability that any one shot will hit a 2-cm radius target is in this case 0.015. Out of 10,000 shots we shall get (on the average) only 150 hits.

The situation is much the same in the scattering experiments. If we have N_a atoms per square meter in a foil, each with an effective target area of πb^2 meters, the probability of a hit within a distance b from the center will be $N_a \pi b^2$. We know that if we hit within a distance of b we shall get a deflection through an angle in excess of θ_b, related to the aiming error b. For instance, with a Coulomb force we have:

$$b = r_o \Big/ \left(2 \tan \frac{\theta}{2}\right), \quad r_o = \frac{kQ_\alpha Q_N}{E\alpha}.$$

The probability of a deflection of more than θ_b is then given by

$$N_a(\pi b^2) = \frac{N_a {r_o}^2}{\left(2 \tan \frac{\theta}{2}\right)^2}.$$

Figure 24–13 in the text is a graph of this probability vs. θ.

The dependence of the probability of scattering of an alpha particle on its energy is contained in the above result. The probability is proportional to r_0^2 and therefore inversely proportional to the square of the kinetic energy of the alpha particle.

The dependence of the scattering on the energy is another test of the validity of the hypothesis that the scattering is due to a force which varies inversely as the square of the distance. The fact that this test was carried out reveals the thoroughness of the investigations of Rutherford and his co-workers.

It should be clear, qualitatively, that since the force exerted by a nucleus on an alpha particle depends on the nuclear charge, a scattering experiment can be used to determine this charge.

The scattering probability varies as r_o^2, and r_o, varies directly with the nuclear charge Q_N; the scattering probability therefore varies as the square of the nuclear charge.

Since an atom as a whole is electrically neutral, the number of elementary charges on a nucleus is equal to the number of electrons which surround the nucleus. Chadwick's measurements, therefore, give the number of electrons in each atom. This number turned out to be identical to the "atomic number," which had been previously used to classify elements on the basis of their chemical properties. This is closely related to the fact that the chemical properties of atoms are due to the arrangement of the atomic electrons.

Rutherford's alpha-particle scattering experiments gave only an upper limit to the size of nuclei, since no departures were observed from what is calculated under the assumption that the alpha particles do not penetrate the nucleus. It may be interesting to point out that later, scattering experiments (many of them going on today) particularly with high-energy electrons, lead to quite a detailed knowledge of the size of nuclei and about the distribution of charge within the nuclei. These experiments are even giving information about the size and charge distributions of the nucleons (neutrons and protons) themselves.

Film

"Rutherford Atom," by Robert I. Hulsizer of the University of Illinois, is a performance of the classical Rutherford Scattering Experiment of alpha particles by a thin gold foil. The experiment and its results are thoroughly discussed

so that this film can well bear the major burden of developing this chapter. Running time: 40 minutes.

Demonstrations

1. *Caution.* With radioactive alpha sources now readily available from several places, it may be tempting to think of demonstrating alpha-particle scattering. *Do not try it* unless you are highly expert and are very well equipped. Any source strong enough to do this experiment would be extremely dangerous. Alpha sources are dangerous because the alpha particles cannot penetrate the wall of a Geiger-counter and are therefore not readily detected if spilled. Often these sources "creep" and get spread around their containers —at least if packaged by an amateur. Confine yourself to observing the scintillations from very weak sources, such as radioactive watch dials.

2. The mechanical model shown in Figs. 24–5 and 24–6 should be very helpful in providing an understanding of the dependence of the scattering angle θ on the "aiming error" b. To avoid possible disappointments, however, it must be pointed out that it is a difficult task to build such a model with sufficiently high precision and with sufficiently low friction to yield anything but crude results. It is possible to play the game on the figures themselves.

If a crude model which is easy to make will suffice, stretch a rubber sheet over a frame and poke a piece of dowel rod or a pencil up in it to raise a hump. There will be a good deal of friction when a marble is rolled on such a model, and the force law it duplicates is $1/r$ rather than $1/r^2$, but it is easy to make and illustrates many of the features of alpha-particle scattering.

CHAPTER

25

Photons

LABORATORY. Experiment 46 Randomness in Radioactive Decay should be done with this chapter. It may be used at any point, but would be most appropriate with Section 25–5.

The first section of this chapter describes some experimental observations which cannot be readily understood from the point of view of a wave model of light, and thus introduces a basis for a reconsideration of a particle model. Students who have seen the film on the Millikan oil drop experiment should have little trouble picturing the experiment described in this section. The interpretation, however, may present some problems. The most difficult idea is that all the drops should lose electrons at the same time *if* the X rays were behaving like waves.

The photon model of light is now invoked to understand the surprising result described in Section 25–1. The arguments are very similar to those for scattering in the previous chapter. The frequency of occurrence of the encounters, that they occur randomly in time, and that they may occur very promptly, are all understood in exactly the same way as in that earlier work, although we now have the incident "particle" (the photon) absorbed by the atom, followed by ejection of an electron.

In the second section, students learn about some of the properties of photons, by examining several manifestations of the photoelectric effect:

(a) When monoenergetic photons are absorbed in a cloud chamber, the ejected electrons all have the same energy.

(b) The ejection of electrons from a metallic surface has a significant and sharp dependence on the wavelength of the light; it must be shorter than a certain critical value called the photoelectric threshold. The actual value of this threshold wavelength depends on the particular surface.

Both of these experiments show that the photons, which constitute the beam of monochromatic light or X rays, have a *unique* energy, not just an energy spread with the appropriate average value. The second experiment also shows that this unique energy depends on the wavelength, shorter wavelengths corresponding to higher energy.

Section 25–3 establishes the fact that the energy of a photon is proportional to its frequency. An experiment is described in which the kinetic energy of photoelectrons is measured as a function of the wavelength (or frequency) of the photons that ejected them. It is found that the maximum kinetic energy of the electron, E_{el}, is related to the frequency as follows:

$$E_{el} = h\nu - B.$$

The quantity B is the amount of energy required to pull the electron away from its atom or from the surface of the piece of material of which it is a part. The quantity $h\nu$ is interpreted as being the energy of the photon. When $h\nu$ exceeds B, the photon can not only pull the electron out of the atom or of the metal surface, but it can also give it kinetic energy. This explanation of the photoelectric effect was proposed by Einstein in 1905.

As with a mystery story, the whole problem of the photoelectric effect is understood in a simple way once an adequate interpretation has been discovered. This is especially true of the energy *distribution* of photoelectrons from a metallic surface, compared with the constant electron energy seen in a cloud chamber. In your development of the material of this section it will therefore be better to get to the conclusion quickly, and then to understand the various intermediate steps in the light of the photon picture.

The photoelectric effect from a metallic surface is sufficiently complicated that it is necessary to keep reminding students just exactly which functional relationships are being investigated in each part of the experiment.

(1) The photoelectric current (the rate at which electrons are ejected) is proportional to the *intensity* of the light under all normal conditions. No experiment has been described here to check this fact, since it tells nothing new about light. It would be expected with the classical wave model, and it is also an immediate consequence of the photon hypothesis.

(2) For constant light intensity, the total photoelectric current stops abruptly for wavelengths longer than the threshold wavelength. The implications of this fact were discussed in the previous section.

(3) For constant light intensity, and fixed wavelength, the ejected electrons have a distribution in energy (Fig. 25–6). As discussed in the text, this complicating feature arises because the target is a solid, which can absorb some of the electron's kinetic energy as the electron escapes from its surface. The distribution in energy under these conditions thus tells us something about the solid material. It is only the maximum electron energy, however, which is simple to interpret, and from which the photon energy properties can be deduced.

(4) For constant light intensity, the *maximum kinetic energy* of the ejected electrons has a simple straight-line dependence when plotted against the *frequency* of the light. Moreover, the straight-line dependence has the same *slope* independent of the material used, i.e., the slope is a property of the photons themselves. The interpretation of these results is that for light of a frequency ν (where $\nu\lambda = c$, the velocity of light), the photons have energy $h\nu$, where h is a universal constant (of dimensions energy times time). This is then the relationship we have been searching for—the relation between the wave characteristics of a light beam and the unique energy carried by its photons.

Section 25–4 considers the order of magnitude of the energies and frequencies involved in the photoelectric effect, and shows that photons carry momentum as well as energy.

The numerical value of h, the slope of the straight-line graph, Fig. 25–7, is obtained, and the energy carried by a photon of visible red light is calculated. Energy relationships become simpler when expressed in electron volts.

Detailed analysis either of the particle model or of the electromagnetic wave theory of light leads through the analysis of collisions of high-energy photons with single electrons, as seen in a cloud chamber. The momentum acquired by the recoiling electron is vectorially just the negative of the change in momentum suffered by the photon deflected in the collision.

This section shows concretely the need for the momentum carried by radiation, particularly photons, and how it preserves the law of conservation of momentum first introduced in Newtonian mechanics. The relation between the momentum and the wavelength will later be generalized to apply to the motion of any particle.

Having in the previous section established the qualitative features of the revolutionary changes in our thinking about light, it is now necessary to acquire some familiarity with the numerical magnitudes involved. The final chapter will go much more easily if students feel able to handle the relationships of this section, and if they obtain some feeling for electron-volt orders of magnitude. Even if you have not been assigning many of the HDL problems so far, it will be wise to go through Problem 11 in class.

We know from our earlier work with light that it exerts a pressure, i.e., momentum, so it is a logical necessity that photons do so. Here we see the experimental evidence. This will bring alive the whole photon concept; photons really act like particles.

Students may want to know when the Compton effect occurs and when the photo effect occurs. Baldly speaking, for low-energy photons (but those with enough energy so that a photoelectric process can go on), the probability of the occurrence of a photoelectric process is far greater than the probability of occurrence of Compton scattering. For photons of higher energy, the probability of a photoelectric process decreases rapidly and becomes smaller than the probability for Compton scattering. Therefore if we look at photons of relatively low energy, we are likely to see photoelectric processes. If we look at photons of higher energy, we are likely to see Compton scattering.

In this section the electron volt is introduced as the "natural" unit of energy for talking about the photoelectric effect. When visible light is used, the energies of the photons turn out to be a few electron volts. This is not pure coincidence. The volt was chosen as the "practical" unit of electric potential difference because it was the right size for talking about the potentials in a chemical cell—flashlight cell, 1.5 volts; lead automobile cell, 2.0 volts. These potentials are generated in chemical reactions which involve the exchange between atoms and molecules of one or two of the outermost electrons. Similarly visible light is generated in the same sort of reaction—the movement of the outermost electrons of an atom or molecule. It is not surprising that the energies are the same. Why this is *visible* light is, of course, a different question; but it is probable that light, to be visible, must make much the same kind of electrochemical reaction in our eyes.

Section 25–5 shows how individually random events, when observed in sufficiently large numbers, will develop a regular and predictable pattern. The motivation is that we must understand how photons can individually behave in a random way, but, when large numbers are studied, seem to follow the regular patterns (interference and diffraction) predicted by the wave theory.

An experiment is proposed to illustrate the statistical behavior of a group of random events. Instead of individual grains of light, a set of marked cards is used. In any one trial, it is not possible to predict which card will be drawn. But after many such drawings, the average number of appearances of each kind of card tends toward a pattern which can be predicted from the original distribution of numbers of each kind of card in the pack.

An experiment of the type described in the text is illuminating and worth doing, if you do not want to use HDL 16. If the experiment described in the text is done each student could make a set of 3 "L" cards, one "C" card and

6 "R" cards. Then each member of the class could draw, record the draw, shuffle, draw, etc., for about ten minutes. Then the individual results could be tabulated. The individual results will show the random fluctuations typical of chance, but the results of the class as a whole should show a pattern much closer to the average of 3 : 1 : 6.

You will not want to carry on the experiment too long especially since very many trials are needed—more than you have time for—to obtain results very close to the predicted ratios. When the main point has been made, you may wish to remind students that they really are familiar with this situation. They toss a coin when a random choice between two alternatives is required, and they all believe that this is a fair way to make the choice. If they tossed many times, heads would come up about as frequently as tails.

Our knowledge of interference phenomena was obtained with intense beams of light, involving many photons all going through the apparatus simultaneously. Is interference a result of the interaction of photons with each other, or is it a property of individual photons? The question is answered by describing G. I. Taylor's experiment in which he photographed the diffraction of light around the shadow of a needle. Even when the intensity of light was so low that only one photon at a time was in the space between the lamp and the photographic plate, a long exposure showed the same diffraction pattern as when the light was intense.

In the rest of this chapter and in the final chapter a ready understanding of wave concepts and properties is assumed. You may need to review the various topics that arise by referring back to Chapters 7, 8, and 9. A little time spent in reminding students of diffraction and interference, and their explanation in terms of the superposition of wave amplitudes, may be necessary.

The film "Interference of Photons" illustrates a modern version of the Taylor experiment. A photo-multiplier is used rather than Taylor's photographic plate so the arrival of individual photons can be heard and seen. One hears and sees very few photons at the known positions of the interference minima, and very many at the maxima. It is clearly demonstrated that "interference" effects are *not* the result of interactions between two or more photons. Note when the light strikes the photo-multiplier, it is not spread out over the interference pattern.

How can a single photon interfere with itself? For interference fringes to occur, light must travel coherently and simultaneously through both slits of a two-slit apparatus, but the individual photons arrive finally, each at only one definite place; and the interference pattern is a statistical accumulation. Can a particle travel through two slits simultaneously? Certainly not a particle such as we have used in the chapters on mechanics. On the other hand, could you find out which slit the photon actually went through? If you cover one slit, to ensure that the photon goes through the other one, then the interference pattern disappears, and you will only see the diffraction pattern of a single slit. In fact, it can be shown that any supplementary apparatus which will detect the passage of the photon through *one* of the slits will also destroy the coherence, and hence the interference pattern. The situation is certainly paradoxical. What is clear is that, with this new result, no detailed discussion in terms of classical waves (as with the ripple tank) or classical properties can tell us what is happening. A theory unlike any we have met before is required. The individual photons, each a particle in some sense, have wave properties. We shall meet an analogous situation with particles of matter in the later sections of this chapter. This dual nature of matter and light is one of the important

facts that must be faced by any thory of modern physics, yet it is most difficult to appreciate. There are some situations in which light behaves as waves. One example is a beam of light passing through a pair of slits and being detected by its heating effect on a thermocouple. There are other situations in which light behaves as particles. An example is the way the X rays knocked charge out of bits of matter in the Millikan experiment.

One approach to this "paradox" which is often effective is the following. Light is light. It is not a wave, and it is not a particle. Our concepts of waves and particles are idealizations of possible kinds of behavior. There is no reason to believe that light will fit in either of these categories uniquely; in fact it is fortunate than any analogy at all is possible. We try to force all new phenomena that we observe into some familiar category, so that we are capable of some kind of description and some degree of understanding. Nevertheless, we should not get in the habit of giving these efforts toward categorization an independent reality. Our ultimate goal is a consistent description of things as they are.

The final section of this chapter shows how the wave model and the particle model of light fit together. Electromagnetic waves of long wavelength (e.g., radio waves) are constituted of streams of photons whose individual energies are so minute that detecting apparatus is insensitive to anything less than enormous numbers of photons. All that is seen is the average effect, and this is what the wave model describes correctly. The experiments with light which established the wave model were also of this kind. The graininess of light will usually not be seen, although special experiments prove that it is there, even for very long wavelengths.

On the other hand, at extremely high energies, where individual photons are easily detected, there is usually no apparatus fine enough to examine their wave nature. A particle model gives most of the features. Our fundamental theory of photons must include both extremes, and neither the wave model nor the particle model is adequate.

If you have been able to show the photon films after going through the first part of this chapter, this section is the place to summarize the new knowledge. The reconciliation of the rather complex, mysterious new picture of photons with the wave model, so laboriously justified in an earlier part of the course, is very necessary to maintain confidence! Problem 20 of HDL will help to put the point across.

It will probably be best, however, not to go so far as to list all of the experiments performed, and precisely the information from each. The main points to be retained are just that light sometimes behaves like waves, and sometimes like particles, but that the picture is more complicated than just either waves or particles. Do not belabor this point. We cannot explain it any more than we can explain why gravity exists. We learn that it does exist, how it behaves, and how to calculate what it will do in all events. The same should hold for light. Learn the facts experimentally; make a simple(!) theory that describes them; learn how to predict what light will do in all circumstances. This we have done.

EXPERIMENT

Randomness in Radioactive Decay 46

Radioactive decay and the emission of light by atoms have the common property that they can be described only in statistical terms. That is, individual atoms or nuclei decay at random; the time of individual decay can be neither

predicted nor controlled. We can predict only the average behavior of a large number of them. This is the only experiment in the entire course where the student can investigate this randomness. That is the sole purpose of the experiment. The experiment may be done during the study of Chapter 25 if one limits the emphasis to the randomness, or it may be postponed until one wants to study radioactivity in particular.

To get a reasonable number of counts per second without running the danger of missing counts in a burst, have the radioactive sample at such a distance from the counter as to give about one count per second.

One counter can be used and the clicks relayed through a loudspeaker for the whole class. (If you have a counter that does not include an amplifier and loudspeaker, use the sound amplifier in a movie projector.) In this case, students should start their graphs of average counting rate versus total number of counts at different initial times, say a minute apart. Comparing these graphs, they will then find that, although the individual graphs look very different at the beginning, they all converge to the same average counting rate.

It is not necessary to count continuously for 20 minutes. A few breaks will make it easier, and the duration of the breaks will have no effect on the final counting rate.

The analysis takes a long time. To speed up the numerical work, have students work in pairs, one adding and one dividing, to find the averages. Fig. (a) and (b) show the results of an actual experiment.

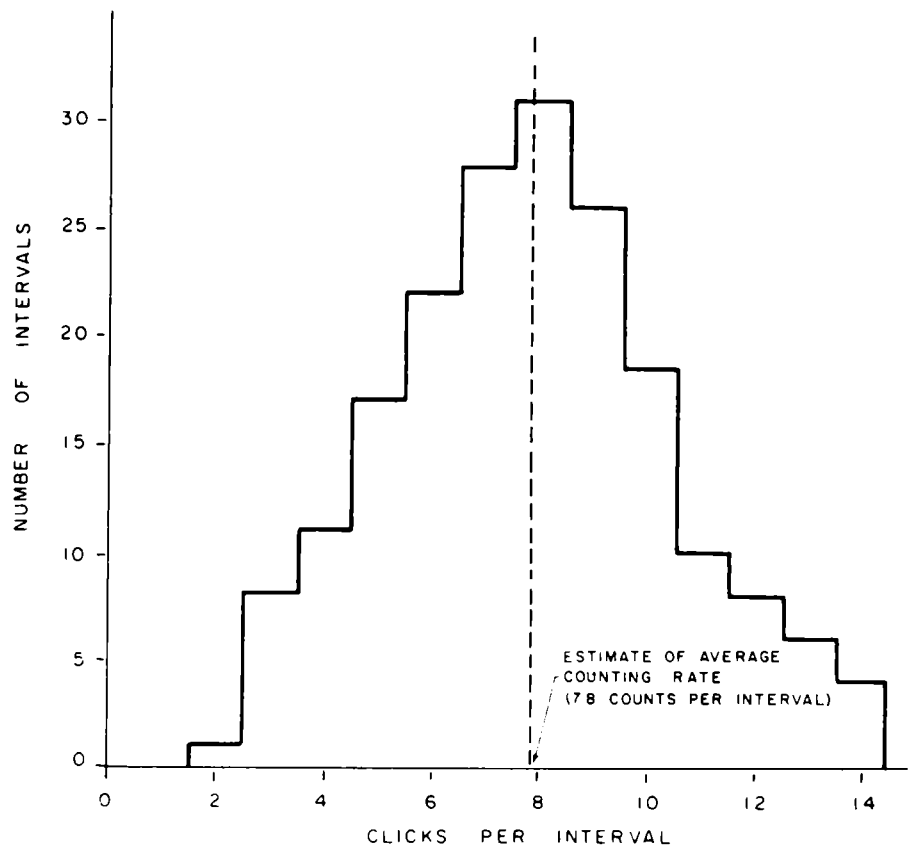


Figure (a)

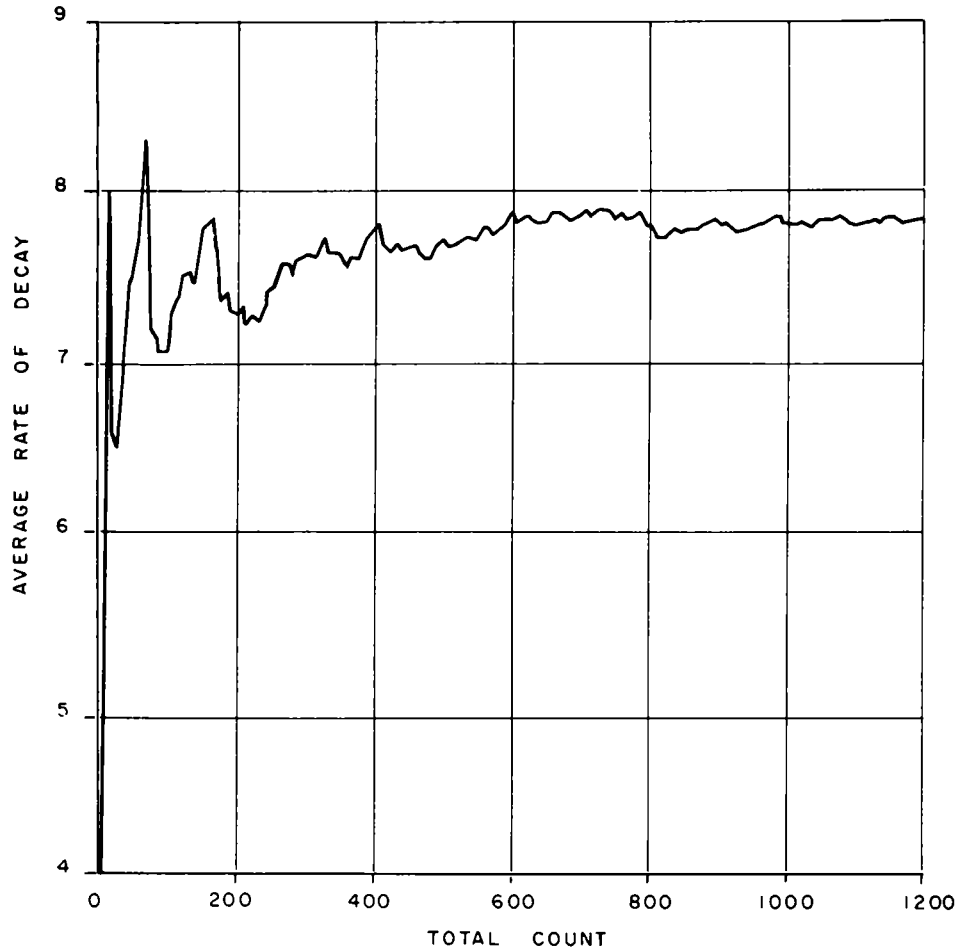


Figure (b)

Answers to Questions

The average counting rate obtained from the bar graph is within about 10 percent of the computed value.

For a small number of counts, the average counting rate changes drastically from point to point; as the number of counts increases, the fluctuations in the average counting rate decrease. The accuracy of the determination of the average counting rate increases with the total number of counts used in the computation. From the small and nearly constant fluctuations during the last few minutes, the student can conclude that counting for two more hours will not yield a substantially higher accuracy. He is not required to know that the accuracy is proportional to the square root of the total number of counts.

Assuming that the sample emits particles evenly in all directions, we can estimate the fraction hitting the counter by estimating the solid angle intercepted by the counter. To do this, we draw an imaginary sphere, centered at the sample, whose surface passes through the counter. We then estimate the cross-sectional area of the counter and divide it by the area of the imaginary sphere $4\pi R^2$. The fraction of the total number of emitted particles that hit the

counter will be equal to this ratio. Another point to consider is the efficiency of the counter (though most students should not be expected to know about this). Only a fraction of the particles hitting the counter register counts, and the efficiency varies widely with different counters and different sources. Alpha particles are not counted by most Geiger counters.

We cannot determine the half-life of the sample from our measurements. The half-life of the sample is too long to show a measurable change in the rate of decay in an hour. To find it, we must know the number of radioactive nuclei present in the source as well as the overall efficiency of the counting. Knowing the number of nuclei N in the source and the number of disintegrations per second $\Delta N/\Delta t$, the half-life is

$$T = \frac{0.693N}{\Delta N/\Delta t}$$

if ΔN is small compared to N. Students, of course, are not expected to know this equation.

A half-life of the order of an hour can be found directly by plotting the counting rate as a function of time. It can then be seen from the graph that the time it takes for the counting rate to drop to half its value (its half-life) is independent of the counting rate itself. Plotting the log of the counting rate yields a straight line, the slope of which is 0.693/(half-life).

Apparatus

1 Geiger counter
1 Radioactive source (wrist watch with radium dial)
2 Sheets rectilinear graph paper
1 Meter stick
1 Stopwatch or watch with sweep second hand

Amplifier and loudspeaker
Cloud chamber

Films

"Photons," by John G. King of M.I.T., demonstrates the particle nature of light with a photo-multiplier and an oscilloscope. Short bursts of very weak light are passed through an intermittent shutter and fall on the photo-multiplier. The output is examined with the oscilloscope showing that light does not come as an "even stream," but in "packages" at random intervals. Running time: 18 minutes.

"Interference of Photons," by John G. King of M.I.T. Using the same detection apparatus as in the film "Photons," a movable double slit is used to move an interference pattern across the photo-multiplier. The light intensity is reduced until calculation shows that at any given time, there is rarely more than one photon between the light source and the photo-multiplier. The resultant observation of the characteristic interference pattern shows the coexistence of wave and particle behavior in light. Running time: 13 minutes.

"Photoelectric Effect," by John Strong of the Johns Hopkins University (in color), shows a number of qualitative demonstrations of the photoelectric

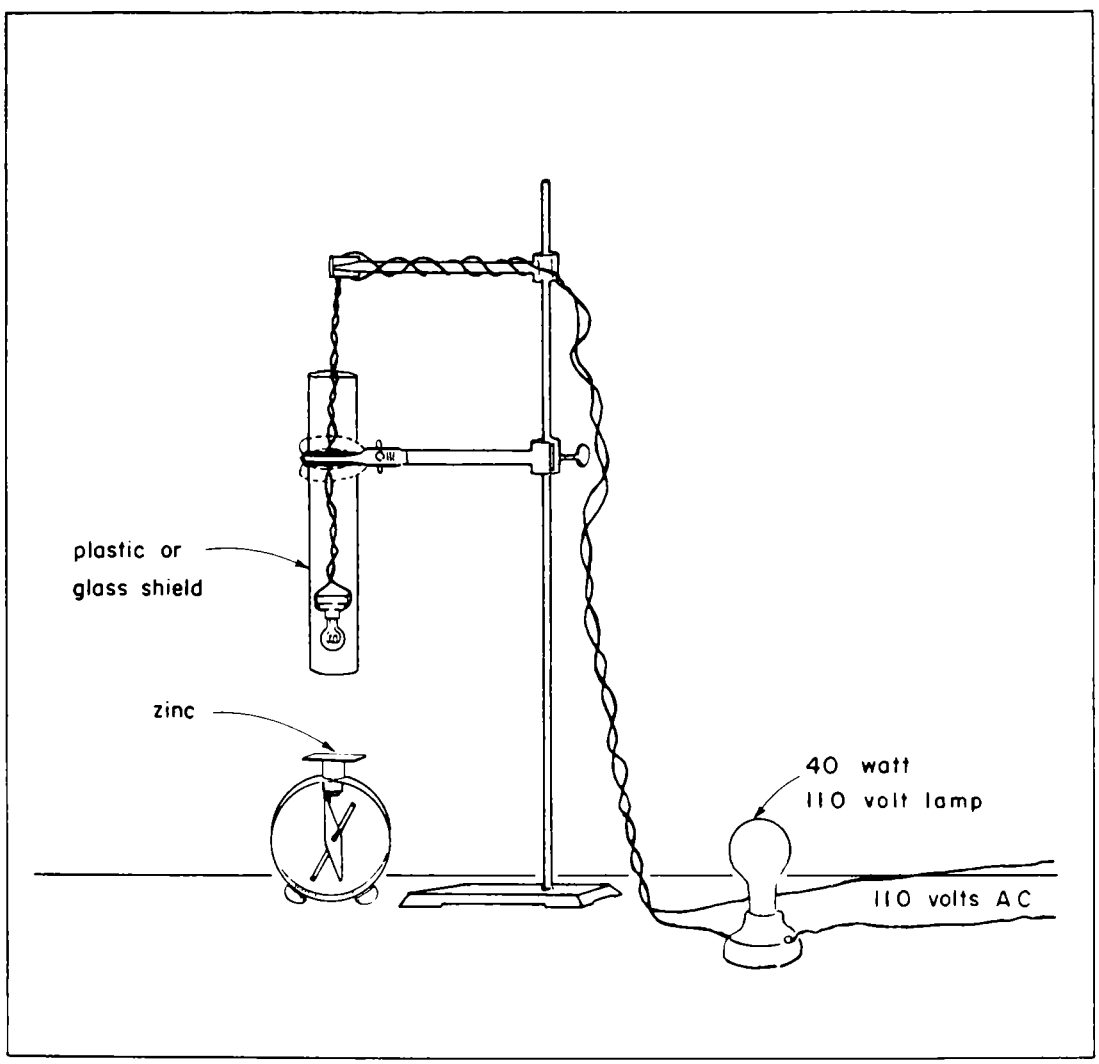
effect and performs a quantitative experiment in which the kinetic energy of the photoelectrons is measured as a function of the incident photon energy. This film can carry the discussion of Sections 4 and 5 very nicely. Running time: 28 minutes.

"Pressure of Light," by Jerrold R. Zacharias of M.I.T. could be used with Section 6 of this chapter if it has not been shown in connection with the work of Chapter 14. The film gives a remarkable demonstration of light pressure. Running time: 21 minutes.

Demonstration: The Photoelectric Effect

This demonstration shows that ultraviolet light ejects electrons from zinc, whereas visible light does not. An electroscope with a zinc or galvanized-iron top is used to show that ultraviolet light from a mercury-vapor sterilizing lamp will eject electrons from zinc.

The ultraviolet lamp must be connected in series with a 40-watt, 110-volt incandescent bulb to limit the current when operated from 110 volts A.C.



The bulb of the ultraviolet lamp, made of special glass, transmits the near-ultraviolet emitted by mercury vapor. It must be surrounded, except at one end, by an ordinary glass tube or by a thin sheet or tube of plastic, to prevent harmful ultraviolet light from reaching the eyes. The zinc plate of the electroscope should be rubbed with steel wool until shiny to remove any oxide or other impurities from the surface.

Charge the electroscope negatively and show that it does not discharge when moved close to a 100-watt incandescent light. Now move the electroscope beneath the ultraviolet lamp. The negatively charged electroscope will discharge rapidly. If, while it is discharging, a piece of glass or plastic is held between the ultraviolet light and the electroscope, the discharge will cease because the glass acts as a filter which absorbs the ultraviolet light.

Now repeat the experiment with the electroscope charged positively. No discharge takes place when the zinc is illuminated by the incandescent light. When illuminated by ultraviolet, the electroscope discharges considerably more slowly than when it was charged negatively.

An entirely different mechanism is responsible for this behavior. When the electroscope is negative, the electrons ejected from the zinc are pushed away. When the electroscope is positively charged, they are quickly pulled back to the plate and do not contribute to the discharge. Since they have only about 1 electron volt of kinetic energy, they cannot escape. The slow discharge from a positively charged plate is the result of ionization of the air by the ultraviolet light. This ionization is, of course, also present when the electroscope is negatively charged, but then its contribution to the total discharge is negligible.

The near-ultraviolet from the sterilizing lamp will not release electrons from copper or aluminum.

A General Electric OZ4S11 ozone lamp that takes a 40-watt, 110-volt incandescent lamp as a current limiter and fits an intermediate screw-type socket can be obtained from home-appliance stores. It is used in electric dryers and washing machines.

CHAPTER

26

Atoms and Spectra

This chapter begins by reviewing two difficulties which confronted physicists prior to the development of the quantum theory. These problems, resolved later in the chapter, are: (1) Electrons, orbiting around a nucleus, do not radiate energy and spiral in toward the nucleus, as classical physics would predict. (2) All atoms of the same chemical species are found to be identical in all respects. For instance they all have the same size. Classical theory permits orbiting systems to have an equilibrium at any radius and thus an arbitrary size.

Earlier chapters led to a kind of climax of classical electromagnetic theory —that accelerated charges give rise to electromagnetic radiation. Experiments with accelerating electrons and protons in gas discharge tubes seem to indicate that these atomic particles obey the laws of classical physics (Section 20–1). If Rutherford's atomic model is accepted, the atomic electrons should, according to this same classical theory, radiate light at a truly enormous rate. An ordinary-sized light bulb (about 100 cm^3) filled with hydrogen should emit several hundred watts of light. According to the same calculation it would not take very long (about 10^{-11} second), before our "light bulb" would go out, each atom having collapsed to the size of its nucleus (about 10^{-15} meter). All of this obviously does not occur. Hydrogen atoms emit no light at ordinary temperatures, and atoms apparently have stable sizes of about 10^{-10} meter. A quite drastic revision of the whole structure of physics is required to resolve these difficulties; classical physics is adequate to explain what happens on a macroscopic scale, but apparently fails at the atomic scale.

A short review of the properties of a classical atom, compiled from material of some of the previous chapters, is a useful way to begin the discussion of the chapter. The concepts of potential and kinetic energy of the electrons within an atom are crucial for an intuitive understanding of the atomic energy level picture which is developed in this section. The wave picture developed in the final chapter uses many of the characteristics of the classical picture.

Consider the classical model of the hydrogen atom:

(1) The proton has 1800 times the mass of the electron. As with a satellite around the earth, or the planets around the sun, we can neglect to a good approximation the motion of the heavy body, the proton, and regard it as fixed. (See Chapter 14 in general, and Chapter 16, Section 3.)

(2) The Coulomb force between the positive proton and negative electron is a force of attraction. The potential energy of the system when the electron is a distance r from the proton is $-ke^2/r$, if the potential energy is taken as

zero when the electron is far from the proton. (Chapter 19, Sections 3 and 4.)

(3) The total energy of the system is:

$$E = \tfrac{1}{2} m_e v^2 - \frac{ke^2}{r}.$$

This expression is similar to that for planetary motion, since the gravitational force and the Coulomb force have the same $\frac{1}{r^2}$ dependence. (See Chapter 16, Section 4.)

(4) We can have a circular orbit for *any* value of r. The velocity of the electron must conform to Newton's law:

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}; \quad v^2 = \frac{ke^2}{mr}.$$

(5) This last equation gives the surprisingly simple result that

$$\frac{1}{2} mv^2 = \frac{1}{2} \frac{ke^2}{r};$$

the kinetic energy is equal to just half the magnitude of the potential energy. Then, for a circular orbit, the value of r thus determines the total energy:

$$E = \frac{1}{2} mv^2 - \frac{ke^2}{r} = -\frac{1}{2} \frac{ke^2}{r}.$$

The total energy is negative as a result of our choice of the zero of the potential energy. $E < 0$ means that energy must be supplied to the electron to remove it completely from its atom. (See Section 16–5 on earth satellites and escape velocity.) By (4) above, the radius r can take any value. Then E, the internal energy of the atom, can have *any* (negative) value we want. The fact that we have restricted this part of the discussion to circular orbits need not be stressed. The results are more complicated in the case of elliptic orbits, but they are qualitatively the same.

The above points are all either familiar, or simple developments of earlier results. Only a little more time will be needed to expound them than is required to make them. One does not want to overemphasize the results of the classical model, since we know that it does not work, and we know vaguely that waves must be brought in. But it is important that the students have some concrete place from which to start. An atom is a dynamical system. It therefore has internal energy, connected in some way with the motion of the constituent parts. But we have not yet learned any details about this motion.

The Franck-Hertz Experiment

The PSSC film on this experiment will be useful. It helps get a good general picture of the experiment and will answer some of the likely questions concerning experimental techniques. Note, however, a point of difference between the filmed experiment and the one described in the text: The apparatus differs in that the gas occupies the region across which the potential V_A is applied. When the applied potential is increased to, say, 10 volts, only a few electrons ever acquire enough energy to excite the 8.84 ev level, as described in the text.

Instead, almost all undergo an inelastic collision as soon as they have more than 4.86 ev of kinetic energy, thus exciting the first state. They are thereby brought almost to rest and accelerate again over the remaining part of the gap, once again gaining enough energy to excite another atom to its first excited state. Thus what is detected is excitation to the first excited state. The processes described in the text do occur, of course. We see that from the emission spectrum (Section 26–3). But the higher excited states are not detected directly in a Franck-Hertz experiment of the type illustrated in the movie.

A discussion of the results of the experiment from a classical point of view, considering solely circular orbits for the atomic electrons, may be helpful. Classically one can see that it would be nearly impossible to hit an atom without producing changes in internal energy, since the bombarding electron would surely disturb the orbits of the atomic electrons. The fact that no change in internal energy occurs until the energy of the bombarding electrons exceeds some threshold value has no classical explanation. The natural explanation is that the atom can exist only in certain states of internal energy, and we can change the energy of the atom from one to another only by providing a large enough amount of energy.

Great pains are taken in the text to avoid saying that the modern picture of an atom is that of an electron moving in a well-defined orbit. The early attempts at quantum theory by Bohr were in terms of well-defined paths, the so-called quantized orbits. This picture has been superseded by the wave model. Therefore, any references to electrons moving in circular or elliptical orbits should be firmly linked to a classical view. It is useful to emphasize to the student that we do not, at this point in the text, understand exactly what is happening within an atom, and the Franck-Hertz experiment shows us that, whatever is taking place, the atom is able to accept energies only in certain amounts. Our natural conclusion is therefore that the atom can exist only in states of certain energy. In some manner which has not yet been discussed, these states of energy must be related to the state of motion of the electrons about the nucleus of the atom.

Several of the early problems in HDL involve considerations such as the above. In order to keep class discussion from getting too abstract and to aid the students' understanding of the exact process, you might ask such questions as:

(a) An electron with an energy of 6.20 electron volts collides with a mercury atom. With what energy might it bounce off? Answer: It might bounce elastically, retaining all 6.20 electron volts. It might bounce inelastically leaving the mercury atom excited to 4.86 electron volts and retaining $6.20 - 4.86 = 1.34$ electron volts of energy.

(b) A beam of 10.00-ev electrons are shot through mercury gas. With what energies could they come out? Answer: $10.00 - 8.84 = 1.16$ ev; $10.00 - 6.67 = 3.33$ ev; $10.00 - 4.86 = 5.14$ ev; and don't forget an electron that collides twice, $10.00 - 2(4.86) = 0.28$ ev.

(c) Students might rightly ask, "If one electron bounced off a mercury atom and left it excited by 4.86 ev, could another electron hit it and raise it to 6.67 ev, thus, in example (b) above, giving rise to electrons of $10.00 - (6.67 - 4.86) = 8.19$ ev?" In principle this could happen, but in practice it does not. Once the mercury atom is excited to 4.86 ev it radiates this energy away as a photon of light in about 10^{-8} second. Therefore very few atoms are in any state but the ground state at any one time, and the electrons can only raise them to excited states from the ground state.

Once the idea of atomic energy levels has been accepted, the hypothesis of this section should appear to the students to be a very reasonable application of photon properties. In Section 25–3 it was seen that an electron acquired energy to escape from a metallic surface by absorbing a photon. It is then reasonable to suppose than an electron may *lose* energy by *emitting* a photon. We expect the photon's frequency to be that given by energy conservation. The detailed results of observations on the emission spectra from mercury atoms are displayed in Fig. 26–5. The energy-level diagrams show what levels may be excited at the particular energy of the incident electrons, and the photon frequencies expected. The beautiful agreement with the observed line spectra should be completely convincing.

It may be helpful to remind your class of the experimental arrangement for observing spectra. One needs a line source, a prism or grating to spread the colors, and some means of observing the position of the lines. A photographic film is frequently used as a detector. The vertical lines on the film result from the fact that a line source is used. In the photographs in the text, lines of longer wavelength appear on the right-hand side. Note that it is the photographic *negative* which is shown—the black line represents the light, seen against the gray background.

Section 26–4 presents additional evidence for the picture of photons and the transitions between atomic energy states by a consideration of dark-line (absorption) spectra. Photon absorption, producing excitation of an atom, is in a sense the same process as that discussed in Section 26–3, reversed in time. The photon just passes by the atom unless its energy is exactly that which is required to excite one of the atomic levels; photons may excite atoms only if they have *exactly* the energy required. The photon vanishes completely, its energy (and momentum) being taken up by the atom.

Note the information in the caption for Fig. 26–6. Since the illustration is of a photographic negative, the light lines correspond to an *absence* of photons of that wavelength.

One question which frequently arises in discussions of absorption spectra is the following: Since an atom absorbs energy by being raised to an excited state, and then must decay back to the ground state by emission of a photon or series of photons of the same energy, why do we see dark lines? Wouldn't the re-emitted photon just make up for the one which was absorbed? The answer lies in the fact that the re-emitted photon goes off in a completely arbitrary direction, but our optical system is sensitive only to light traveling in one direction, that of the incident photons. Thus only very rarely do we see the re-emitted photons. Most of the re-emitted photons go off in some direction other than the one in which we are looking. If we could look in all directions at once and count each photon, it would appear that the incident photon was scattered by the atom.

The final section of this chapter introduces the numerical results for the energy levels of hydrogen, the simplest atom. These results will be explained with a wave model in the next chapter.

From now on, the discussion concerns the hydrogen atom. Classically it is the simplest atom. The level scheme of hydrogen turns out to have a relatively simple numerical description unlike that of the mercury levels.

Limitations of space and time force the text to omit the fascinating story of the discovery of the formula for the energy levels of hydrogen. The major break-through was made in 1885 by Balmer. Many atomic spectra had been observed by that time, but the frequencies of the lines observed did not appear

to obey any simple laws (such as being harmonics of a basic frequency, like the various possible vibrations of a string). Balmer studied the spectrum of hydrogen, which was particularly simple since only 9 lines were known. He found that the wavelengths, in angstroms, were given by the formula

$$\lambda = 3645.6 \frac{n^2}{n^2 - 4} \quad n = 3, 4, 5, \ldots, 10, 11$$

This formula was accurate to better than 1 part in 1000. We now know that it describes the frequencies of hydrogen lines in which the atom drops from the $n = 3, 4, 5, \ldots, 10, 11$ down to the $n = 2$ state. Rydberg, in 1890, found that rather similar formulas described the wavelengths of the spectra of other atoms. In particular, he found it simplest to express the spectra in terms of the *frequency* of the lines, as

$$\nu = \nu_0 - \frac{Rc}{n^2} \qquad n = 3, 4, \ldots$$

Where ν_0 is a constant, c the velocity of light, and R a constant called the Rydberg constant. This tantalizing simplicity could not be exploited for another twenty years. It required an understanding of photons, given by Planck (1903) and Einstein (1905), and the nucleus-plus-planetary-electrons model of the atom proposed by Rutherford (1911), before Bohr (1913) could devise a model for electron motion which would make some physical sense out of this simple expression. An understanding of the results in terms of wave properties of electrons was only achieved ten or more years later, starting with de Broglie.

Film

"The Franck-Hertz Experiment," by Byron L. Youtz of Reed College with an epilogue by James Franck. This film provides a useful illumination of the text material, although the apparatus used is somewhat different from that described in the text (see Development of Section 1). Running time: 30 minutes.

Demonstration

It will be desirable for your students to observe some line spectra. If you wish to demonstrate to an entire class at once, give each student a piece of the inexpensive grating material which is now available (PSSC laboratory kits, and other sources) and let them observe a colored flame set up in front of the class. An electric discharge tube rather than a flame spectrum perhaps brings the demonstration closer to the material of the chapter. Tubes of various gases are available from scientific supply houses, or perhaps a local neon sign shop will make you several. Tubes which will show the hydrogen spectrum are now available from several scientific suppliers, though they tend to be rather expensive. Burning ordinary hydrogen gas will *not* work because what will be seen is the molecular spectrum of H_2, not the atomic spectrum. A small high voltage transformer will be needed to excite the tubes. You should not try to reproduce the effects of Fig. 26–5. When the voltage applied is low, the spectra are present, but *very* faint.

Demonstration

If you have a fairly good spectroscope, you can illustrate absorption spectra for your class. By means of mirrors, reflect the sun's light into your instrument. Using a fairly narrow slit, you should see dark lines crossing the otherwise continuous spectrum. These are the absorption lines. There is a very prominent dark line in the orange-yellow. It is caused by absorption by sodium atoms. You can prove this by looking at a sodium flame spectrum at the same time. Reflect it in with another mirror. The bright yellow line will appear at the same place. This also proves, incidentally, that the yellow sodium line is caused by a transition from an excited state directly to the ground state, otherwise it would not show up in absorption.

CHAPTER

27

Matter Waves

At the end of this chapter, we shall be able to predict the energy levels of the hydrogen atom—and hence its spectrum—from the knowledge of the values of certain fundamental physical constants. That this can be done is the result of the whole sequence of experiments, laws, and logic that have been presented throughout the course—and which at times have seemed to be divided into unrelated compartments. But now the synthesis is evident.

The chapter begins in the introduction and Section 27–1 with the following ideas: (1) de Broglie's speculation that particles have properties of waves whose wavelength, λ, is related to the particle momentum, p, by the equation $\lambda = h/p$. This is the same relation that holds for light, with the same constant, h (Planck's constant). (2) Experiments with neutrons and helium atoms show that these particles do indeed exhibit interference properties characteristic of waves.

That matter, indeed *any* particle, possesses wave properties will be rather harder to accept than the particle behavior of light, since the idea of a particle of matter is deeply rooted in our common sense experience of the world. There is nothing wrong with that idea, but there are circumstances in which we see that Newtonian mechanics is inadequate, and probabilities and wave effects take the place of the definite Newtonian orbits to which they reduce in ordinary experience. It should be possible to offset this feeling that particles cannot have wave behavior by pointing out the symmetry that this new knowledge brings to our picture of the physical world.

The purpose of Section 27–2 is to see, quantitatively, when the wave nature of matter is important, and to reconcile it with our everyday experience.

It is important that students catch their breath here and digest this material. Much of the significance of the chapter would be lost if this section were skimped.

For various ordinary motion problems the appropriate numerical values for λ, h, and p could be inserted into de Broglie's relation, $\lambda = h/p$. (See Table 1 for one case.) In these situations, the masses are large, so that even for the small velocities usually encountered, p is tremendous on an atomic scale. Thus λ will almost always be so minute that any easy observation of the wave properties is impossible.

Electrons, with velocities characteristic of atomic phenomena (about 10^6 m/sec) have wavelengths of the same order of magnitude as atomic dimensions. (This is, of course, not an accident. See COMMENT on this section.) Thus wave properties of *electrons* are important in *atomic physics*. A ripple-tank picture of waves encountering obstacles (Figure 27–6) illustrates how the relative sizes of wave and obstacle determine how the wave will be affected by the obstacle.

Similarly protons and neutrons with energies characteristic of nuclear phenomena have wave properties which are important in encounters with *nuclei*.

Neither of these kinds of phenomena are readily observable in a direct way. One has to perform experiments of great complexity to see the wave properties.

The apparatus required for obtaining a suitable beam of electrons and investigating the wave properties is very complicated and expensive (beyond most college laboratory equipment). The film on "Matter Waves" uses a piece of research quality equipment in the hands of Lester Germer (one of the three persons who first gave experimental evidence for the existence of matter waves) to display the effect of electron diffraction in a convincing way. Alan Holden carries out the discussion and provides an optical analog experiment, producing the same pattern with light.

The compelling line of argument adopted in the figures in the text follows from juxtaposing results of light-wave experiments with the results of analogous experiments with electron beams. The results are closely similar, so we are led to conclude that particles of matter, in motion, exhibit wavelike characteristics. For most students this argument will be sufficient. A commentary in the figures, together with information about some of the detailed features of the processes involved in each case, is included in the comments. Your students should leave this section *convinced* that in some situations particles (all particles) show wave-like properties. They will spend the rest of the book in seeing how this wave-nature of particles enables us to understand some of the hitherto puzzling features of atomic physics.

Any theory of the wave-nature of particles must have the property that when the particle wavelength λ becomes very small compared with the significant dimensions of the problem (the size of the scatterer, for example) the results predicted must be exactly the same as those obtained by classical mechanics. We know this must be so, because experiment shows that ordinary-sized objects do obey Newtonian mechanics. The requirement that the new wave theory must correspond to classical mechanics where λ is negligible is a very important check on any new theory that is suggested, and in quantum mechanics it has been very helpful in deciding which avenues to follow.

For those of your students who have forgotten some of the characteristics of wave phenomena as discussed in Chapters 15–17, and for those who by now would like to see something "real," some ripple-tank demonstrations on the scattering of bodies of various size may be refreshing. Students might better understand the importance of the ratio of wavelength to size if you could reproduce the results illustrated in Figure 27–6, and the intermediate cases.

That electrons with energies characteristic of atomic phenomena—a few electron volts—have wavelengths comparable to atomic size is no accident. We shall examine the properties of atoms using the hypothesis that the atomic electrons move in a way governed entirely by wave dynamics. Atoms are the size that they are because they contain electrons, and electrons with energies

of a few electron volts have just that wavelength. The same is true, with altered numerical values, for protons and neutrons interacting with nuclei.

Section 27–3 ties together the new picture of matter and the earlier work on light.

Content

For both material particles and photons:

(a) Energy and momentum are conserved in every individual event.

(b) Waves determine the probable location.

(c) When λ is small compared to other relevant dimensions, a Newtonian approximation can be used.

(d) When λ is comparable to other relevant dimensions, a Newtonian approximation is not adequate.

Despite their similarities, photons are distinguished from material particles by the ease with which they can be created and destroyed.

We are now at the stage where the symmetry between the modern pictures of light and matter has been shown. These classically completely distinct entities both behave in similar ways. For example, all of the results and implications of the Taylor experiment in photons (Section 25–6) apply equally to electrons. The interference pattern of Fig. 27–5 is painted by even the lowest intensity electron beam. We can never tell, when we fire an electron into some apparatus, exactly where it will appear at the other end. The shaded pattern of light and dark bands calculated by the matter-wave theory only predicts where we shall *probably* find it, on the average.

All of this is true for light, too. But photons themselves were a new phenomenon, anyway, so it is hardly surprising that they do not behave in an accurately determinable way. With electrons—particles of matter—this is a radical departure from the comfortably certain Newtonian mechanics. It has been, and still is, a cause for discontent and speculation among philosophers and a few physicists. At present, however, the only mechanics we have—quantum mechanics—predicts the probability pattern of any given experiment, but can say no more about any individual trial and its outcome.

The following remarks are more for your information than for the student's instruction. It is fortunate that some problems involving the wave properties of electrons are amenable to wave arguments of the kind used in the chapters on light. We are now concerned with these, and a good plan at this stage is to minimize the philosophy, and move on to useful calculations which will reinforce the belief in the matter-wave theory. These comments have been included for use in answering searching questions as and when they arise.

Particles and Photons We have seen that some of the properties of particles are the same as some of the properties of light. The similarity is in some cases remarkable. It is difficult to show the particle properties of electromagnetic radiation, and difficult to show the wave-nature of particles, at least with ordinary laboratory apparatus. But let us be careful to make a fair comparison: between photons and particles of the *same wavelength*. For the electrons which made the interference pattern of Fig. 27–5, $\lambda = 8.6 \times 10^{-12}$ m. A photon with that wavelength has an energy of 20,000 electron volts, and X rays with this energy exhibit quite marked particlelike properties in atomic collisions. But the photon and the electron, each with $\lambda = 8.6 \times 10^{-12}$ m, *are* distinguishable. For one thing, an electric field will deflect the electron, but not the photon. The electron has a charge, the photon is electrically neutral. They are

different. So let us compare photons and *neutrons*. . . . We can go on in this way, comparing the behavior of photons with that of all the other particles we know about. There will always be some experiment which shows a difference between them. A photon is one kind of entity; we can call it a particle, provided we do not take that word in its classical sense. There are many other kinds of particles whose properties have been ascertained by experiments, and they all differ in some respect, although sometimes the differences are very small and hard to detect. One marked way in which photons differ from all other particles is in the ease with which the less energetic ones can be created. Compare a light bulb, which is creating many photons with wavelengths in the visible region, with an electrical circuit. In the circuit, passage of electrons is observed. You can never observe effects unless you make a continuous circuit for current to *flow* around. Electrons cannot be created at one end of a wire and destroyed at the other, producing a current. Their number stays constant. On the basis of what we have seen in the lab, this is then a very marked difference between photons and electrons. The other particles we have met—neutrons, protons, helium atoms—share the electron's property of being conserved in ordinary situations.

It is in fact possible to create electrons, neutrons, protons, etc. Readers of popular science magazines may have encountered antimatter; antielectrons (called positrons), antiprotons, antineutrons. It is not possible to create a single electron, for example, but a pair consisting of an electron and a positron can be created, provided the right conditions exist. A positron is a quite different particle from an electron—it has the opposite electric charge. More important, on meeting any other electron, both of the particles disappear, leaving behind energy in the form of photons. To understand the situation one needs Einstein's theory of relativity, but the facts are as follows. For the creation of a pair, an amount of energy equal to their combined mass times the square of the velocity of light is needed. Even for such light particles as electrons, an electron-positron pair requires an energy of 10^6 electron volts. So it is not *easy* to create particles. On the other hand, photons can be created *singly*, and the creation of a photon with wavelength in the visible region requires only about two electron volts of energy. With the right experimental arrangement, such photons are quite easily created; a light bulb does quite well. So in the distinguishing feature mentioned in the text, the emphasis is on the *ease* with which photons can be created. The antiparticles, by the way, are not only hard to create, they are very easy to destroy. That is why one does not encounter them in ordinary situations. The particles with which they annihilate are present in great abundance in ordinary matter.

We have met waves, in the ripple tank for example, as a type of disturbance of a system. The system, or substance, has many other measurable properties. We now know about electromagnetic waves and matter waves, both of which are propagated in a vacuum. It is natural to ask about other properties of the vacuum, and to think that perhaps it has properties like those of our well-known wave carriers (the water surface). But there is no need for such properties to exist. The vacuum has the property that it can carry electromagnetic and matter waves, and that is sufficient.

The attitude among most present-day physicists toward this question is described in the text. It can be elaborated as follows: The theories of light (electrodynamics) and of the atomic particles (quantum mechanics) are complete mathematical and physical schemes, so far as they go. The result of any proposed experiment can be predicted by them, and that is what we want of

any theory. The quantities in terms of which the theory is formulated—electric field strengths, say, for electrodynamics, or "probability amplitudes" for quantum mechanics—exist (i.e., can be nonzero) throughout all space. These are the quantities we manipulate—the field strength or probability amplitude—and the "vacuum" is just the name we give to all of empty space. It is not an entity in our theory. Its properties are solely that it can support these field strengths or probability amplitudes. Nothing more *need* be said of it in order to use electrodynamics or the quantum theory of particles. Whether or not the "vacuum" has other properties is at present not known. Are there other kinds of waves which can be propagated through a vacuum, waves associated with phenomena which have been so far undetectable? Are there measurements which can be made which involve the vacuum in a more active way than those we know how to make? There could be. Future scientists may find such things. We don't know.

Nonetheless, students will be puzzled by the question "What is it that waves?" The only examples of wave motion that they have encountered occur in systems with many other properties, and it is natural to wish to transfer all attributes of the models to the actual system under study. Clearly, in using a model you have to know where the similarity between model and actual system stops. Any answer of this kind is somewhat unsatisfying, unfortunately, until the students understand and can work with the actual theory of electrodynamics. They will then no longer need models and will gradually realize that the question "What is it that waves?" does not have to be answered in order to understand all that we know at present about the universe.

Section 27–5 illustrates how wave motion entails the possibility of discrete states of motion.

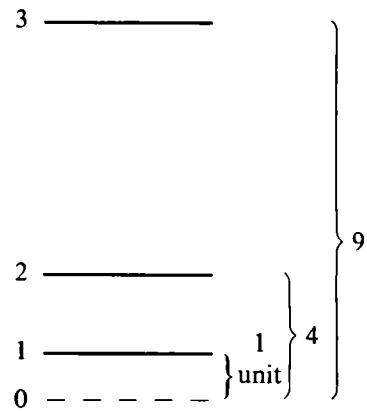
The wave properties of electrons, discussed in the last chapter, are brought forward as a possible explanation of the observed discrete energy levels. The levels must correspond to quite distinct wave patterns. Stationary or standing waves, which appear on a vibrating string and in many other situations, provide just this behavior. Detailed practical suggestions for their observation are given.

As emphasized in the text, some actual experiments with tubes or suspended slinkies are essential. The student who has determined for himself the unwillingness of the stretched tube to vibrate at any but its natural frequencies will not require further persuasion.

The connection between standing waves and the traveling waves that have occurred hitherto is necessary in order that λ, the wavelength, may be identified for later use in the de Broglie relationship. The fact that a standing wave may be viewed as the superposition of two equal traveling waves moving in opposite directions is sometimes rather hard to grasp. The experimental approach suggested in the text is sure to be helpful.

The wave mechanics of a particle in a box illustrates the methods we shall use in the Section 27–6. Classically, the particle would be reflected at each end of the box without change in speed. Thus its momentum would be constant in magnitude, and the de Broglie relationship can easily be applied. (In the next section the situation is less clear.) The most important point to be emphasized is that there is an energy state below which it is impossible to go. This suggests a possible solution to the puzzling problem of atomic *stability*. It may also be worthwhile to work out the energies of a few of the higher energy levels. The situation is clear-cut here, with no potential energy to worry about. In addition, this will help clarify the point that $1/n^2$ dependence of the energy states

of the hydrogen atom is very special. Other systems, such as this one, have different level schemes. To obtain the energies corresponding to the waves shown in Fig. 27–7, we note that the wavelengths are respectively $\lambda_1 = 2d$,



$\lambda_2 = \frac{2d}{2}$, $\lambda_3 = \frac{2d}{3}c$, $\lambda_4 = \frac{2d}{4}$, etc. In general, $\lambda_n = \frac{2d}{n}$. Substitution in the de Broglie relationship, $E = \frac{h^2}{2m\lambda^2}$ produces the result that $E_n = \frac{h^2}{8md^2}n^2$.

The expression is mathematically simple, but the relative positions of the energy levels are quite different from those of the hydrogen atom.

The last section presents a simplified wave picture of the hydrogen atom and a plausible derivation of the energy level expression.

This is the climax of the chapter. Spend enough time to get over the arguments. Remember, however, that to simplify the mathematics, the arguments are made approximate and not completely rigorous. The accuracy of the results is so striking that the qualifying statements may be overlooked.

The problem of finding the correct standing wave patterns is now more complicated; the wave is in three dimensions, and there is no well-defined boundary as there was for a stretched string and a particle in a box. By considering only circular orbits, however, and by assuming that the wave motion is confined to the region surrounding the classical orbit, a standing wave can be found by the methods we have developed. Instead of a wave between two boundaries, we now require that the wave must fit onto itself, i.e., that it goes around the inside with no detectable joint. This determines λ, and, by de Broglie's hypothesis, we can find p and the kinetic energy. The potential energy is calculated from the expression $V_e = \frac{kq_1q_2}{r}$, assuming that the electron is at a distance r for which the wave exactly fits onto itself. This determines the total energy of a set of discrete standing waves which represent the electron's motion about the proton. The expression obtained for the energy levels is correct. The level ordering, $E_n \propto -\frac{1}{n^2}$, is obtained, and the constant factor, 13.6 ev, turns out as a simple combination of e, k, m, and h.

The explanation can be checked against other atoms which should behave similarly. They do. Even the tiny effects of recoil of the nucleus can be accounted for by this simple model.

Laboratory

Experiment 47, The Spectrum of Hydrogen and Planck's Constant, allows students to measure accurately the wavelengths of the three visible spectral lines of hydrogen. Using these measured values with the theory of Section 27–6 provides confirmation of the validity of the predictions of the wave theory of hydrogen energy levels and also allows a determination of the value of Planck's constant by a method quite independent of the photoelectric determination of Chapter 25.

With the necessarily simplified treatment of the hydrogen atom given here it is somewhat fortuitous that exactly the right answers are obtained. The same approach applied to other problems—the energy levels of a rotating diatomic molecule for instance—gives only approximate answers. We can really expect no better, for we have applied a one-dimensional wave model to a truly three-dimensional problem. The students have not been shortchanged.

Film

"Matter Waves," by Alan Holden and Lester Germer of the Bell Telephone Laboratories, actually exhibits the wave nature of electrons in a modern version of the original Davisson-Germer experiment. The electron diffraction patterns are explained in terms of wave behavior and an optical analog experiment is performed to show that a nearly identical pattern can be obtained with light. This film presents about the only direct experience that a student is apt to have with the wave nature of matter and therefore is highly recommended. Running time: 28 minutes.

EXPERIMENT

The Spectrum of Hydrogen and Planck's Constant 47

The study of the spectrum of hydrogen offers a unique opportunity to combine precise measurements with the checking of a theory and the determination of a universal constant.

Students should be given sufficient time to align the spectrometer carefully and practice using it to locate not only strong but weak lines, particularly at the ends of the visible spectrum. A hard and sharp pencil is a must if the maximum precision of the instrument is to be exploited. A slide rule is not sufficiently accurate for the computations.

Care has to be taken to focus the telescope on infinity and to place the grating directly over the pivot point of the telescope.

Slight errors in alignment are reduced and the general accuracy is increased by observing all lines on both sides of the central maximum.

Particular care should be used in measuring the position of the mercury green line, since this line is used to calibrate the spectrometer. Any error in measuring its position will result in an error in the proportionality constant and thus be reflected as a systematic error in all hydrogen wavelength determinations.

The useful life of the hydrogen-tube source is only a few hours. After this time, lines in addition to the hydrogen lines will begin to appear, owing to contamination of the gas by elements present in the materials of which the tube is constructed. It is wise, therefore, to warn students to operate the tube only while actually making measurements with the spectroscope.

Answers to Questions

The image of the slit is at infinity if the telescope has been properly focused on infinity beforehand. When the image is sharpest, the slit is in the focal plane of the collimator lens, and the light from the slit emerges from the lens as a parallel beam.

The grating is parallel to the slit when the spectral line under observation is diffracted by the largest angle. When the grating is tilted with respect to the slit, the effective distance between the lines is increased, and therefore the diffraction angle is reduced.

From Fig. (a),

$$\lambda = d \sin \theta = d \cdot \frac{x}{L}$$

where L is the distance from the center of rotation to the notch at the center of the base, and x is the displacement perpendicular to the central maximum. The constant of proportionality is d/L.

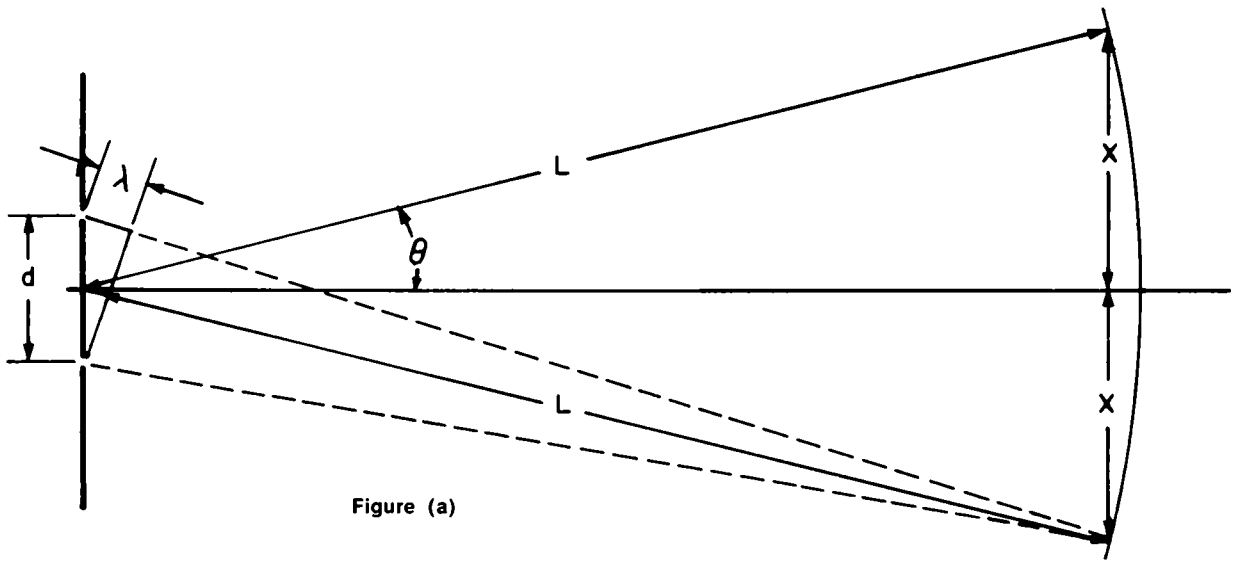


Figure (a)

We can express the constant of proportionality in terms of the calibration wavelength, in this case the green mercury line:

$$\lambda_{Hg} = x_{Hg} \cdot \frac{d}{L}$$

$$\frac{d}{L} = \frac{\lambda_{Hg}}{x_{Hg}}$$

Data obtained with the spectrometer are shown in the table. Error in λ may be about 0.3 percent, but should never be above 1 percent.

DATA OBTAINED WITH SPECTROMETER

SOURCE	COLOR	DISTANCE	λ (MEASURED)	λ (HANDBOOK DATA)
Mercury	Green	18.66 cm	Standard	5461 A
Hydrogen	1. Deep violet	14.03	4107 A	4102
	2. Violet	14.81	4335	4340
	3. Blue-green	16.62	4865	4861
	4. Red	22.39	6554	6563

Wavelengths from a handbook were included for comparison.

One can readily see three of the bright lines of hydrogen. By darkening the room and waiting until the eyes have become accustomed to darkness, one may be able to see a fourth line of hydrogen in the far violet. Some students may have physiological difficulty in seeing the deepest violet line.

The table below shows both the calculated ratios of the reciprocals of the wavelengths and the ratios obtained from wavelengths measured with the spectroscope. The calculated ratios were found from

$$\frac{\frac{1}{\lambda_1}}{\frac{1}{\lambda_2}} = \frac{\frac{1}{n_i^2} - \frac{1}{n_f^2}}{\frac{1}{n_i'^2} - \frac{1}{n_f^2}}$$

n	n'_i	n_f	CALCULATED RATIO	MEASURED RATIO
2	3	1	.8438	
2	4	1	.8000	
2	5	1	.7813	
3	4	2	.7407	.7426
3	5	2	.6614	.6615
3	6	2	.6250	.6267
4	5	3	.6836	
4	6	3	.5833	
4	7	3	.5359	

Planck's constant is determined from the relation

$$\frac{1}{\lambda} = \frac{2\pi^2 k^2 m}{h^3 c}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Using each value of measured wavelength in turn with the appropriate values of n_i and n_f gives an average value for h of 6.622×10^{-34} joule/sec. The error introduced from the measurement of λ is very small—about 0.3 percent. Since $h \alpha \lambda^{1/3}$, the error in determining h in this experiment is about 0.1 percent. This is comparable with the experimental errors in the other physical constants used in the above equation as given in handbook tables of physical constants.

Apparatus

1 Spectrometer
1 Ruler
Masking tape
1 Sheet plane paper
1 Hydrogen spectrum tube
1 Mercury spectrum tube
1 High-voltage source kit

Solutions to the Problems

CHAPTER 1

1–1*

Which of these objects are luminous (when in normal operation)? (Section 1.)

Camera	Chrome trim on car
Firefly	Electric stove heating element
Flash bulb	Diamond
Mirror	

Firefly, flash bulb, and electric stove heating element.

1–2*

How can you tell whether an object is luminous or nonluminous? (Section 1.)

Look at it in the dark.

1–3

We have seen that glass, although transparent, does not transmit all of the light that enters it, some of the light being absorbed. Is this also true of clear water? Be prepared to discuss what evidence you would look for to support your answer.

All known substances absorb some light; pure water does not absorb much. Students may give answers from their own experience, such as the darkness deep in the ocean or the reduced illumination at the bottom of a swimming pool. However, the absorption in these cases may be due partly to impurities in the water. Students can measure the absorption of light by a column of distilled water in a large graduate (40 cm high), either by comparing the intensity of a light source at the bottom of the graduate when it is empty and when it is filled, or by lowering a white disk into the water. Since the light absorbed by 40 cm of water is slight, a sensitive photometer would be required for the first method.

1–4

The color of the sun appears to change. For example, it looks redder at sunset than at noon. What can you conclude from this regarding the passage of light of different colors through the air? Can you relate this phenomenon to the fact that the sky is blue?

The sky appears blue because blue light is more easily scattered than other colors. At sunset, when the air path is longer and more blue light is scattered, there is a higher ratio of red to blue in the direct beams of the sun. Some students may suggest that blue light is absorbed more than other colors, and therefore more blue is missing at sunset when the path is longest. This would not, however, account for the blue sky.

1–5

Figure 1–11 shows clearly that smoke scatters light toward our eyes, even though the light was originally traveling in such a direction that it would not reach us. In view of this, why does a dense cloud of smoke overhead appear dark, rather than light? Write briefly. Be prepared to discuss.

The overhead cloud of smoke appears dark because it is illuminated from behind and some of the light is absorbed. In Fig. 1–11 the smoke also absorbs some light, but it appears bright in comparison with its dark background.

1–6

We have seen that when light from a white object falls on a photographic film it produces a deposit of black silver after development. This results in the familiar negative image on the film. If you shine light through the negative onto photographic film, after development what kind of image will you have?

A suitable class discussion should include a demonstration, if possible.

A positive image or "print" is made by allowing light to pass through the negative to another piece of film or photographic paper. The two should be in contact to avoid blurring the final image. Since more light passes through the clear portions of the negative than through the black portions, a dark region on the negative will become light on the positive or print. Thus a white object produces a dark image on the negative and a light image on the positive. (A Polaroid camera does not necessarily produce a conventional negative.)

1–7

Move this book toward you, with one eye closed or covered. When the book has just reached the point at which the print blurs, have someone measure the distance of the book from your eye. Repeat the experiment with the other eye. Are the two distances approximately equal? Try this with a few people of different ages, and record the results along with their ages.

(a) Is there a limit to the ability of the eye to adjust itself for clear vision?

(b) Is this limit the same for all persons, or for the two eyes of any one person?

(c) Does it vary, in general, with the age of the person? Compare your answers with those found by other people in the class.

This problem again brings to the student's attention the fact that he must focus his eyes.

The shortest distance for clear vision is about 4 inches for high school students, and may be several feet for older persons. Usually the two eyes are more or less alike, but not in all cases. The distance varies with eye fatigue.

1–8

Consider the changing appearance of the moon in the course of a month. Can you explain why a full moon rises close to sunset and a new moon rises close to sunrise?

The plane of the moon's orbit nearly coincides with the plane of the earth's orbit. (The angle between the two planes is about 5°.) A full moon would be seen when the sun, moon, and earth are almost in line, with the moon on the opposite side of the earth from the sun; hence the full moon would rise about at sunset. The new moon would be seen when the sun, earth, and moon are almost in line, with the moon on the same side of the earth as the sun; hence the new moon would rise about at sunrise.

1–9†

(See the Appendix in the text.)

1–10

(a) How long does it take light to reach the earth from the sun?

(b) If the light from the nearest star takes 4.3 years to reach us, how far away is the star?

(a) The distance of the earth from the sun is about 1.5×10^{11} meters:

$$t = \frac{d}{v} = \frac{1.5 \times 10^{11}\ \text{m}}{3 \times 10^{8}\ \text{m/sec}} = \underline{500\ \text{seconds.}}$$

(b) Since the speed of light is known in m/sec, 4.3 years must be converted to seconds.

$$d = vt = 3.00 \times 10^{8}\ \text{m/sec} \times 4.3\ \text{years} \times 365\ \text{days/year} \times 24\ \text{hours/day} \times 3600\ \text{sec/hour}$$

$$= \underline{4.1 \times 10^{16}\ \text{meters}} = 2.5 \times 10^{13}\ \text{miles.}$$

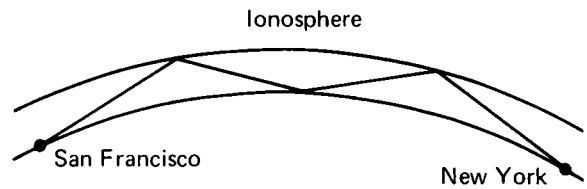
This would be a good time to talk with your students about checking units when they are solving problems.

1–11

Radio waves travel at the same speed as light in empty space or in air.

(a) How long does it take a radio signal to travel from New York to San Francisco, a distance of about 4.8×10^{3} km?

(b) A radar transmitter, which sends out radio signals of a particular type, when pointed at the moon receives a reflection 2.7 sec after the signal is sent. What does this experiment give as the distance of the moon from the earth?



A light beam cannot be sent directly from New York to San Francisco because of the curvature of the earth. However, a radio wave in the frequency range between 5 and 30 Mc will be reflected between an ionized layer of gas in the upper atmosphere and the earth several times as the radio wave travels between the two cities.

(a) $$t = \frac{d}{v} = \frac{4.8 \times 10^{3}\ \text{km} \times 10^{3}\ \text{m/km}}{3 \times 10^{8}\ \text{m/sec}}$$

$$= \underline{1.6 \times 10^{-2}\ \text{second.}}$$

(b) The total light path d is

$$d = vt = 3 \times 10^{8}\ \text{m/sec} \times 2.7\ \text{sec}$$

$$= 8.1 \times 10^{8}\ \text{meters.}$$

The distance to the moon is $d/2$, or

$$\underline{4.0 \times 10^{8}\ \text{meters.}}$$

1–12

(a) How does the 2.7 sec required for a radio wave to travel to the moon and back affect the conversation between an astronaut on the moon and ground station personnel?

(b) Extend (a) to astronauts on Mars when it is at its nearest or farthest position from the earth. (The distance from the earth to the sun is 1.5×10^{11} m and from Mars to the sun is 2.3×10^{11} m.)

(c) Extend (a) to an astronaut traveling close to the next nearest star, from which the light requires 4.3 years to reach us.

(a) The 2.7 seconds is the minimum time between the sending of the end of a question and the reception of the beginning of the answer.
(b) When the problem is extended to the distance between Earth and Mars, the times are 540 seconds and 2540 seconds. When Mars and Earth are on the same side of the sun and closest together, they are separated by $2.3 \times 10^{11} - 1.5 \times 10^{11} = 0.8 \times 10^{11}$ meters; when on opposite sides of the sun and farthest apart they are $2.3 \times 10^{11} + 1.5 \times 10^{11} = 3.8 \times 10^{11}$ meters apart.
(c) It will take 8.6 years to get an answer.

1–13*

In Fig. 1–9, is it possible to place the screen in such a way as to obtain the penumbra without the umbra? (Section 7.)

Yes; if the screen is moved back far enough, there will be only a penumbra on it.

1–14

Figure 1–9 shows how a fuzzy shadow is produced. How could you produce a sharper shadow of the same obstacle?

This is a good problem for class discussion; it can be discussed in class without being assigned.

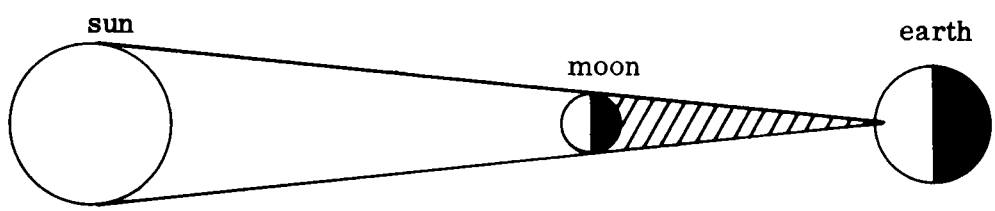
For a given light source, the shadow can be sharpened both by increasing the distance between the source and the obstacle, and by decreasing the distance between the obstacle and the screen. A smaller light source would also sharpen the shadow.

1–15

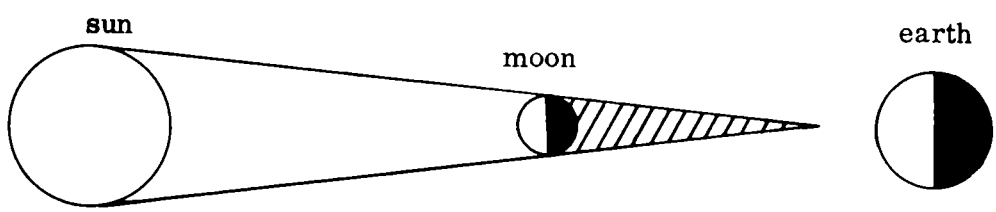
We sometimes see total eclipses of the sun by the moon, and sometimes annular eclipses. In the latter, a ring of light from the sun is seen around the edge of the moon.

(a) By drawing a diagram of the earth, moon, and sun, explain why two different kinds of solar eclipses occur. Do the distances of the moon from the earth and of the earth from the sun remain the same?
(b) The moon is about a quarter of a million miles from the earth, and the sun is about 93 million miles away. If the moon has a diameter of two thousand miles, what is the approximate diameter of the sun?

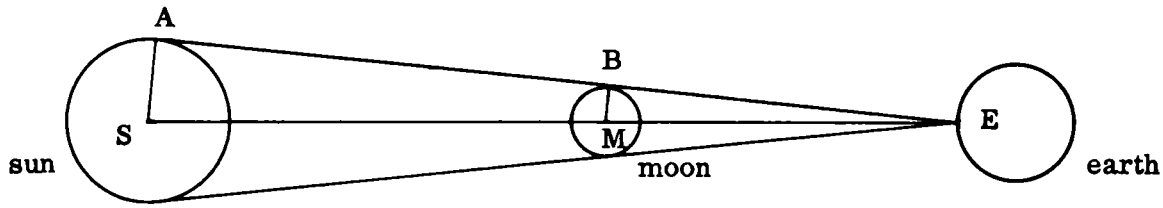
(a) A total eclipse occurs only when a part of the earth's surface is in the umbra, because it is only from points within the umbra that no part of the sun can be seen. The relationships between the sizes and distances of the earth, moon, and sun are such that, in some eclipses, the tip of the umbra just grazes the earth's surface. But in some eclipses, the earth is far enough from the moon (or the moon is



case 1 - total eclipse



case 2 - annular eclipse



close enough to the sun) that the tip of the umbra does not touch the surface of the earth. In this case, an observer sees the outer rim of the sun. Such an eclipse is called annular.

Most eclipses of the sun are partial eclipses, in which not only is no part of the earth's surface in the umbra, but the extension of the line between the centers of the sun and moon (this line also goes through the tip of the umbra) does not touch any part of the earth.

The fact that we have both annular and total eclipses shows that the distance from the moon to the earth or the distance from the earth to the sun must change. Both do.

(b) Since in the total eclipse, the tip of the umbra usually just grazes the earth's surface, we may draw the triangles ASE and BME. If the umbra extended over any large region of the earth's surface, the geometry would become indeterminate. Then $\frac{AS}{BM} = \frac{SE}{ME}$ and $AS = BM \times \frac{SE}{ME} = 10^3 \text{ miles} \times \frac{9.3 \times 10^7 \text{ miles}}{2.5 \times 10^5 \text{ miles}} = 3.7 \times 10^5$ miles. Thus the diameter of the sun $= 7.4 \times 10^5$ miles. (The accepted value of the diameter is 8.7×10^5 miles.)

1–16*

In the upper photo of Fig. 1–12 what would happen to the cone of light if the source were moved farther away? (Section 8.)

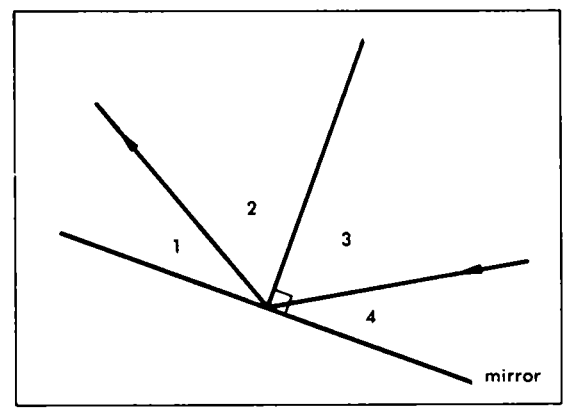
It would get narrower.

CHAPTER 2

2–1*

In Fig. 2–24, which is the angle of incidence and which is the angle of reflection? (Section 1.)

Angle of incidence is angle 3.
Angle of reflection is angle 2.



2–2*

How many different rays can strike a mirror at the same point and with the same angle of incidence? (Section 1.)

An infinite number of rays, which together form a hollow cone whose axis is perpendicular to the mirror surface.

2–3*

For a given incident ray of light striking a mirror, how many possible reflected rays are there? (Section 4.)

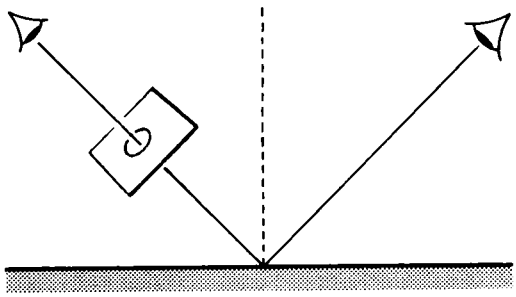
One.

2–4

If you can see the eyes of someone in a complicated system of mirrors, is it possible for him to see your eyes?

The laws of reflection imply that light paths are reversible. It is easy to see this experimentally in

a single mirror. Suppose, looking through a small opening in a barrier, you can see the eye of someone else reflected in a mirror; it turns out that the opening is placed so that he can see your eye also. This idea can be extended mirror by mirror through any system, however complex.



2–5

Place a plane mirror (one about 30–40 cm high is convenient) with its center approximately at eye level. Hold a meter stick vertically just in front of your face, with the middle of the stick at eye level; stand in front of and facing the mirror.

(a) Move toward and away from the mirror. Does your motion change the amount of the stick that you can see?

(b) Formulate a general rule connecting the length of the stick that can be seen with the height of the mirror. By making a ray diagram, show that this rule holds for all distances from the mirror.

(c) Clothing stores often have mirrors that extend all the way to the floor, designed to allow a customer to see his or her full length. Is it necessary for this purpose for the mirror to be as long as it is?

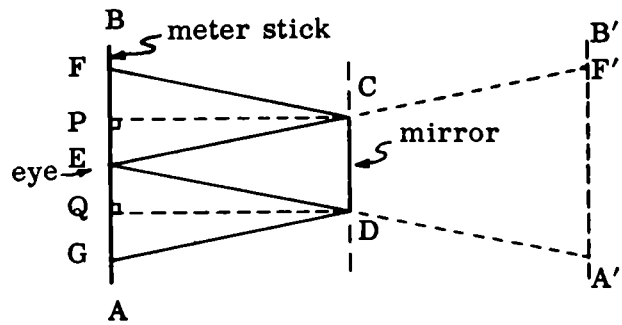
(d) If the shortest customer has eyes at a height of 5 ft. 0 in., what is the maximum allowable height of the bottom of the mirror from the floor if the customer's feet are to be visible?

(a) and (b) The length of the stick that can be seen is just twice the height of the mirror, regardless of how close the mirror is. To prove this, we draw the two rays, FCE and GDE, which hit the mirror at its upper and lower edges. From the second law of reflection, $\angle FCP = \angle ECP$ and $\angle EDQ = \angle GDQ$, and triangles FPC and EPC and triangles EQD and GQD are identical. Thus $FP = PE$ and $EQ = QG$. But $PQ = CD = 40$ cm and since E is centered in front of the mirror, PE equals EQ; and FG, which is the length of the meter stick which can be seen in the mirror, is twice the height of the mirror, or 80 cm. The same result is found by constructing the image $A'B'$ of the entire ruler and seeing how much of the image the "window" CD lets us see.

(c) No. The rays of light from the feet to the mirror and then to the eyes determine the necessary lower edge of the mirror.

(d) The maximum allowable height of the store mirror from the floor is 2′6″; half the "eye height" of the shortest viewer.

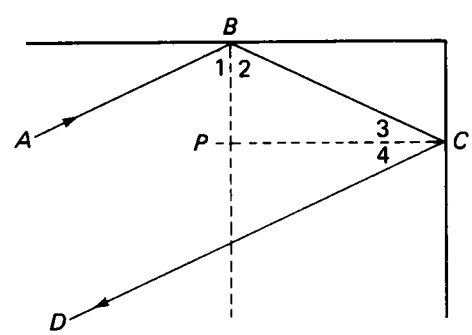
Since an image is always twice as far from the eye as the mirror is, by proportionality, the height of the image that can be seen is always twice the height of the mirror.



2–6

Two mirrors form a right angle (Fig. 2–25). A light ray AB is reflected from both mirrors. Prove that the reflected ray CD is always parallel and opposite in direction to AB, independent of the angle of incidence of AB. (This result is also true when three mirrors are at right angles like the corner of a box. This is how light from a laser is reflected from the moon.)

Your students will probably turn up with several different proofs, all acceptable. Here is one way:



Construct lines perpendicular to the mirror surfaces at points B and C; these perpendiculars will intersect at some point P. Evidently the angle BPC is 90°, and hence $\angle 2 + \angle 3 = 90°$. From the laws of reflection, $\angle 1 = \angle 2$, and $\angle 3 = \angle 4$; hence $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$. Rays AB and CD are crossed by a transversal (BC) so that the interior angles on the same side of the transversal are supplementary; therefore AB and CD are parallel.

2–7*

Measure the focal length in millimeters on the drawing of the parabolic mirror in Fig. 2–12. (Section 3.)

$f = 12$ mm.

2–8*

What would happen to the rays of light in Fig. 2–13 if a small plane mirror were placed at F facing the mirror? (Section 3.)

The rays would be reflected back to the parabolic mirror, where they would then be reflected out parallel to the axis.

2–9*

Why do we need to use only two rays in Fig. 2–14 to locate the image of the star? (Section 4.)

Two rays are enough, because the others pass through almost the same spot.

2–10

(a) What will happen if, in Fig. 2–18, the real light bulb is placed at the position where the real image was previously formed?

(b) Can you state a general rule about moving an object to the position of its real image?

(a) If a light bulb of the same size as the image in Fig. 2–18 were placed where the present image is shown, a new image of the same size as the present light bulb would be formed where the light bulb now is. The light would travel along the lines shown on the figure, but in the reverse direction.

If the actual light bulb were moved to the position now occupied by the image, a new image would be formed at the present position of the light bulb; this image would be larger than the actual light bulb.

(b) In general, if an object is moved to the position of its real image, a real image is formed at the original position of the object. (This would *not* be true if an object were moved to the position of its *virtual* image.)

2–11*

In Fig. 2–18, if you put your eyes at the position of the letter S_i, could you see the image? (Section 5.)

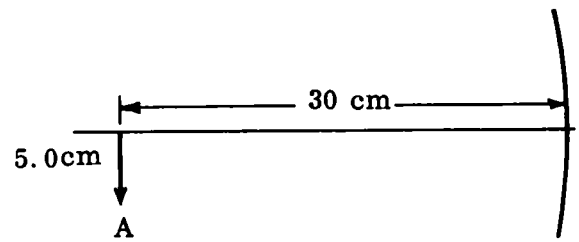
No; none of the light from the image will pass into your eyes.

2–12†

(See the Appendix in the text.)

2–13

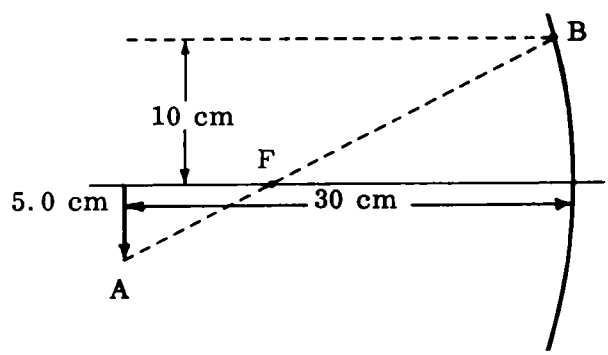
The image of a candle is 30 cm from the center of a concave mirror. The candle is 10 cm long and its image is 5.0 cm long. What is the focal length of the mirror?



This is a good problem to show how easy it is to use ray diagrams to derive a formula from the geometry of a problem.

We know that there is a 5-cm image 30 cm from the *mirror*.

We also know that the ray which passed through the focus to reach the tip of the image approached the mirror parallel to the axis, 10 cm above it (because the object is 10 cm high). Therefore we can locate B.



The line AB crosses the axis at F. From similar triangles f is 20 cm:

$$\left(\frac{10}{f} = \frac{5}{30 - f}, \text{ or } 60 - 2f = f, \text{ and } f = 20 \text{ cm}\right).$$

This problem could also be done by formula.

Since $\frac{H_i}{H_o} = \frac{S_i}{f}$, and we know that $S_i + f = 30$,

$$\frac{H_i}{H_o} f + f = 30 \text{ cm},$$

$$f = \frac{30 \text{ cm}}{1 + \frac{H_i}{H_o}} = \frac{30}{1.5} = 20 \text{ cm}.$$

The geometry is more instructive and easier than the algebra. Be sure to show it to your class if you assign this problem.

2–14

How large an image of the sun will be formed by the Palomar telescope, whose focal length is 18 m? The sun's diameter is about 1.4×10^9 m, and it is 1.5×10^{11} m away.

$$\frac{H_i}{H_o} = \frac{f}{S_o}; \qquad H_i = H_o \frac{f}{S_o}.$$

$$H_i = 1.4 \times 10^9 \text{ m} \times \frac{18 \text{ m}}{1.5 \times 10^{11} \text{ m}} = 0.17 \text{ m}$$

$$= 17 \text{ cm}.$$

2–15

What is the focal length of a plane mirror?

This problem should not be considered until after students understand Section 3. It is best used for class discussion. Students can be asked what happens to the focal length as a parabola is flattened. Eventually, the parabola will be so nearly flat that the difference between a parabolic mirror and a plane mirror will be negligible—the focal length of a plane mirror is infinite. Properly speaking, the focal length of a plane mirror is undefined. It is only in the limit of a nearly flat parabola that we can find a focal length.

2–16

The distances of an object and its image in a concave mirror are often measured from the center of the mirror, instead of from the principal focus. We call these distances D_o and D_i respectively. We have $S_o = D_o - f$ and $S_i = D_i - f$ where f is the focal length. Using these relations, show that from $S_o S_i = f^2$ follows $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}$.

This problem shows that the Gaussian form of the "lens" equation follows algebraically from the Newtonian form employed in this book.

$$S_o S_i = f^2$$

$$(D_o - f)(D_i - f) = f^2$$

$$D_o D_i - f D_i - f D_o = 0.$$

Divide by $D_o D_i f$ to get:

$$\frac{1}{f} - \frac{1}{D_o} - \frac{1}{D_i} = 0; \qquad \frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}.$$

2–17*

Is a real image formed by a parabolic mirror ever larger than the actual object? (Section 6.)

Yes; whenever $S_o < f$.

2–18*

Is a virtual image formed by the parabolic mirror of Fig. 2–23 ever smaller than the actual object? (Section 2–10.)

No.

2–19

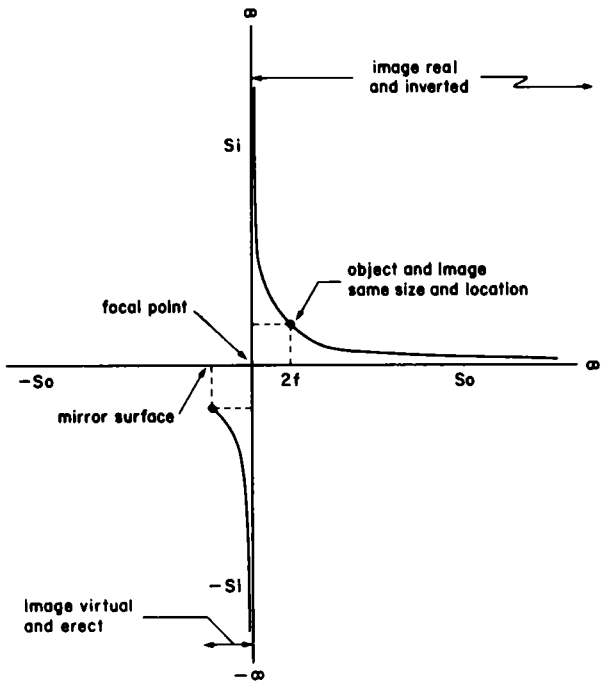
In Section 2–5 the question is asked: At what distance from the principal focus must the object be placed so that the image will be at the same place? Show that this question has two answers and explain the second answer, which was not discussed in the chapter.

This problem on virtual images forces the students to think about the properties of a concave mirror.

We can write down in a straightforward way $S_o S_i = f^2$. If object and image are to be in the same place, then surely $S_o = S_i$ and it follows $S_i = S_o = f$. Students on their algebraic toes will see that there is another solution to the equation $S_o S_i = f^2$, i.e., $S_i = S_o = -f$. Do not introduce sign conventions in discussing this problem, but merely point out that the object could be placed one focal length toward the mirror from the principal focus, i.e., at the mirror itself. The image will be virtual and at the mirror itself. You should probably have a concave mirror (a shaving mirror will do) handy to defend this answer experimentally!

2–20

Be prepared to discuss what happens to the image as the object is moved along the principal axis of a concave mirror from infinity to the mirror.



When the object is at infinity, the image is at the principal focus. As the object moves in, the image moves out from the principal focus, becoming gradually larger, and is real and inverted. The object and image meet at a distance f in front of the principal focus (that is, a distance $2f$ in front of the mirror), where both are the same size. As the object moves in toward the principal focus, the image moves farther out, continuing to increase in size. When the object reaches the principal focus, the image is at infinity. As the object moves closer to the mirror, the image moves in from *behind* the mirror and is virtual and right-side up. When the object meets the mirror, the image is also at the mirror and is the same size as the object.

The graph shows the position of the image versus the position of the object, using the principal focus as the origin. Positive positions are in front of the principal focus, negative positions behind it.

2–21

Our discussion of curved mirrors has been limited to the inner, or concave, surface. The outer, or convex, surfaces of these curves will also produce images. Such mirrors are called convex mirrors and are often used as side mirrors on a car or as ornamental mirrors in a room. Using the laws of reflection and assuming the surface to be parabolic, demonstrate the following facts by suitable constructions.

(a) The area reflected to the eye by a convex circular mirror is larger than that reflected by a plane mirror of the same diameter in the same position.

(b) Light rays parallel to the axis reflect as though they were coming from a point behind the mirror. This is the principal focus of the mirror. It is called a virtual focus. Why?

(c) Rays starting from a fixed point on the axis are reflected in such a way that they seem to come from a point on the axis behind the mirror. The image is therefore virtual.

(d) The image formed is smaller than the object and is not inverted.

(e) As the object moves in from a great distance, the image moves toward the mirror.

(f) There is a limit to the distance of the image from the mirror—that is, this distance can never be greater than a certain value. What is this value? Try drawing ray diagrams.

This problem requires the student to reproduce, for a convex mirror, a large part of the development which the text does for a concave mirror.

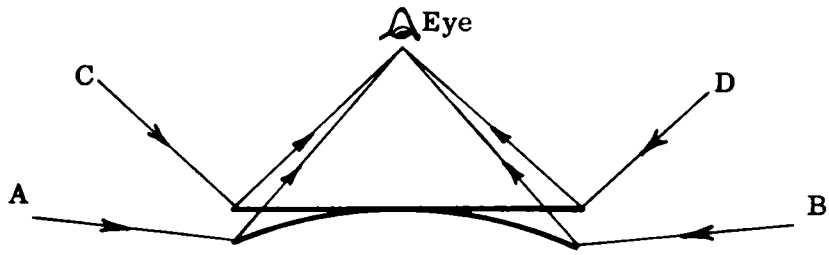
This exercise may be assigned after students have gotten a thorough understanding of the entire chapter. Convex mirrors are not considered again in the course, and it is not at all necessary to assign this problem if you are short of time. It does, however, serve as a high-level check on understanding. If a student can do this one, you can be pretty sure he understands the essentials of the chapter. Under no circumstances should sign conventions be introduced here.

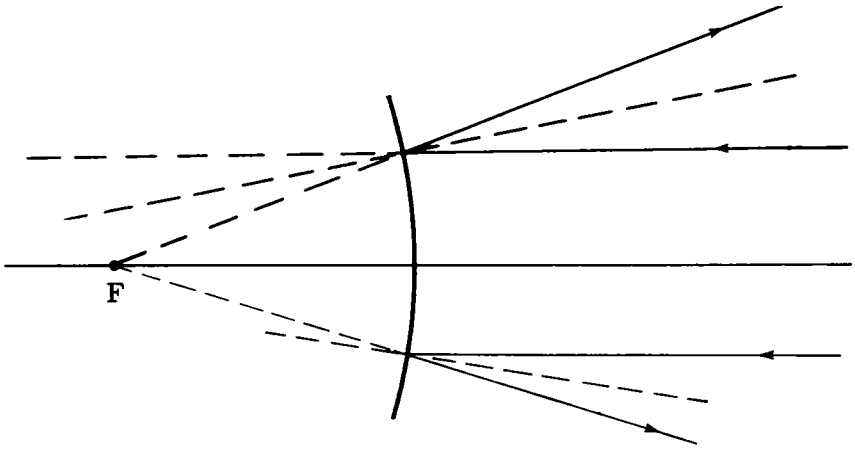
(a) As is evident from the diagram, light in the arc from A to B can reach the eye from the convex mirror, while only light in the arc from C to D will reach the eye from the plane mirror. Thus the convex mirror reflects a larger area to the eye than the plane mirror.

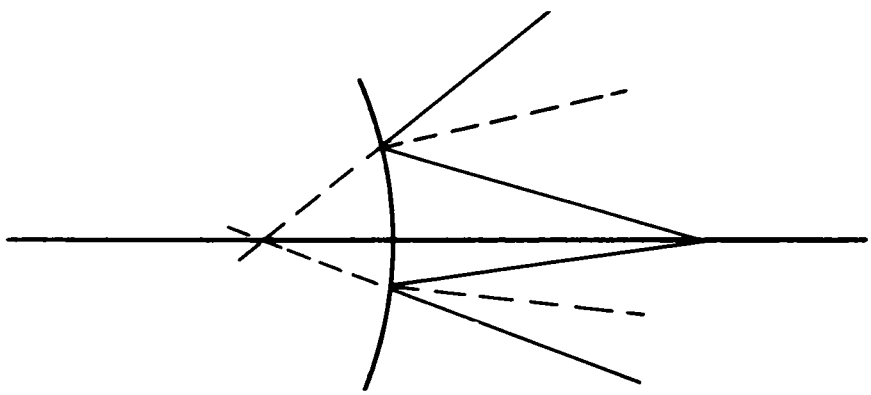
(b) After drawing a set of light rays parallel to the axis and constructing their corresponding reflected rays, the student will observe that the reflected rays appear to come from a point behind the mirror. This point is called a virtual focus because light rays never really pass through the focus.

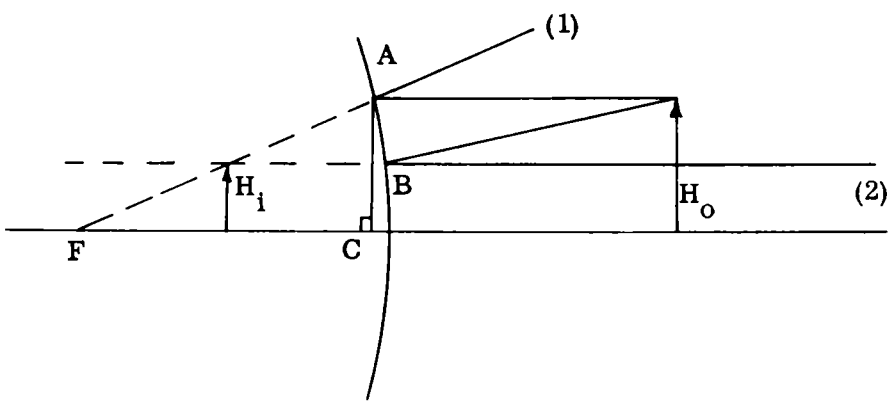
(c) A rough construction will suffice here, and it is obvious that the rays will diverge so as to appear to come from a virtual image on the axis extended behind the mirror.

(d) By drawing principal rays, we see that the top


Eye
C
D
A
B


F




(1)
A
H_i
B
H_o
(2)
F
C

of the image must always lie on the line FA, so that H_i is always less than AC. But since $AC = H_o$, the image is always smaller than the object. It is also upright since the principal ray (2) (parallel to the axis) is always above the axis if the object is above the axis.

(e) In the diagram for part (d), we see that as the object moves in closer to the mirror, the point B where principal ray (2) hits the mirror moves nearer A. Since the distance from B to the axis is the same as the height of the image, the image becomes larger. But to do this, it must move in toward the mirror, because its top must always be along the line AF.

(f) Since, as the object comes closer to the mirror, the image also comes closer to the mirror, the distance of the image from the mirror must be greatest when the object is very far away. When the object is very distant, the rays reaching the mirror are almost parallel, and the image is just about at the principal focus. The image can never be farther from the mirror than the principal focus.

2–22

Compare Fig. 2–11, a paraboloid which reflects all rays parallel to its principal axis through its principal focus, with Fig. 2–21 and explain why all parallel rays striking the surface of the sphere would not reflect through F.

The intention of this problem is to stimulate the students to think qualitatively about spherical aberration. It can be used as a quantitative problem, if extended to include ray diagrams.

The spherical surface is bent toward the principal axis with respect to the paraboloid. This increases the angle of incidence and thus the angle of reflection, so the reflected rays cross the principal axis between F and the mirror.

CHAPTER 3

3–1*

Which is the angle of incidence in the drawing in Fig. 3–26? Which is the angle of refraction? (Section 1.)

Angle of incidence = ∠2.
Angle of refraction = ∠4.

3–2*

Compare Fig. 3–1 with the upper right-hand photo of Fig. 3–2. In which is the light bent more? (Section 1.)

In the photo of Figure 3–2.

3–3

Can you give a reason why the incident ray, refracted ray, and the normal should all be in the same plane? Can you imagine a material in which this would not be true?

In order to *prove* that in ordinary optical media the two rays and the normal lie in a plane, one must assume Maxwell's equations or their equivalent. Obviously, students will not be able to give such an answer, but thinking up arguments to support their own answers is a good intellectual exercise for them. It is important that they realize they are not giving *proofs*.

There are two good responses to this question. First, if the two rays and the normal did not lie in one plane, the refracted ray would have to have some reason for traveling on one side or the other of the plane formed by the incident ray and the normal. Usually there is no such reason. Second, we believe that the two rays and the normal lie in one plane because this is what we observe when we test materials in the laboratory.

In doubly refracting materials such as a calcite crystal, the internal structure provides a reason for deviating from the plane defined by the incident ray and the normal. A detailed discussion of the exceptions is not necessary.

3–4*

Figure 3–27 shows the path of light traveling from air into glass. Is the glass on the right or the left in the drawing? (Section 2.)

On the left.

3–5*

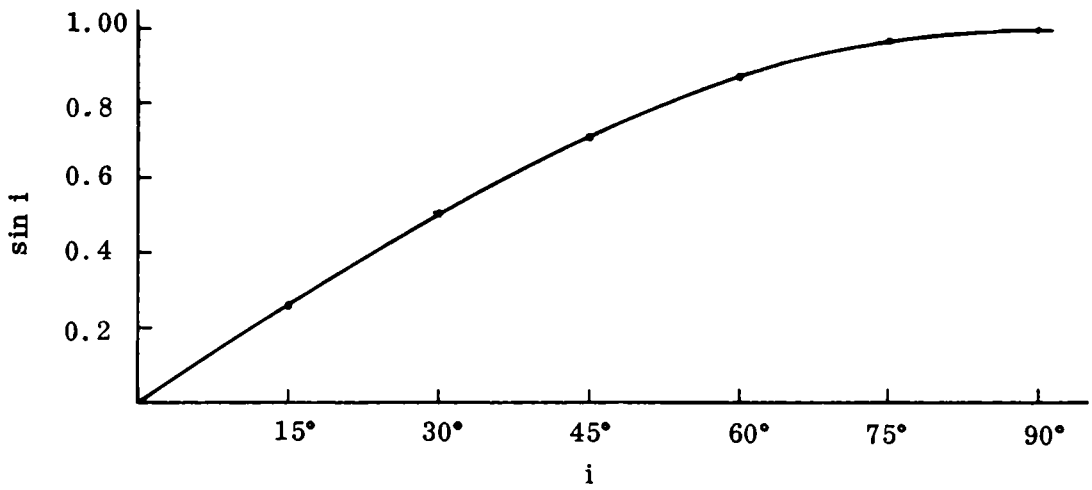
In the nearly straight portion of the curve for water (Fig. 3–3), what is the approximate relation between r and i? (Section 2.)

$r = \frac{3}{4}i$.

3–6

(a) What are the sines of the following angles: 4°, 30°, 45°, 60°, 73°, 17.8°, 37.3°, 90°?

(b) What are the angles that have the following sines: 0.1045, 0.0000, 0.3090, 0.8660, 1.000, 0.5000, 0.5225, 0.9636?



(c) Plot sin i versus i from 0° to 90°.

(a)

Angle	Sine	Angle	Sine
4°	0.0698	73°	0.9563
30°	0.5000	17.8°	0.3057
45°	0.7071	37.3°	0.6060
60°	0.8660	90°	1.0000

(b)

Sine	Angle	Sine	Angle
0.1045	6°	1.000	60°
0.0000	0°	0.5000	30°
0.3090	18°	0.5225	31.5°
0.8660	60°	0.9636	74.5°

(c) See graph above.

3–7*

In Fig. 3–28 what is the index of refraction for light going from air into substance X? (Section 3.)

$n = \sqrt{3}/\sqrt{2} = 1.2$.

3–8*

Check Fig. 3–1. Do the incident and refracted pencils correspond to what they should be if the refractive material really is water? *Must* it be water? (Section 3.)

Yes, since the measured n is 1.3 to 1.4. The material is not necessarily water.

3–9*

For a given (non-zero) angle of incidence, which substance listed in Table 3 causes the greatest change in direction of light entering it? (Section 3.)

Diamond.

3–10

How could you predict from Fig. 3–4 that the index of refraction of water is close to 1.33?

The index of refraction of water is the ratio of the sine of the angle of incidence to the sine of the angle of refraction for a ray of light entering water from air. When angles are sufficiently small, the ratio of the sines of two angles is closely equal to the ratio of the angles themselves. Hence, by reading the left-hand side of the graph for air and water in Fig. 3.4, you can readily predict that the index of refraction will be close to 1.33.

3–11

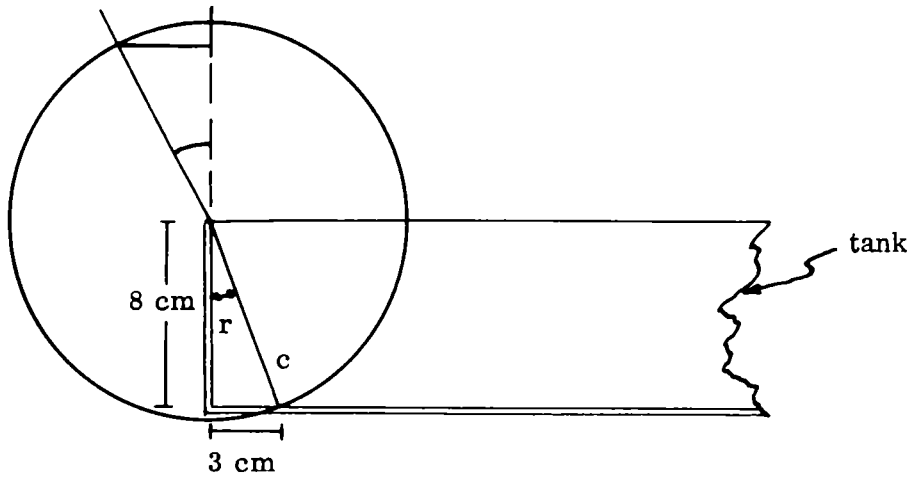
A rectangular tank 8 cm deep is filled with water. A light ray enters the top surface of the water at a point just touching the side of the tank. After refraction it falls on a point on the bottom of the tank 3 cm from the same side of the tank.

(a) What is the sine of the angle of refraction? What is the angle of refraction?

(b) What is the sine of the angle of incidence? What is the angle of incidence of the entering ray?

(c) Suppose that the same tank were filled with a liquid other than water and you found that in order to fall on the same point 3 cm from the side the angle of incidence of the entering ray had to be 31°. What is the index of refraction of the liquid?

Students can do this by a scale drawing and construction, or they can use the tables to find r. Using the second method,



$c^2 = 8^2 + 3^2$, and $c = \sqrt{73}$.

(a) $\sin r = \dfrac{3}{\sqrt{73}} = 0.3511.$

$\underline{r = 20.6°}$

(b) $\sin i = 1.33 \sin r = 0.4670$

$\underline{i = 27.8°}$

(c) $n = \dfrac{\sin i}{\sin r} = \dfrac{\sin 31°}{\dfrac{3}{\sqrt{73}}} = \underline{1.47.}$

3–12

A narrow pencil of light enters the top surface of the water in a rectangular aquarium at an angle of incidence of 40°. The refracted pencil continues to the bottom of the tank, striking a horizontal plane mirror which reflects it back again to the surface, and it is again refracted as it emerges into the air.

(a) What is the angle between the incident ray entering the water and the refracted ray emerging from it?

(b) If the water in the tank is 10 cm deep, what is the distance between the points on the water surface where the ray enters and where it emerges?

This problem may be solved either by graphical construction or by computation.

(a) $\angle 1 = 40°$

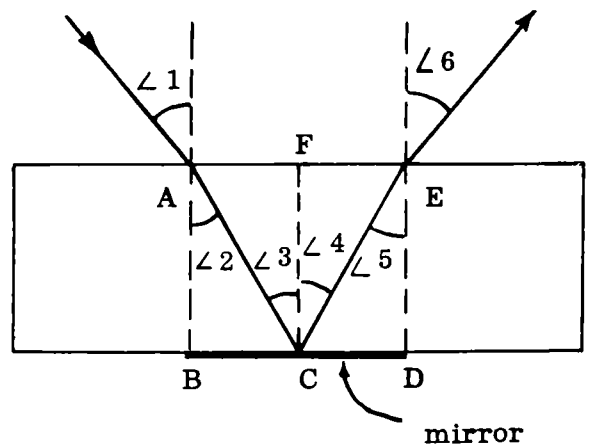
$n \sin \angle 2 = \sin \angle 1$, by Snell's law

$\angle 2 = \angle 3$, and $\angle 4 = \angle 5$, by geometry

$\angle 3 = \angle 4$, by second law of reflection

$n \sin \angle 5 = \sin \angle 6$, by Snell's law

Since $\angle 2 = \angle 5$, $\sin \angle 1 = \sin \angle 6$, and $\angle 1 = \angle 6 = 40°$.



The angle between the incident ray and the ray leaving the water is $\angle 1 + \angle 6 = \underline{80°}$.

A follow-up situation to discussion of this answer would be a glass mirror silvered on the back.

(b) The distance AE on the water surface between the points where the ray enters and leaves the water is, since $\angle 3$ and $\angle 4$ are equal, just 2 AF. We know $CF = 10$ cm.

$$\sin \angle 3 = \frac{AF}{AC} = \sin \angle 2 = \frac{\sin 40°}{1.33}$$

$$= 0.4833.$$

By the Pythagorean theorem

$$AC = \sqrt{\overline{AF}^2 + \overline{FC}^2},$$

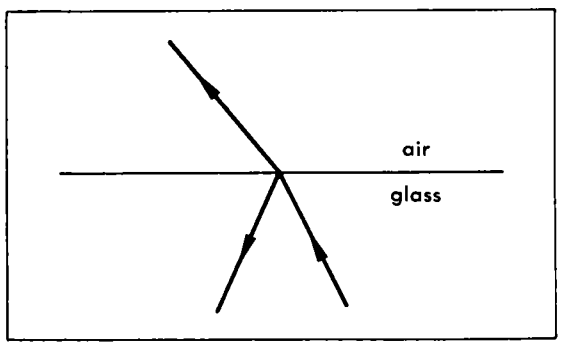
$$AF/\sqrt{\overline{AF}^2 + 10^2} = 0.4833, \qquad AF = 5.52.$$

Thus $AE = \underline{11 \text{ cm.}}$

If students know trigonometry, tangents may be used to make the problem less involved.

3–13*

Sketch one of the photographs of Fig. 3–2 and show by arrowed rays how the picture would look if the light were sent from below. (Section 5.)



3–14

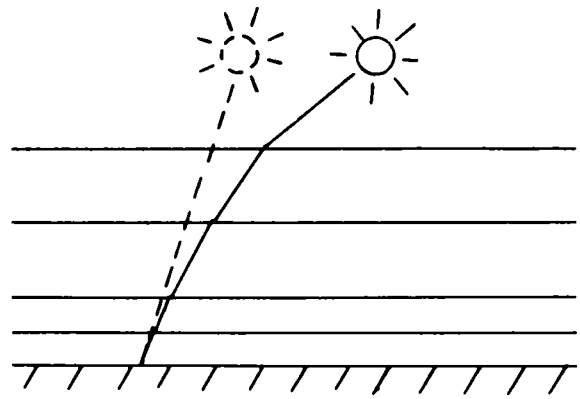
If the angle of incidence were changed in Fig. 3–7, would the two rays corresponding to the ones at the top of the photo still be parallel to each other?

The two reflected beams will always be parallel to one another since the two refractions which the internally reflected beam has experienced are just the opposites of one another.

3–15

Light from the setting sun comes through the earth's atmosphere along a curved path to your eye, so that the sun looks higher in the sky than it really is. How do you explain this? Illustrate your answer with a diagram.

This problem, involving the bending of a light ray in air of varying index of refraction, may provide an interesting class discussion.



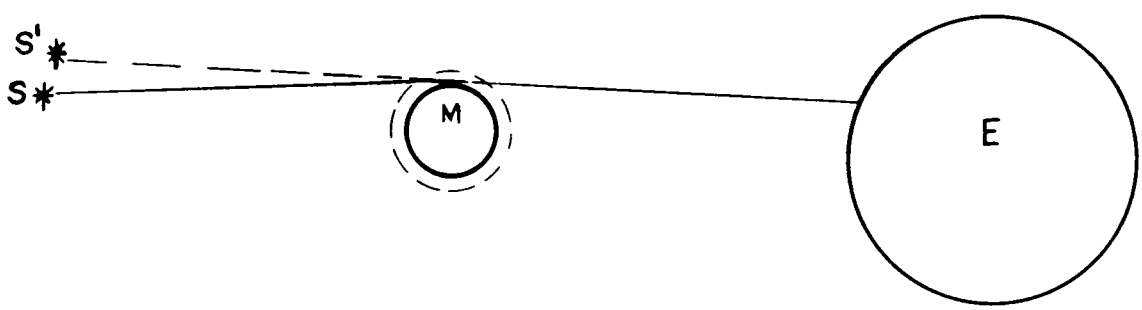
Since the index of refraction for air is different from that for a vacuum, the index for rarefied air is in between the index for sea-level air and the index for a vacuum. The pressure of air drops as the altitude increases, and the index also drops as altitude increases. Assume, for simplicity, that the atmosphere is made of many layers, each higher layer having a slightly smaller index than the layer below, but with each layer having a constant index within that layer. Then we get the following diagram in which the refraction has been greatly exaggerated.

Thus a setting sun is actually already below the horizon; it is only through refraction that we see it. Also, many mirages are caused by refraction between layers of hot, thin air and cold, dense air.

3–16

As the moon moves across the sky, it passes in front of stars. Compare what is seen as the moon passes in front of a star with what you would see if the moon had an atmosphere like that of the earth.

If the moon had an atmosphere which passed between the observer and the star, light from the star would be refracted as shown in the diagram, and



the star would seem to shift from S to S'.

Thus, as the moon approached the star, the star would seem to move ahead of the moon with respect to the fixed stars and to redden and dim just before the solid edge of the moon cut off its light.

3–17*

In Fig. 3–29, is the relative index for light going from medium A into medium B greater or less than 1? (Section 6.)

$n = \sin\theta_A/\sin\theta_B < 1.$

3–18

If the *relative* index for light going from glass into diamond is 1.61 and the *absolute* index of glass is 1.50, what is the *absolute* index of diamond?

If $\frac{n_d}{n_g} = 1.61$ and $n_g = 1.50$, then $n_d = n_g \times \frac{n_d}{n_g} = 1.50 \times 1.61 = \underline{2.42.}$

3–19

If the relative index for light going from oleic acid into water is 0.91 and the index of water is 1.33, what is that of oleic acid?

If $\frac{n_w}{n_o} = 0.91$ and $n_w = 1.33$, then $n_o = \frac{n_w}{n_{ow}} = \frac{1.33}{0.91} = \underline{1.46.}$

3–20

What will you see when you look at a piece of fused quartz submerged in oleic acid? (Refer to Table 3.)

Fused quartz and oleic acid both have the same index of refraction, and both are colorless. Since light will pass through the interface without being refracted or reflected, there is no way of "seeing" the quartz at all, unless there are internal imperfections. You might mention that a truly invisible man would have to be colorless and transparent with an index of refraction equal to that of air.

Demonstration. Commonly available materials with nearly the same index of refraction are ordinary mineral oil and lucite or Plexiglas. A mixture of benzene and alcohol, in the right proportions, has the same index as Pyrex. Some of your students who have studied biology may have seen tissue specimens mounted in Canada balsam which has the same index of refraction as the glass used for microscope slides and cover glasses.

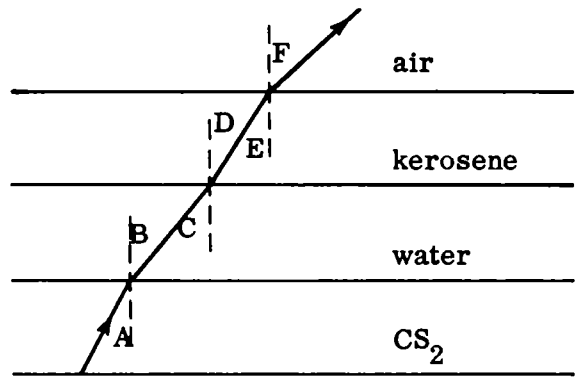
3–21

It is possible to place carbon disulfide, water, and kerosene in separate layers in that order, since they do not mix. A container having these three liquids in layers of equal depth has on its bottom a light source which projects a pencil of light upward through the liquids at an initial angle of 5° from the vertical.

(a) Calculate the angle of refraction of the ray as it finally emerges from the kerosene.

(b) Would a pencil of light travel over the same path in the reverse direction from kerosene through water into carbon disulfide?

(c) Suppose the cylinder had been filled with water only. How would the angle of refraction of the emerging ray compare with the angle calculated above?



If you tell students in advance that they do not need the index of refraction of kerosene (or provide them with a value), you will discourage them from the desirable practice of solving problems algebraically before substituting known values.

(a) From Snell's law, . . .

$$n_{CS_2} \sin 5° = n_w \sin\theta_w = n_k \sin\theta_k = \sin\theta_{air}.$$

Therefore,

$$\sin\theta_{air} = 1.63 \sin 5° \quad \text{or} \quad \theta_{air} = \underline{8.2°.}$$

(b) The answer is <u>yes,</u> but it should be emphasized that reversibility is an empirical law.

(c) For water only, the angle of the emerging ray would be <u>smaller</u> than in part (a) and can be computed from Snell's law:

$$1.33 \sin 5° = 1 \times \sin\theta_{air},$$
$$\sin\theta_{air} = 0.1160, \text{ and } \theta_{air} = 6.7°.$$

3–22*

What is the critical angle for light passing from glass to water? (Section 7.)

63°.

3–23

In Fig. 3–30 continue the light ray until it comes out of the glass.

It can be seen immediately that the original ray cannot enter the air, for the critical angle for water-into-air is less than 60°. The presence of the glass has no effect on this—in the glass the ray must strike the glass-to-air surface at an incident angle greater than the critical angle for glass-to-air.

$$(n_{\text{glass}})(\sin\theta_{\text{glass}}) = (\sin\theta_{\text{water}})(n_{\text{water}}) > 1$$

$$\therefore \sin\theta_{\text{glass}} > \sin\theta_{\text{critical for glass-to-air}}$$

$$\sin\theta_{\text{glass}} = (\sin 60°)\left(\frac{1.33}{1.5}\right)$$

$$= \frac{(0.87)(1.33)}{1.5}$$

$$= 0.77$$

$$\theta_{\text{glass}} = 50°.$$

3–24†

(See the Appendix in the text.)

3–25

Why is it easier to observe dispersion in a prism rather than in the setup shown in Fig. 3–2?

The dispersion produced by a prism will be much easier to observe because of the second refraction which the light must undergo in getting out of the prism. The second refraction roughly doubles the angular spread of the colors.

3–26†

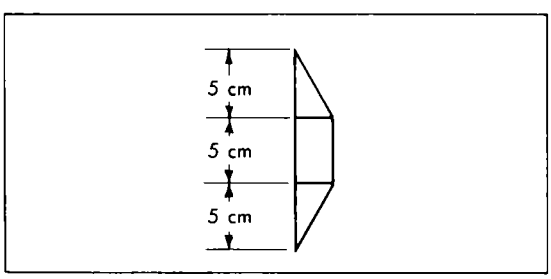
(See the Appendix in the text.)

3–27

A crude converging lens can be constructed by placing two 30°-60°-90° glass prisms together with a glass block as shown in Fig. 3–32.

(a) What is the focal length of this "lens" to one significant figure?

(b) Would such a lens form a clear image? Explain.

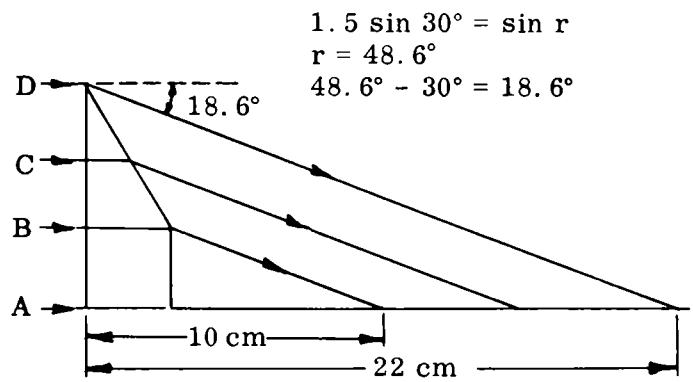


Students can solve this problem by finding the angle of refraction through the application of Snell's law, then either computing the "focal length" using a trigonometric function or measuring the "focal length" from a scale drawing. The results are dependent upon whether the light is incident on the flat face or the opposite faces, but, to one significant figure, this does not matter.

(a) The central ray A goes through undeflected. If an index of refraction of 1.5 is used for the glass, the three rays B, C, and D cross A at distances of approximately 10, 16, and 22 cm from the flat face. An answer of 15 cm is pretty good, but, to one significant figure, either 10 cm or 20 cm should be accepted.

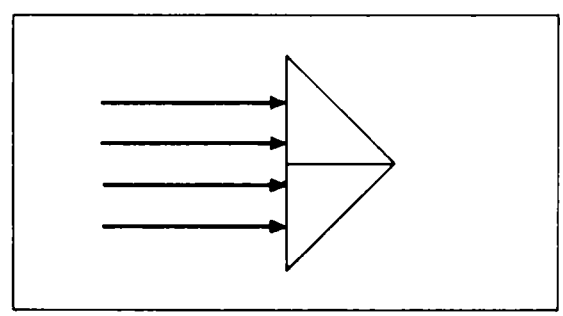
If the student has read Section 2 he may try to use the lensmaker's formula, using an approximate radius of curvature. The "value" of the radius of curvature is about 10 cm, giving a focal length of about 20 cm.

(b) This "lens" would not form a clear image since parallel light is not focused to a single point, but is merely somewhat concentrated.

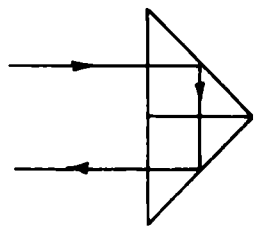


3–28

If two 45° prisms of glass (index = 1.50) are arranged as in Fig. 3–33, they will not converge parallel light. Why not? What will happen to the light?



The two prisms shown will not converge light since light entering one will be totally reflected twice and emerge parallel to its initial direction. After this problem has been done, make it clear that total internal reflection is almost never important in lenses. It does prevent the making of some very bulgy types.



3–29

A lens (index = 1.50) has a focal length in air of 20.0 cm. Is its focal length in water greater or less than in air?

The relative index of refraction for a ray going from glass into water is n_{glass}/n_{water}; 1.50/1.33 = 1.13. Consequently, the bending of rays parallel to the axis of the lens is less in water than in air, and the focal length of the lens in water is greater than it is in air.

3–30*

An object is 1 meter in front of the principal focus of the lens of a camera. How far behind the other principal focus will the image be if the focal length of the lens is 5 cm? (Section 11.)

0.25 cm.

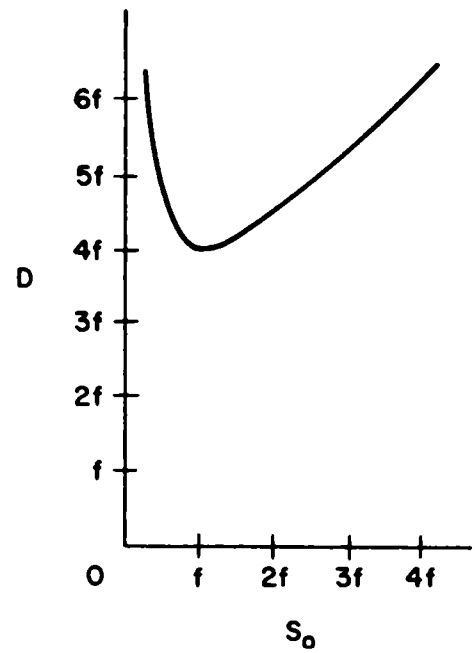
3–31

What is the minimum distance between an object and the real image formed by a converging lens? (Make a graph of the total distance versus the object distance. Use the focal length f of the lens as the unit of distance.)

This problem is a good way of introducing the students to the idea of the minima of functions. The total distance, D, is $S_o + S_i + 2f$. Substituting from $S_oS_i = f^2$,

$$D = S_o + \frac{f^2}{S_o} + 2f.$$

If f is used as the unit of the graph, the result is as shown at the top of the next column.



If your students know differential calculus, they can solve the problem analytically,

$$D = S_o + \frac{f^2}{S_o} + 2f.$$

Differentiating D with respect to S_o,

$$\frac{dD}{dS_o} = 1 - \frac{f^2}{S_o^2} + 0.$$

The minimum of the curve is when

$$\frac{dD}{dS_o} = 0, \quad \text{or} \quad \frac{f^2}{S_o^2} = 1.$$

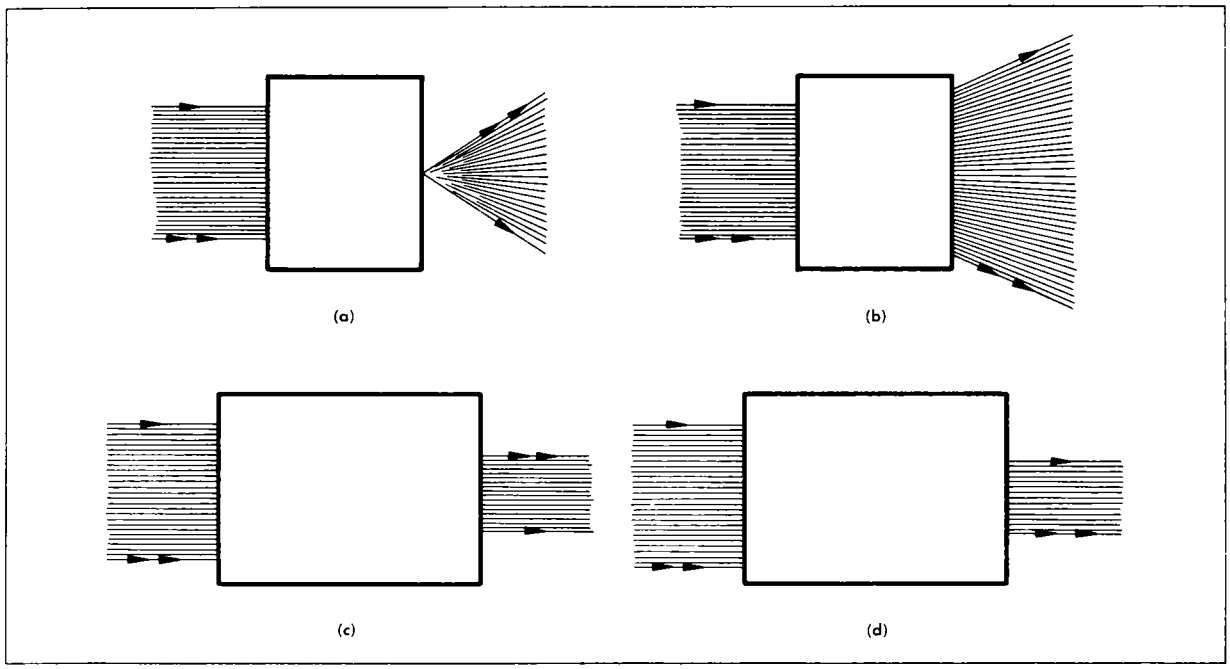
Thus, $S_o = f$ and $D = f + \frac{f^2}{f} + 2f = 4f$.

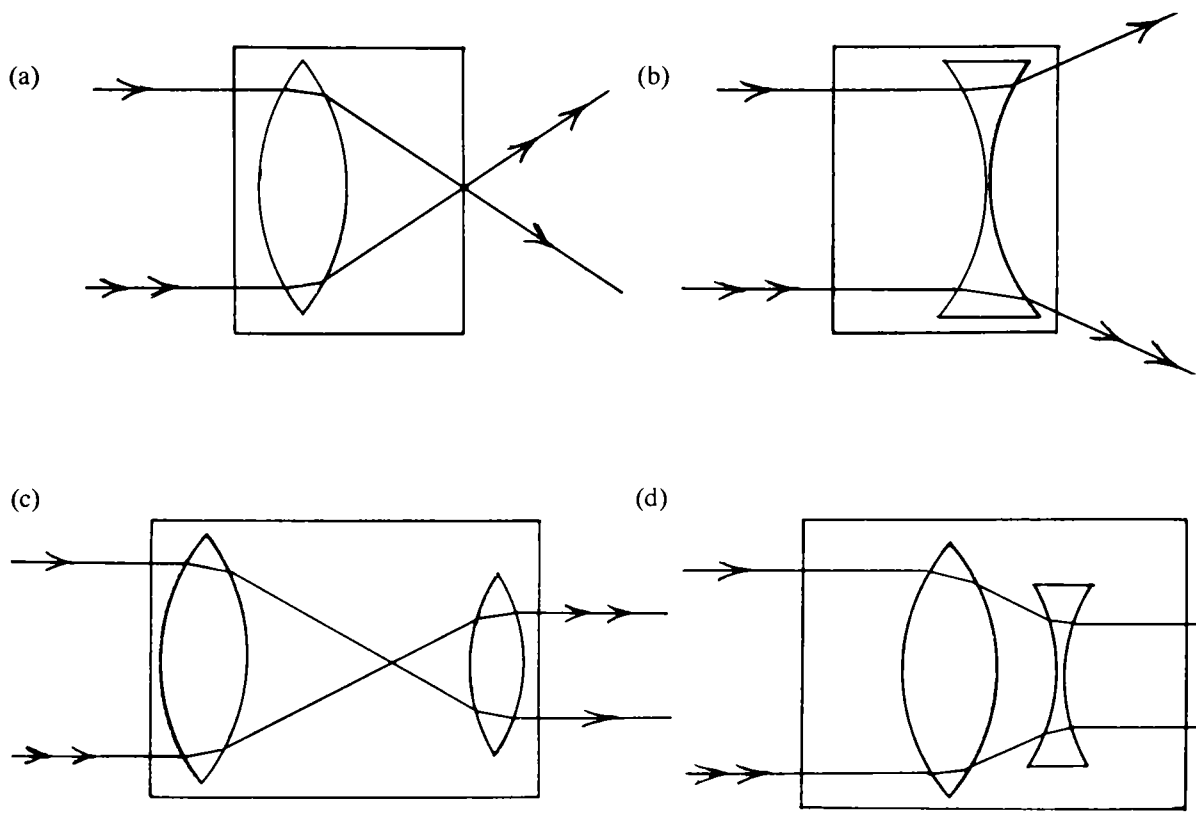
3–32

In Fig. 3–34, a parallel beam of monochromatic light (light of a single color) enters each box from the left. Draw what could be in each box to produce the effects shown. The single and double arrows on the emerging beam show the corresponding edges of the entering beam.

You should make it clear to your students that each of the diagrams refers to a real physical situation—they are not "trick" problems.

The following diagrams show one way of accomplishing each part.


(a)
(b)
(c)
(d)


(a)
(b)
(c)
(d)

CHAPTER 4

4–1*

How can you check several light sources to see if they are equal sources of light? (Section 1.)

Isolate each one in a dark room where there are no reflections, and see if each gives the same reading when placed at a given distance from a light meter.

4–2*

Suppose a light meter gives a reading of 1 unit when it is a distance r from a light bulb. At what distance from the same source will the meter give a reading of 2 units? (Section 1.)

At $r/\sqrt{2} = 0.71r$.

4–3†

(See the Appendix in the text.)

4–4

What does the particle theory predict about the intensity produced by an *extremely* weak light source —so weak that only a few particles are emitted per second? How might you test this prediction?

The particle theory predicts that light from very weak sources has so few particles that the light appears to come in bursts. This aspect of the particle theory can be tested by letting light from a weak source fall on a very sensitive light meter and looking for large fluctuations in the reading. Of course, if no fluctuations are found, then light may still be particles—only smaller ones than were supposed when the experiment was designed. (One of the nice things about this particular "thought" experiment is that indeed, when we look at a sufficiently weak source, we do count individual "particles" of light. It is hard to reconcile this observation with a wave model.)

4–5

It is found that a 40-candle source placed 3.0 feet from a light meter gives the same scale reading as an unknown source 1.2 feet from the meter.

(a) What is the strength of the unknown source?

(b) What is the light intensity in foot-candles read on the meter?

This is an easy problem involving the inverse-square law. Part (a) requires no knowledge of illumination units; part (b) does.

(a) $I = \dfrac{40c}{(9\text{ ft})^2} = \dfrac{K}{(1.2\text{ ft})^2}$,

$$K = \frac{57.6}{9} = \underline{6.4 \text{ candles.}}$$

(b) $I = \dfrac{40c}{(9\text{ ft})^2} = \underline{4.4 \text{ foot-candles.}}$

In class discussion be sure to point out that part (a) could be worked if the 40-candle source had had a strength of 40 "gizmos," and no one knew what a "gizmo" was except that it measured the intensity of a light source.

4–6*

In Fig. 4–9, how far to the left of A is the light source, if it gives an intensity of 8 foot-candles at A and $\frac{1}{2}$ foot-candle at B? (Section 31.)

2 meters.

4–7

A light meter calibrated in foot-candles is 2.0 feet from a mirror. A very small 25-candle-power source is placed midway between them. What is the reading of the meter if the mirror is

(a) a plane mirror?

(b) a concave mirror with a diameter of 6 inches and a focal length of 1 foot?

We can replace the mirror by a virtual source 1 foot behind it. Then,

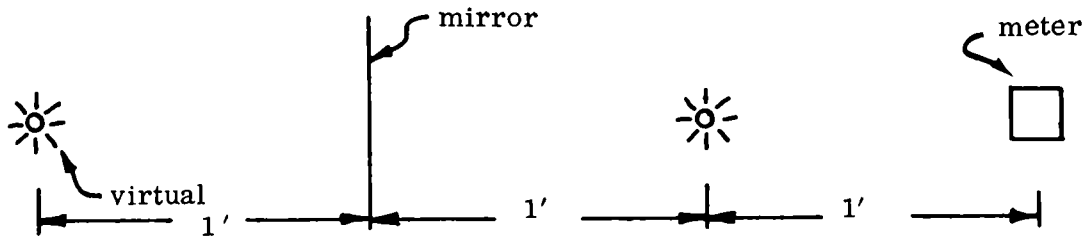
$$I_{\text{at meter}} = \frac{25\text{C}}{(1\text{ ft})^2} + \frac{25\text{C}}{(3\text{ ft})^2} = \frac{250}{9}$$

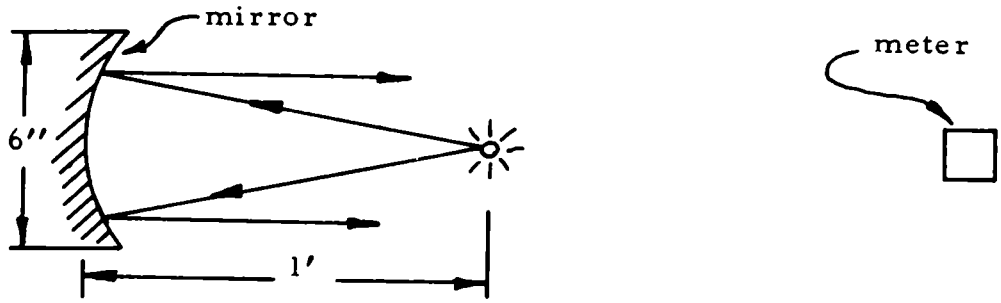
$$= \underline{27.8 \text{ foot-candles.}}$$

Since the light source is at the principal focus of the concave mirror, the rays after reflection are parallel to each other. This gives a cylinder of light six inches in diameter that does not change in intensity with distance if scattering and absorption are neglected. For the short distances involved in this problem, scattering and absorption of light in clear air would be very small indeed. Thus the intensity in this cylindrical beam of light is the same as at the surface of the mirror itself.

$$I_{\text{at meter}} = \frac{25\text{C}}{(1\text{ ft})^2} + \frac{25\text{C}}{(1\text{ ft})^2}$$

$$= \underline{50 \text{ foot-candles.}}$$





4–8

In the use of radar, a beam of radiation is sent out from a source. Some of this radiation falls on a distant object, such as an airplane, is reflected by the object, and is detected when it returns to its starting point. The radiation used behaves like light. Assuming that the source of the radiation is equivalent to a point source of light and that the object reflecting the radiation is a diffuse reflector, can you convince yourself that the intensity of the returning radiation varies inversely with the fourth power of the distance from the source to the reflecting object?

The intensity of "light" arriving at the distant object is proportional to $1/r^2$, i.e., $I = K/r^2$. The amount of "light" it reflects is proportional to the incident intensity. It therefore becomes a source of strength CK/r^2. But the intensity of "light" arriving back at the radar station is

$$I = \frac{CK/r^2}{r^2} = \frac{CK}{r^4},$$

as was to be shown.

4–9

Upon which object would light exert the greater pressure, a clear piece of glass or a mirror?

Light would exert the greater pressure on the mirror since, upon reflection of the light, the particles of light would have their momenta *reversed* rather than just increased or decreased a little bit.

4–10

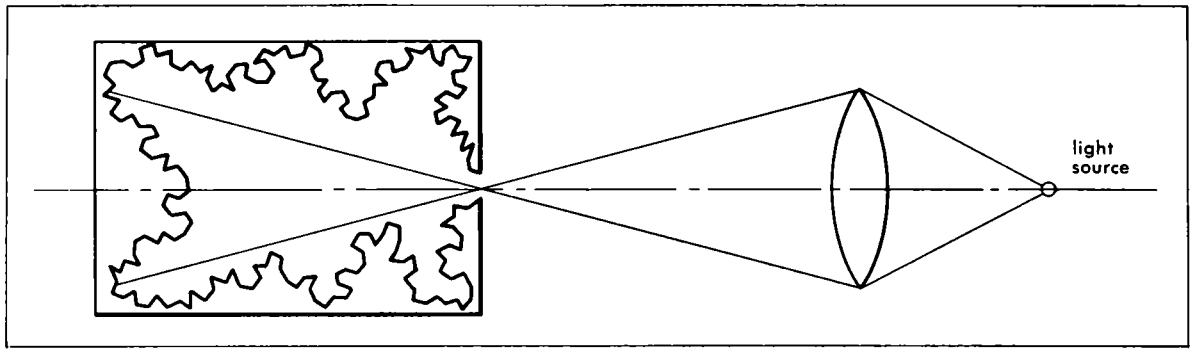
A sensitive thermometer placed in the different parts of the spectrum from a prism will show a rise in temperature. This shows that all colors of light produce heat when absorbed. But the thermometer also shows a rise in temperature when its bulb is in either of the two dark regions beyond the two ends of the spectrum. How can the particle theory account for this?

The particle theory would account for this heating by saying that there are particles of light not detected by the eye which are refracted by a prism and which cause a rise of temperature in a thermometer placed at the extremes of the visible spectrum.

4–11*

Why should a Ping-Pong table have a more even surface than the floor of a basketball court? (Section 4.)

The smaller Ping-Pong ball is deflected by small bumps and holes that would not affect a basketball.



4–12

Consider the box shown in Fig. 4–10, whose inside walls are perfectly reflecting. What do you predict will happen when you shine a beam of light into the box? (Use the particle model of light to arrive at your answer.)

In applying the particle model of light, we should restrict ourselves to the essential features of the model, rather than seek an exact analogy, say, with steel balls or gas molecules. Imagine that you are shooting a stream of particles into the box and that no other particles are present at the beginning. Because the particles will be reflected many times off the wedges on the walls, the number inside will increase steadily. But as the particles accumulate, more and more, by chance, bounce out of the hole. When a sufficient number have accumulated, the number of particles finding their way out will equal the number coming in. We predict, therefore, that after enough light has accumulated in the box, the same amount of light will be shining out of the box as is coming in, although not in so concentrated a beam.

It must be kept in mind that we are dealing with a highly idealized situation in which there is *no* absorption. Since the light would undergo a large number of reflections, a tiny amount of absorption on each reflection would greatly reduce the amount of light that can come out.

If the walls are of polished silver they will reflect 93% of the incident light. Assuming our box to be about 1 m on each side, the average light path inside may be about 0.5 m. After 100 reflections (0.93^n) the intensity is down to 10^{-4}, and after 1000 it is down to 10^{-42}. One thousand reflections would require a path length of about 500 meters which would take light about 10^{-6} sec to travel. A real box would show very little light coming back out of the hole.

4–13*

In Fig. 4–6 if you keep the speed of the ball bearing on the upper level constant by launching the ball always in the same manner, at what angle of incidence to the hill will you get the highest speed on the lower level? (Section 5.)

The speed on the lower level is the same for all angles of incidence.

4–14*

What does the experiment of particle model refraction suggest about the speed of light in a material, that was not apparent in our refraction experiments with light? (Section 5.)

The speed of light should be greater in a refractive material than in a vacuum.

4–15

The index of refraction of carbon disulfide is about 1.63. What should the speed of light be in this liquid, according to the particle model of refraction given in Section 4–5?

According to the particle model, the speed of light in a medium of refractive index n is

$$v = nc = 1.63 \times 3 \times 10^8 \text{ m/sec}$$
$$= \underline{4.89 \times 10^8 \text{ meters/sec.}}$$

During discussion, remind the students that this formula comes from the particle analog of Experiment 6. Depending on how you are developing the particle model, you may wish to remind them that this is an incorrect answer.

4–16

The speed of light measured in a certain medium is 2×10^8 m/sec.

(a) According to the particle model of light, what should be the index of refraction of this medium?

(b) Using the index of refraction from part (a),

what would be the path of a light ray which passes from air into the medium at an angle of 30° from the normal to the surface of the medium?

The purpose of this problem is to illustrate the inconsistency of the particle model when relating speeds of propagation and index of refraction.

(a) $n = \dfrac{2 \times 10^8}{3 \times 10^8} = \underline{0.67}$

(b) $\frac{2}{3} \sin r = \sin 30° = \frac{1}{2}$, $\sin r = \frac{3}{4}$, $r = \underline{49°}$.

4–17

Occasionally there occurs in the heavens an explosion of a star, producing what is known as a super nova. The star suddenly becomes many times brighter than before. As you know, the stars are so far away that the light from them takes many years to reach us. An explosion that we observe must have occurred a long time ago, and the light has been traveling toward us ever since. We see the explosion as a bright white light, not as a series of different colors arriving at different times.

(a) What does this show about the speed of light of different colors in vacuum?

(b) Try to suggest a particle model for dispersion in prisms consistent with the single speed of light of all colors in vacuum.

This is a straightforward problem; the student is to analyze a simple fact about light in terms of the properties of a model.

4–18*

Why is it possible to see light through the hole in the strobe disc in Fig. 4–7 when the disc is rotating very slowly? (Section 7.)

The light is able to travel through the hole to the distant mirror and back through the hole before the disc has turned far enough to move the hole out of position.

4–19

Using the data given in Section 4–7 about Cornu's application of the Fizeau method, compute the shortest distance from the rotating disc to the mirror that will permit the returning beam to pass through the opening immediately following the one through which it started.

Section 7 states that in Cornu's stroboscope there were 200 openings and that it could turn at 54,000 rpm.

Time for 1 revolution

$$= \frac{1}{5.4 \times 10^4} \text{ min} = \frac{6}{5.4} \times 10^{-3} \text{ sec.}$$

Time for $\frac{1}{200}$ of a revolution $= \dfrac{6}{1.08} \times 10^{-6}$ sec.

Distance light travels in that time

$$= \frac{1.8 \times 10^3}{1.08} \text{ meters} = 1.67 \times 10^3 \text{ meters.}$$

Distance to mirror

$$= \frac{1.67 \times 10^3}{2} = 8.33 \times 10^2 \text{ meters} = \underline{833} \text{ meters.}$$

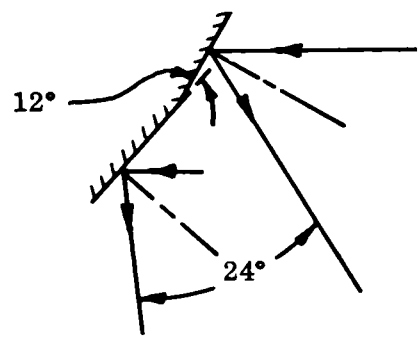
4–20

In the Foucault-Michelson modification of the Fizeau method, rotating mirrors are used in place of the rotating disc. (Fig. 4–8). Assuming that the mirrors are rotating 500 times per second and that the distance from them to the fixed mirror is 10.0 km, show that a mirror face rotates through an angle of 12° before it reflects the beam a second time and that the angle between this reflected beam and the original direction of the light is therefore 24°.

$$t = \frac{d}{v} = \frac{2 \times 10^4 \text{ m}}{3 \times 10^8 \text{ m/sec}} = \frac{2}{3} \times 10^{-4} \text{ sec}$$

Angular rotation

$$= \frac{500 \text{ rev}}{\text{sec}} \times \frac{360°}{\text{rev}} \times \frac{2}{3} \times 10^{-4} \text{ sec} = 12°$$



24° between initial and final beams

If the mirrors turn through an angle of 12°, the normals to the mirror will also change direction by 12°. The angles of incidence and reflection will both change by 12°. Thus there is 24° between the initial and final beams.

4–21†
(See the Appendix in the text.)

4–22
Can you modify your proposed experiment (Problem 21) to test the assertion that the speed of light in vacuum is independent of previous refractions?

To test the hypothesis that the speed of light in vacuum is independent of previous refractions, one lets the light go through a number of refractions before it *first goes* through the toothed wheel.

4–23
Suppose that someone were to propose to you that sound consists of small, rapidly moving particles that are emitted by a source and that affect your ear when they strike it. What evidence could you use to support or to refute it?

One could check to see if sound obeys an inverse-square law, if it reflects and refracts like the particle model, and if it exerts pressure. The following evidence from the student's experience will refute the particle model: (1) Sound does not travel through a vacuum and (2) Sound travels around obstacles. (The sound of a truck can be heard on the other side of a house.)

4–24
We have said that light particles must be very small. Suggest an experiment to show that they must also have an extremely small mass.

A flashlight on a sensitive balance does not appear to lose mass continuously when it is turned on. Also, objects are not knocked about when light shines on them.

4–25
Is it possible that the slowing down of light as it goes from a vacuum into a transparent material is some kind of friction effect?

If the slowing down of light as it passed from a vacuum into a transparent material was due to a friction effect *in* the material, the speed of light in air or in glass would not be constant. The speed in glass would depend upon how much glass the light had traveled through.

If we assumed some kind of friction effect that was effective only *at the surface of* the transparent material, but not within it, we would have to invent some kind of "reverse friction" which would increase the speed of light as it left the transparent material. Otherwise, light re-entering a vacuum after traversing a transparent material would be further slowed, and would be refracted toward the normal instead of away from it. We know that the speed of light in a vacuum is constant, no matter what the light has traversed. We know that light entering a vacuum from a transparent material is refracted away from the normal.

4–26*
Why does the measurement of the speed of light in water cast doubt on the validity of the particle model as developed in this chapter? (Section 7.)

The particle model predicts that the speed of light is greater in water than in air, while actual measurements show just the reverse.

CHAPTER 5

5–1*
What is similar in all examples of waves discussed in Section 1? (Section 1.)

The waves all travel through some medium, but the medium itself does not travel with the wave.

5–2
Suppose you look out your window and see your neighbor across the street sitting on his porch. In how many ways could you do something to attract his attention, make him move, or otherwise influence his actions? Which ways involve mass transmission and which ways wave motion?

It is not the aim of this exercise merely to inform students that some phenomena (e.g., sound) are wave motions. The use of this exercise should be to give the students a better idea of how wave motion differs from the motion of an object. It should be made clear that one must know something of the detailed mechanism of a communication process in order to classify it. Shouting to a friend is a good example of wave motion, if, and only if, students realize that individual air molecules oscillate back and forth, but do not travel from mouth to ear.

A clear example of mass transmission is throwing a rock. A pulse sent along a rope is an equally clear example of waves. Discussion of the mechanisms of such familiar wave phenomena as sound and radio waves should be avoided at this stage. It will be sufficient to note that, for these ex-

amples, it is difficult to find evidence of mass transmission. Many students will be uncertain about telephoning. (The electrical signal which moves along the wire is an electromagnetic wave; the electrons in the wire move only slightly.) Sending the heat from a fire is ambiguous. Radiation is an example of waves; convection involves mass transmission; conduction is most appropriately an example of waves.

Remember that, at this stage, the classification of light is uncertain. Since the simple particle model did not work, "mass transmission" is unjustified. On the other hand, some particle model might work. At least until we get some wave evidence, listing light under waves should be considered as tentative.

5–3*

According to Fig. 5–4, along what part of the pulse is the spring momentarily at rest? (Section 2.)

At the point of maximum displacement.

5–4*

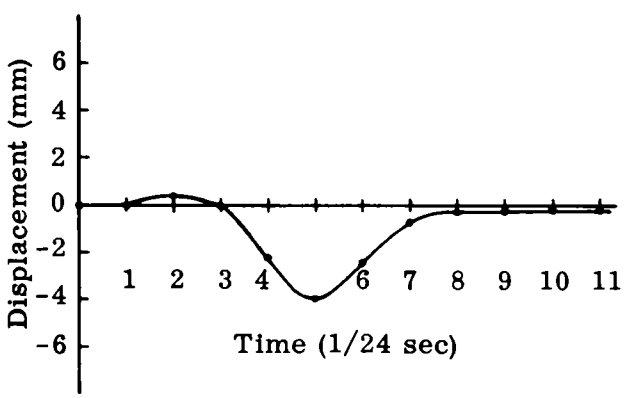
In Fig. 5–2, the ribbon first moves up and then down as the wave moves past it from right to left. Would the ribbon move up first or down first if the same pulse traveled from left to right? (Section 2.)

It would still move up first.

5–5

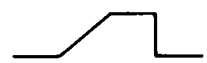
Fig. 5–2 shows the displacement of a point on a spring as a pulse goes by. Make a graph showing the displacement of this point as a function of time. Plot displacement vertically and time horizontally with $\frac{1}{24}$-sec intervals (the interval between pictures of Fig. 5–2).

Displacements in Figure 5–2 were found by measuring the distances from the bottom of the coil spring to the tip of the reference arrow.



Frame number	Time (1/24 sec)	Distance (mm)	Displacement (mm)
1	0	5.2	0.0
2	1	5.2	0.0
3	2	5.6	0.4
4	3	5.2	0.0
5	4	3.0	−2.2
6	5	1.3	−3.9
7	6	2.8	−2.4
8	7	4.5	−0.7
9	8	5.0	−0.2
10	9	5.0	−0.2
11	10	5.0	−0.2
12	11	5.0	−0.2

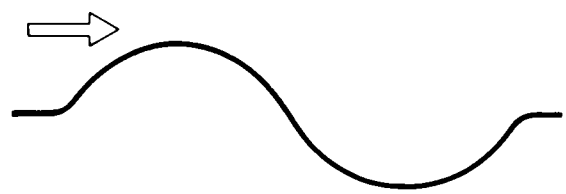
Supplementary note: This problem can be supplemented by considering a pulse, moving from left to right, with a displacement which, as a function of distance, looks like this:



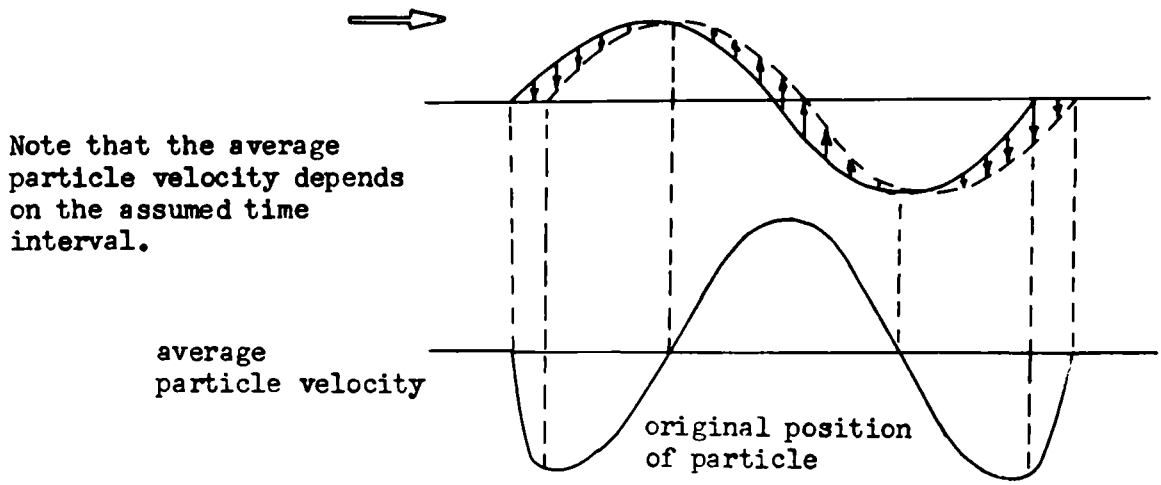
Have students draw the displacement of a point as a function of time when this pulse passes by. This will show the similarities and differences between graphs of displacement vs. distance and graphs of displacement vs. time.

5–6

Sketch the motion of the spring for the pulse in Fig. 5–14.



This problem provides a good simple test of students' knowledge of the relation between wave shape and particle speed. (See *Development* in Teacher's Guide, Section 2, Chapter 5). Make it clear to students whether you want only two successive positions, the "velocity arrows," or a graph


Note that the average particle velocity depends on the assumed time interval.
average particle velocity
original position of particle

of average particle velocity as a function of initial particle position.

5–7*

If two pulses traveling toward each other on a coil spring have displacements in the same direction, can they cancel each other when they cross? (Section 3.)

No.

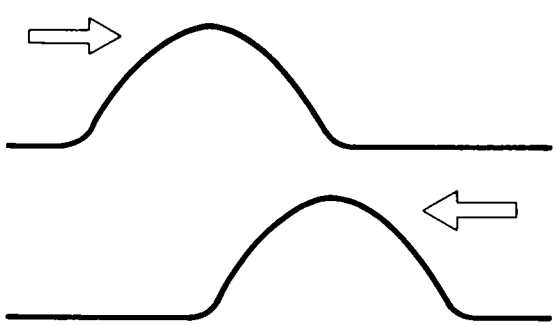
5–8*

Two pulses have maximum displacements of 3 cm and 4 cm in the same direction. What will be the maximum displacement when they pass? (Section 3.)

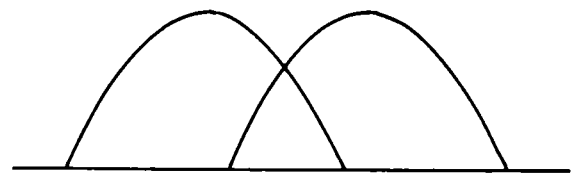
7 cm.

5–9

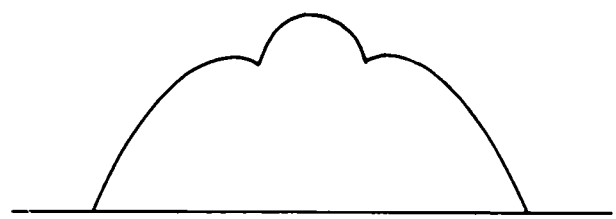
Using the two pulses shown in Fig. 5–15 determine the size and shape of the combined pulse at this moment. Do the same thing for several other positions of the pulses.



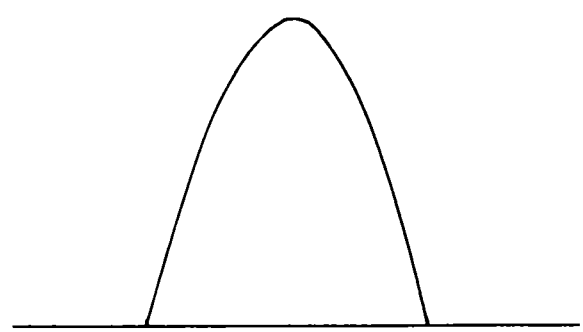
Individual pulses as they would look if only one were present at a time.



Add the displacements graphically to get



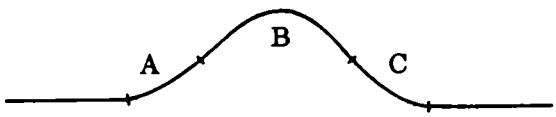
When the pulses are on top of one another, the combined pulse is twice as high as either pulse alone.



5–10

The seventh picture from the top in Fig. 5–5 shows two pulses at the moment of crossing. Specify the pieces of spring that are moving and their direction of motion.

In the seventh picture from the top of Figure 5–5, the lack of blur on the left side and only slight blur on the right indicates that, at this instant, there was either no motion or that the motion was very slow.

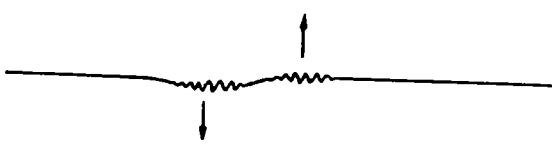


In studying the sequence of pictures (top to bottom of page) it can be seen that sections *A* and *C* of the rope have moved downward during the interval between the 6th and 7th frames. In the 7th frame, sections *A* and *C* are at rest. Between the 7th and 8th frames, these sections will move upward. Between the 6th and 7th frames, section *B* moved up and is at rest in the 7th frame. Between the 7th and 8th frames, section *B* will move downward.

5–11

In the fifth picture from the top of Fig. 5–7, which points are moving and in which direction do they move?

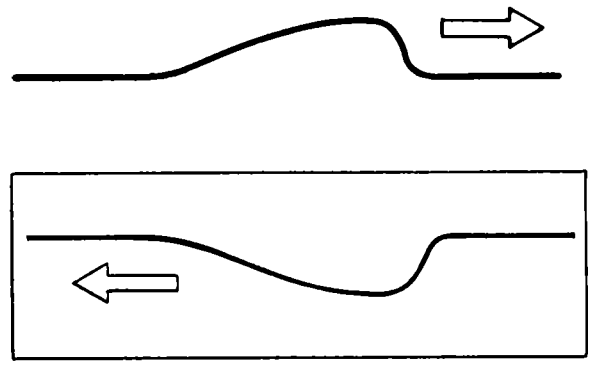
Students can do this problem by examining the pictures whether or not they understand superposition of velocity. However, the problem is particularly instructive if the students have been exposed to the superposition of velocities.



As background, look at the fourth and sixth pictures. In picture 5, the points are moving as shown. Note the unblurred part in the middle which is at rest. This is where the velocity changes direction. (Insofar as the two pulses are exact opposites, the midpoint is a node.)

5–12*

A pulse, shown in Fig. 5–16 is sent along a coil spring toward the right. Draw the pulse traveling to the left which could momentarily cancel the pulse shown. (Section 3.)



5–13

In the sixth picture, Fig. 5–17, we see the superposition of two equal pulses, each of which is symmetrical about its center line.

(a) The absence of blur indicates that there is no motion at this instant. Show that this is true by using the principle of superposition.

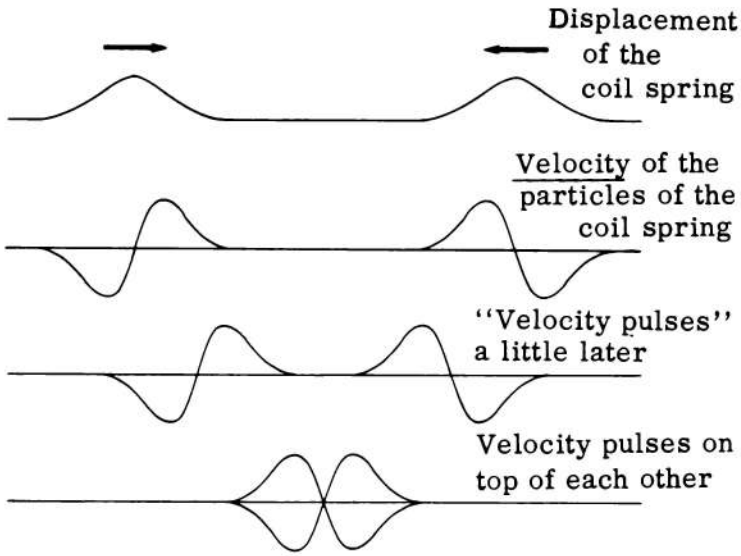
(b) Assume that you deform the coil spring in the same manner as shown in the sixth picture. What will happen when it is released?

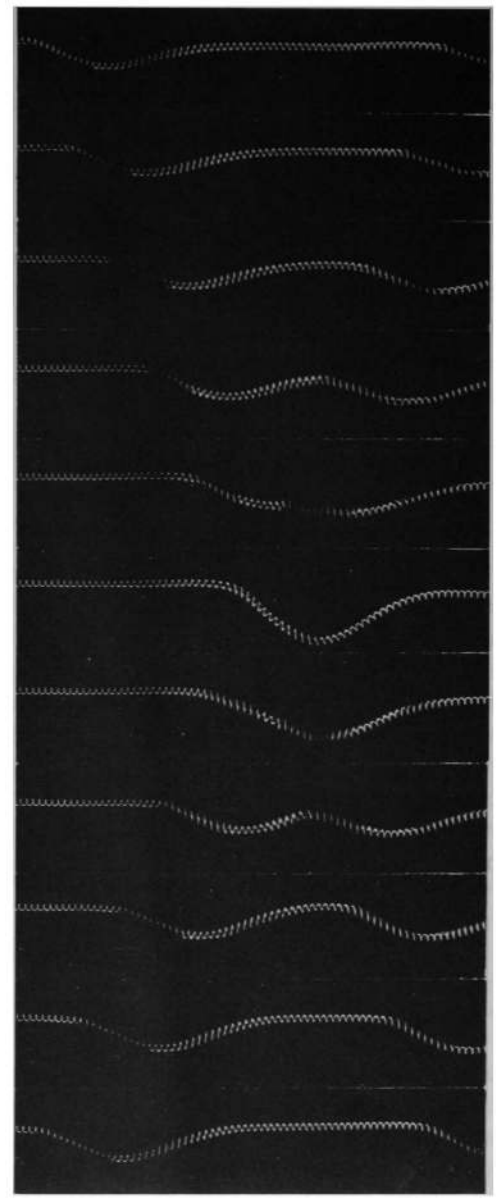
Assignment and Class Discussion can come after Section 3, but only if you have discussed the superposition of velocities.

Students will not find this problem very hard if they realize that velocities superimpose as do displacements.

(a) It is probably best to draw a graph of velocity of the points of the coil spring vs. position on the coil spring. The student should note that the velocity of the leading edges of the pulses is upward. For the trailing edges, the velocity is downward. The principle of superposition holds for velocities as well as for displacements. Add the velocities of the two waves shown in the last figure to get zero. Thus *momentarily* the velocity of the spring is zero.

(b) If the spring is deformed as shown in the sixth picture of Figure 5–17, held at rest, then released, two pulses each half as high as the original deformation will go out in opposite directions. This can be made plausible


Displacement of the coil spring
Velocity of the particles of the coil spring
"Velocity pulses" a little later
Velocity pulses on top of each other



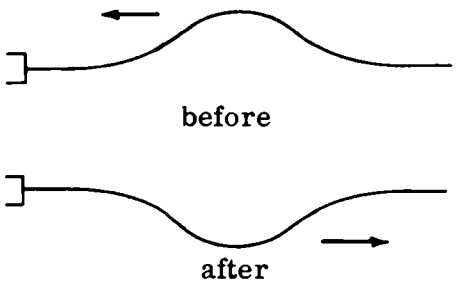
by using the following argument: "Look at Figure 5–17, picture 6. What happens afterwards? Two pulses go out in opposite directions as stated. If we cover up pictures 1, 2, 3, 4, and 5, would we know how the spring came to be deformed as in picture 6? No. Then no matter how it came to be in this condition the same thing must happen afterwards."

Some students may say that we would get the same thing as in picture 6 if a single pulse were traveling along the spring. Although the displacement would be the same, there is obviously a difference because if there were only one pulse, parts (especially the leading and trailing edges of the pulse) of the spring would be *moving*. It might be a good idea to point out that if we wish to predict what will happen at future times to a wave, we must know the displacement and velocity of every point on the spring.

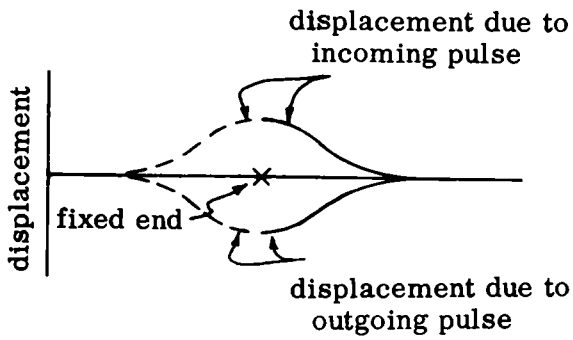
Note that this is a new fact which is by no means obvious. Students might suppose, for example, that the acceleration or some as yet undefined kinematic or dynamic property would help govern the subsequent motion. The essential new fact is that the displacement and the velocity completely determine the future (and incidentally the past) of a wave on a rope if no extra forces act. Students will enjoy experiments in the lab to confirm these ideas.

5–14
The sixth picture of Fig. 5–10 shows the spring at an instant when the spring is almost straight. Explain why there is an instant when this happens.

In Figure 5–10 there is a symmetrical pulse coming into a fixed end and being reflected. It will look like this:



Halfway through reflection the displacement of the spring is the sum of the displacements of the incoming and outgoing pulse.



Note that this graph of displacement should not be confused with a picture of the spring. The spring cannot be in two vertical positions at the same time.

5–15
Consider the asymmetric pulse coming from the left in Fig. 5–6. Draw the shape it will have after being reflected at a fixed end.

5–16*
In Fig. 5–11, which has the larger displacement, the incident or the transmitted pulse? (Section 4.)

The incident pulse.

5–17*
In Fig. 5–11, what is the ratio of the speed of the pulse in the light spring to the speed in the heavy spring? (Section 4.)

Three. (The speeds are proportional to the distances the pulses move between successive pictures.)

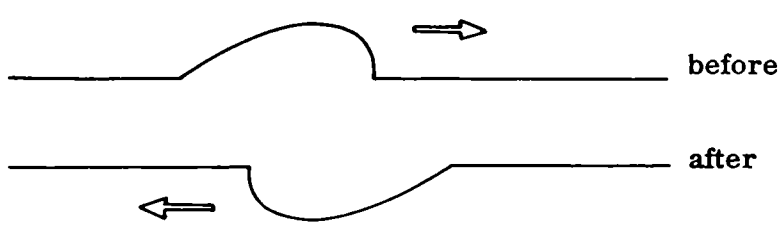
5–18
You send out a pulse at one end of a spring, and it returns to you upside down and smaller in size. What can you deduce about the speed of the pulse on a second spring which is attached to the other end of your spring?

This is just the situation of Figure 5–11, where part of the pulse is transmitted and part returns upside down, but smaller. In this case, the speed of the pulse in the second coil is less than in the first coil.

5–19†
(See the Appendix in the text.)

5–20
When light passes from air to water or vice versa, part of it is reflected. If this situation resembles that of a pulse crossing from one coil spring to another, in which case will you expect the light pulse to be reflected upside down?

If a pulse is moving from one spring to another in which its speed will be lower, the reflected pulse will be upside down. Applying this observation to light, we conclude that the light pulse traveling in air will be reflected upside down from water, since the velocity of light in air is greater than in water. This point is raised again in Chapter 8, Section 9.



5–21
Hold one end of a long rope with the other end tied to a rigid support. Stand looking along the rope and generate a wave by moving your hand through three fast clockwise circles.
(a) Describe the wave generated.
(b) Describe the reflected wave.
(c) Describe the motion of a particle of the rope as the wave passes forward and back.

This might be a good test of students' ability to observe effects when they are not sure of the outcome. One difficulty is that it is hard for students to understand why the circular wave reflects as it does. In particular, if students try to use an argument apparently similar to the one given in Section 4, they will get the wrong answer. Another difficulty of many students is in finding the right vocabulary to describe a circular wave.

It will be hard to generate a good three-turn wave, but a one-turn or two-turn wave shows the necessary reflection properties.
(a) The wave generated is a three-turn "left-hand" helix.
(b) As viewed by the observer, the rope seems to be rotating the same way in the reflected wave. However, since the wave is moving in the opposite direction, the reflected wave would be called a three-turn "right-hand" helical wave.
(c) As the wave passes, a particle moves over a circular path three successive times. As the wave is moving away from the observer, the particle moves over its circular path in a clockwise direction (as viewed by the observer). As the wave moves toward the observer, the particle also moves in a clockwise direction (as viewed by the observer).

Note: For this circular wave, the particle motion after reflection is in the same sense (circularly) independent of whether the reflecting end of the rope is fixed or free.

5–22*
How could you tell in Fig. 5–10 that the sequence of events proceeds from top to bottom and not from bottom to top? (Section 5.)

The pulse is smaller in the bottom frames than in the top frames.

5–23†
(See the Appendix in the text.)

5–24
What do we mean by an ideal spring in the context of this chapter?

By an ideal spring in this chapter we mean a spring for which the frictional losses are very small and which is sufficiently flexible so that superposition of both displacements and velocities holds to good accuracy.

5–25
We can say that the surface of the sea is approximately flat. Give some examples where this is a good approximation and some examples where it is a bad approximation.

A number of examples will occur to you and your students quite readily. A few samples are the following: The surface of the sea is approximately flat to:

Good approximation for statements of height of a mountain above "sea level"; estimating the amount of pipe necessary for undersea oil drilling; describing the average depth of the ocean.

Poor approximation for ships in a storm; man on a raft trying to use a sextant or a telescope; actually drilling deep-sea oil wells.

CHAPTER 6

6–1*
If a stick 10 cm long were placed horizontally in the ripple tank shown in Fig. 6–2, about how long would its shadow be? (Section 1.)

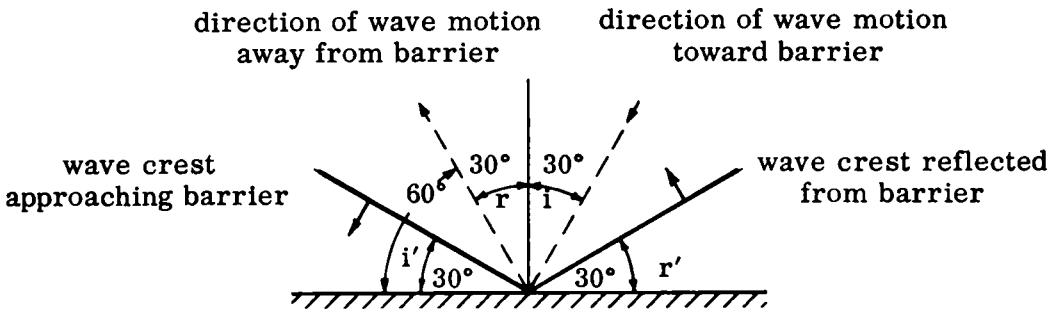
About 20 cm long.

6–2*
In Fig. 6–7, if $i = 25°$, what is the value of r'? (Section 3.)

$r' = 25°$

6–3*
In Fig. 6–22, the heavy black lines are crests; the arrows represent the direction of propagation of the pulses.
(a) Which is the incident pulse and which is the reflected pulse?
(b) What is the angle of incidence? (Section 3.)

(a) Incident pulse: (2).
Reflected pulse: (1).
(b) $i = 70°$.



6–4

A straight pulse approaches a barrier at an angle of 30°. What is the direction of motion of the pulse after reflection? Indicate it on a diagram.

If the wave crest makes an angle, i', of 30° to the barrier, then the reflected wave crest makes an angle, r', of 30° to the barrier. Since the wave travels in a direction perpendicular to the wave crest, the direction of travel is at 60° to the barrier or at 30° to the normal to the barrier.

6–5

In Fig. 6–9, where is the object source? Where is the image source?

The object source is at the pointed end of the dropper. The image source is at a point an equal distance behind the barrier off the bottom of the picture. These points can be confirmed using a compass.

6–6

Describe the wave motion that results when you dip your finger into the center of a circular tank of water. What would be the motion under ideal conditions?

Make sure students realize that the term "ideal" means "ideal reflector" and "ideal wave propagation." (After working with a ripple tank some students may associate an ideal tank with one which has perfectly absorbing sides.) This is a good problem for beginning a discussion on focusing of waves.

If a finger is dipped into the center of a circular water tank, a circular wave front is produced which travels outward, is reflected, converges back to the center, goes through itself and outward again, etc. Students should note that the direction of propagation of each point on the outgoing circular wave is *along* a normal to the reflecting surface. Hence, the direction of the corresponding segment of the reflected wave is back along the same line, toward the center.

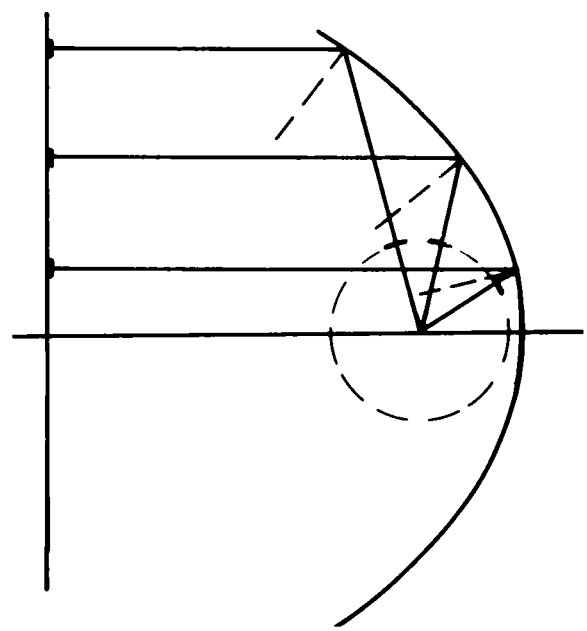
You can see this effect very nicely in a cup or glass of liquid by letting a drop fall at the center.

6–7†

(See the Appendix in the text.)

6–8

A circular pulse generated at the focus of a parabolic reflector is reflected as a straight pulse. What does this tell you about the geometric properties of a parabola? (Hint: Consider tiny segments of the circular pulse originating at the focal point and see what must be the shape of the parabola to produce a straight pulse.)



Let us follow three segments of the circular pulse (see drawing). After reflection, they become segments of a straight pulse which at some time coincides with the straight line in the drawing. The distance each pulse segment travels from the focus to any point on the parabola and then to the straight line must be constant. For the various parts of the circular pulse to be reflected in a direction parallel to the axis, the perpendicular to the parabola at each point must bisect the angle between the line from the focus and the line parallel to the axis.

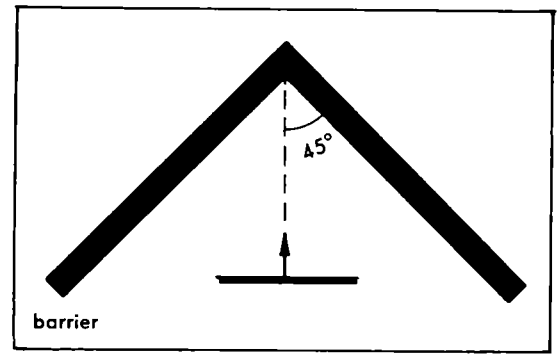
It may be instructive to compare this problem with Problem 7, which involves an elliptical reflector. If we move the focus B more and more to the left, the lines from the reflector to B tend to become parallel, and the reflected pulse becomes straight. Thus, a parabola can be looked upon as an ellipse with one focus at infinity.

6–9

In Fig. 6–24, a straight pulse approaches a right-angled barrier at an angle of 45°.

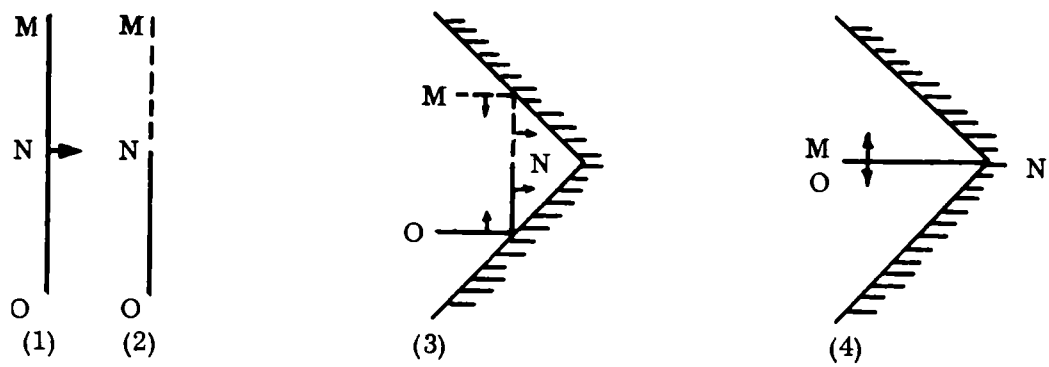
(a) How does it reflect?

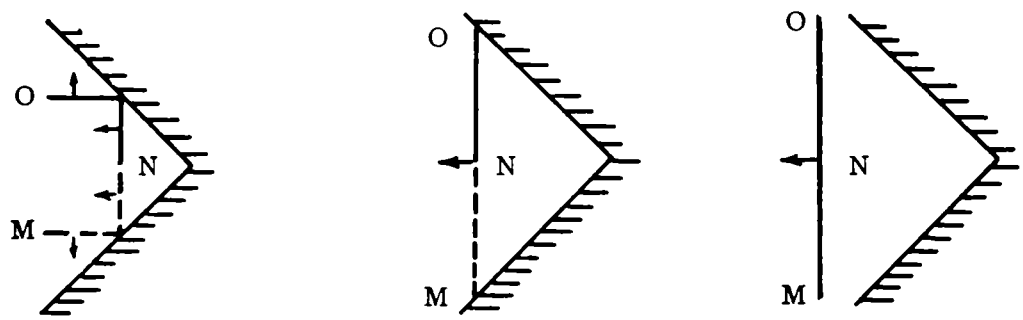
(b) What happens if the wave is incident at some other angle?



(a) The straight pulse is reflected as a straight pulse but interchanged end for end. For a qualitative understanding it is probably best first to consider one half of the pulse such as ON, in the diagrams below, follow it through the reflection, and then add the dashed half. Leave the proof for part (b).

(b) This part of the problem is somewhat difficult. Again consider only the part NP of the pulse in the following diagrams. Point M will follow the indicated dashed track through M', and point P will follow the indicated path through P'.





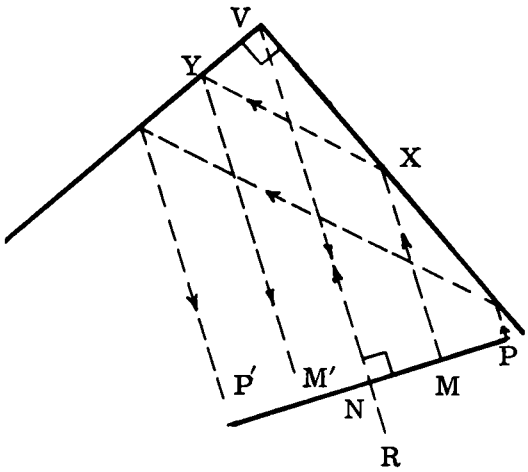


Fig. 1.

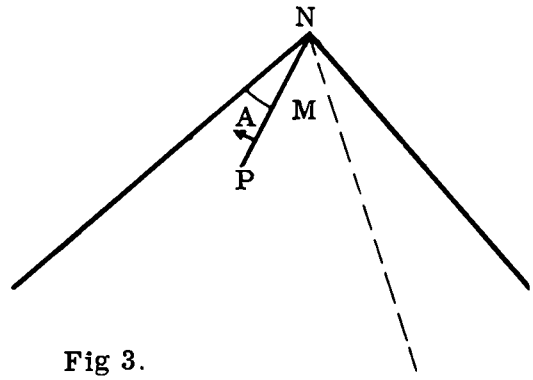


Fig 3.

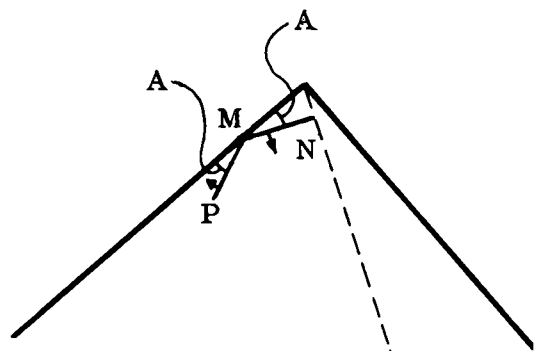


Fig. 4.

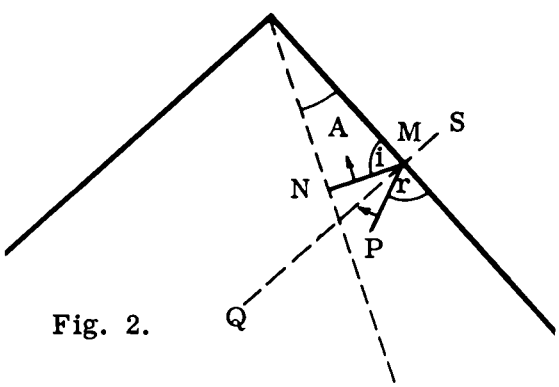


Fig. 2.

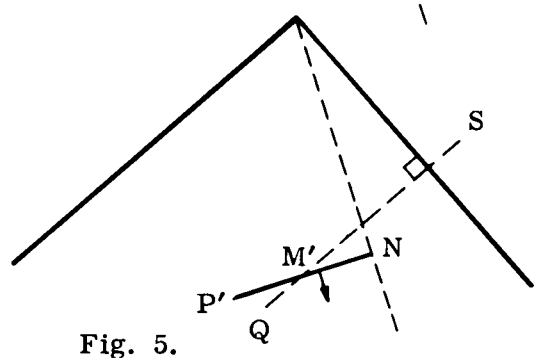


Fig. 5.

Figures 2, 3, 4, and 5 show the position of the pulse segment *PMN* at intermediate stages. In Figure 2, the pulse is shown after partial reflection. *QS* is perpendicular to the first mirror and thus parallel to the second mirror. Since $\angle r = \angle i$, the complementary angles *PMQ*, *NMQ*, and *A* are also equal. The angle of incidence at the second mirror is equal to $\angle PMQ$ and for simplicity has been labeled $\angle A$ in Figure 3. In Figure 4, the angle of reflection is also labeled $\angle A$. In Figure 5, $\angle SM'N$ is equal to $\angle A$ (the angle of reflection of Figure 4) since their sides are mutually parallel. Thus it has been shown that the reflected pulse is parallel to the incoming pulse, but moving in the opposite direction and with the ends *P* and *N* interchanged. You may wish to discuss the path of point *N* of the pulse along the line *RV* to the mirrors and back. This path may be treated as the limit to which the path *MXYM'* approaches as point *M* is moved toward *N* along the wave front *PN*.

Note that in a normal reflection from a plane mirror, left and right sides *do not* cross over. This results in your left hand being your image's right

hand. Using a corner mirror, however, you will find that when you move your right hand, your image will also move its right hand. This is one way to see yourself as others see you!

6–10*

A machine gun fires 10 rounds per second. The speed of the bullets is 850 m/sec.

(a) What is the distance in the air between the flying bullets?

(b) What happens to the distance between the bullets if you step up the rate of fire? (Section 4.)

(a) 85 m.

(b) It decreases.

6–11*

If a wave generator produces 12 pulses in 3 seconds, what is (a) its period? (b) its frequency? (Section 4.)

(a) $T = \frac{1}{4}$ sec.

(b) $f = 4$ per second.

6–12*

Would you increase or decrease the frequency of the generator in a ripple tank in order to produce waves of greater wavelength? (Section 4.)

Decrease the frequency.

6–13

(a) In a ripple tank, when one pulse is sent every $\frac{1}{10}$ sec, we find that λ is 3 cm. What is the speed of propagation?

(b) In the same medium we send two pulses, the second one ½ sec after the first. How far apart are they?

This problem can be used to trigger a discussion of the formula $v = f\lambda$. It can also be worked logically without direct reference to the formula.

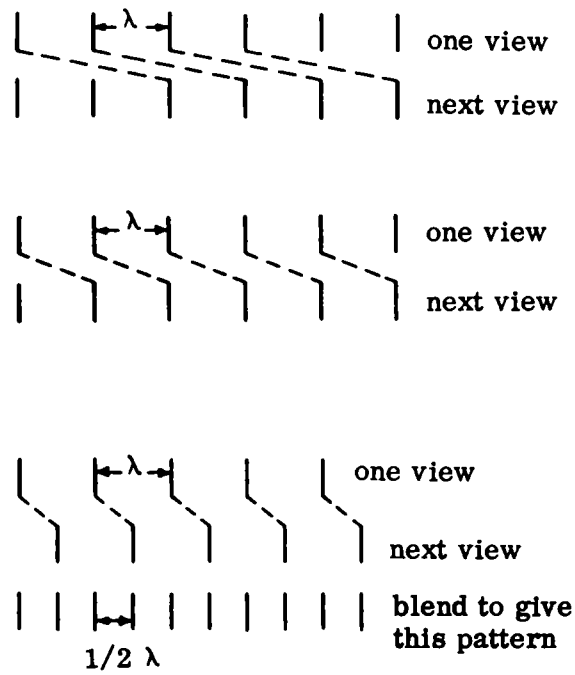
(a) $v = f\lambda = 10 \times 3 = \underline{30 \text{ cm/sec.}}$

(b) $v = f\lambda$; $30 = 2\lambda$; $\underline{\lambda = 15 \text{ cm.}}$

6–14

Assume you are looking at a periodic wave of frequency $f = 4$ per sec through a two-slit stroboscope. What do you expect to see if you rotate the stroboscope with a frequency of 1, 2, and 4 revolutions per sec?

One revolution per second of the stroboscope will give two views per second. Between views any particular crest will have moved exactly two wavelengths. Thus the pattern will be stopped with crests one wavelength apart.



Two revolutions per second of the stroboscope gives 4 views per second, which is the same as the frequency of the periodic wave. You will see a stopped pattern with crests one wavelength apart. The pattern appears to be the same as with one rev/sec.

Four revolutions per second giving 8 views per second will give a stopped pattern with crests apparently one-half wavelength apart.

6–15

A point source in the ripple tank produces circular periodic waves. By using a stroboscope to stop the motion, we measure the difference in radius between the first and sixth circular crests and find it to be 10 cm.

(a) What is the wavelength?

(b) Why did we not calculate the wavelength by using the radius, say, of the fifth pulse only?

(c) Why do we use this method of measurement rather than take the difference between neighboring crests?

(a) $5\lambda = 10$ cm/5. Therefore, $\lambda = \underline{2 \text{ cm.}}$ Some students will make the mistake of thinking there are six wavelengths between six crests.

(b) At any given instant, it is hard to tell whether there is a wave crest at the center (source). Therefore we may make an error of some fraction of a wavelength. In fancier

language, we do not know the phase of the wave at the origin.

(c) If we make a certain error in locating the position of a wave crest, or in measuring, the error in the wavelength is only $\frac{1}{5}$ as great when we measure five waves as when we measure one.

6–16

Circular waves are produced in the ripple tank by drops of water which fall at a constant rate. The wavelength is observed to be 1.2 cm. The experiment is repeated with the source moving at a uniform rate from one end of the tank to the other, along the center line. An observer near the end of the tank toward which the source is moving measures the wavelength and finds it to be 0.8 cm.

(a) What is the ratio of the wave velocity to the source velocity?

(b) What wavelength would an observer at the starting point measure after the source has moved some distance away from him?

(a) The wave travels 1.2 cm between pulses. Since the wavelength is decreased by .4 cm when the source is moving, the source must move 0.4 cm between pulses. Therefore, the ratio of wave velocity to source velocity is 3.

(b) Each pulse will be generated 0.4 cm farther away, and therefore the wavelength will be 1.2 cm + 0.4 cm = 1.6 cm.

Note: The Doppler effect may be easily demonstrated in a ripple tank with a length of hose fitted with a glass tube bent 90° at one end. Hold the end of the glass tube about 1 cm above the water surface and puff at a steady, rapid rate as you move the glass tube along. A shock wave will occur if the speed of the source is greater than the speed of the waves.

6–17*

In Fig. 6–14, where is the boundary between the area of deep and shallow water? (Section 5.)

Near the fifth crest from the bottom.

6–18*

What is the index of refraction in passing from the deep to the shallow water in Fig. 6–14? (Section 5.)

$n \approx 1.8$.

6–19

Measure the index of refraction in Fig. 6–15 by the method you used in the previous problem, and by finding the ratio of the sines of the appropriate angles. Compare the results.

$$n_{12} = \frac{\lambda_1}{\lambda_2} = \frac{0.69 \pm 0.01\text{ cm}}{0.42 \pm 0.02\text{ cm}} = \underline{1.64 \pm 0.10.}$$

$$n_{12} = \frac{\sin i}{\sin r} = \frac{\sin (57.5° \pm 1.0°)}{\sin (29.0° \pm 2.0°)} = \underline{1.74 \pm 0.14.}$$

These results check well within the accuracy of the measurements.

6–20*

If the frequency changed when periodic waves went from deep to shallow water, would a stroboscope be able to stop all of the waves at one time? (Section 5.)

No; not unless one frequency was a whole number multiple of the other.

6–21

A ripple-tank wave passes from a shallow to a deep section with an incident angle of 45° and a refracted angle of 60°.

(a) What is the ratio of speeds in the two sections?

(b) If the wave speed is 25 cm per second in the deep section, what is it in the shallow one?

$$\text{(a)}\quad \frac{v_{\text{deep}}}{v_{\text{shallow}}} = \frac{\sin 60°}{\sin 45°} = \frac{0.866}{0.707} = \underline{1.23}$$

$$\text{(b)}\quad v_{\text{shallow}} = \frac{v_{\text{deep}}}{1.225} = \frac{25\text{ cm/sec}}{1.225} = \underline{20.4\text{ cm/sec}}$$

6–22

(a) A tire on an automobile wheel has a circumference of 7.0 feet. When the wheel is turning 200 times per minute, what is the speed of the automobile in feet per min?

(b) A light wave whose frequency is 6.0×10^{14} per sec is passed through a liquid. Within the liquid the wavelength is measured and found to be 3.0×10^{-5} centimeters. What is the speed of light in this liquid?

(c) What is the wavelength in vacuum (from which the frequency was calculated)?

(d) What is the index of refraction of the liquid for light of this frequency?

Many students will solve this problem without being aware that they made two assumptions: (1) light is a wave, and (2) the frequency of light is not changed as it goes from one medium to another. You may want to call this to their attention in discussing the problem.

(a) The car travels 7 feet each cycle (one revolution); thus, 7 feet/cycle × 200 cycles/min = $\underline{1400\text{ feet/min.}}$

(b) $v = f\lambda = 6.0 \times 10^{14}$ cycles/sec $\times$ 3 $\times 10^{-5}$ cm = $\underline{1.8 \times 10^{10}\text{ cm/sec.}}$

(c) $\lambda = \dfrac{3 \times 10^{10}\text{ cm/sec}}{6 \times 10^{14}\text{ cycles/sec}}$
$= \underline{5 \times 10^{-5}\text{ cm in vacuum.}}$

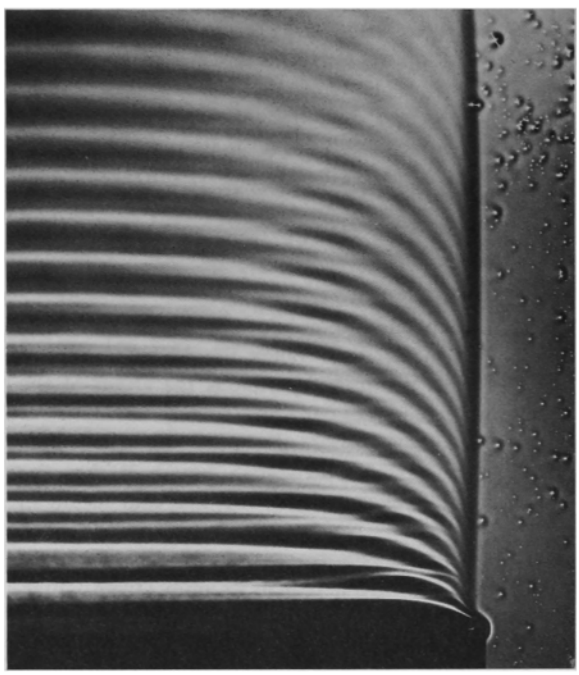
(d) $n = \dfrac{v_{\text{vac}}}{v_{\text{liq}}} = \dfrac{3 \times 10^{10}}{1.8 \times 10^{10}} = \underline{1.7.}$

6–23

The ripple tank is arranged so that the water gradually becomes shallow from one side to the other. Because of this, on one side of the tank the speed of a wave crest is different from that on the other side. As a result, straight waves become curved (Fig. 6–25). In the picture the pulses are moving toward the top of the page.

(a) Which is the shallow side?

(b) Does a similar phenomenon occur with light? Be prepared to discuss this in class.



This problem is more suitable for class discussion than for home assignment.

(a) Because water waves travel more slowly in shallow water, the right side must be the shallow side.

(b) This phenomenon would occur in a medium where the index of refraction gradually changes. Atmospheric refraction is an example of this. See Problem 15, Chapter 3.

Atmospheric refraction is also responsible for the "water" mirages that are commonly seen on roads on hot days. The sun heats the road, and the air near the surface becomes hotter (and less dense) than that above it. In this situation, the speed of light is greater near the road surface than it is a few feet above. Then light from a distant source which is headed toward the road in front of the viewer will gradually curve upward. If the angle is proper, the sky can be seen by looking down slightly toward the road. This effect is interpreted by the observer as resulting from reflection from water on the road.

6–24†

(See the Appendix in the text.)

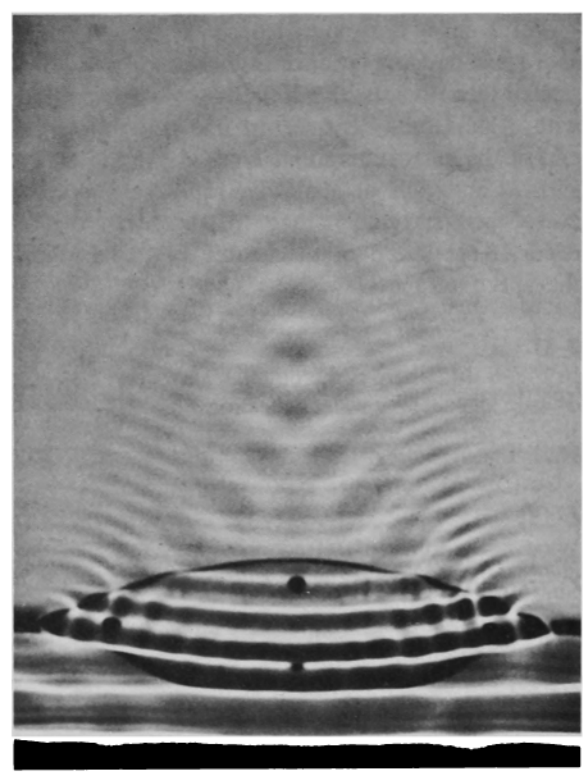
6–25

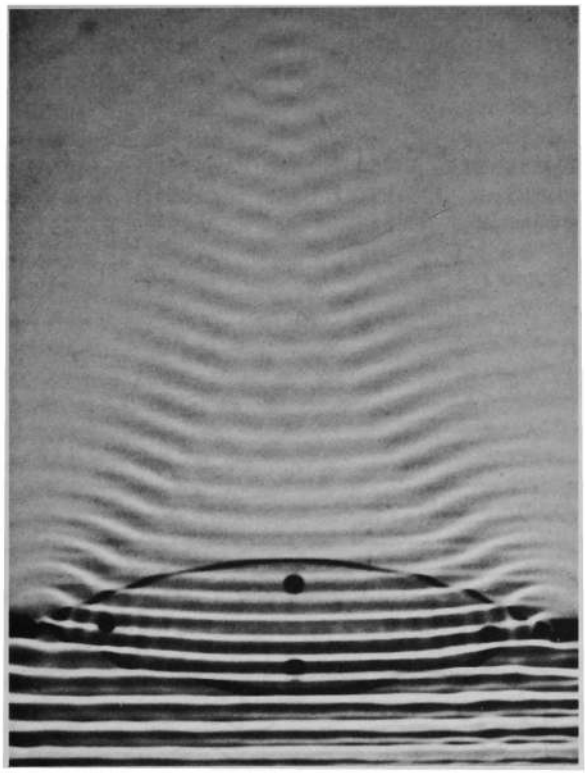
We set up the ripple tank as shown in Fig. 6–26 and generate a periodic straight wave. The resulting wave pattern is shown in the photograph in Fig. 6–27.

(a) Explain what is taking place.

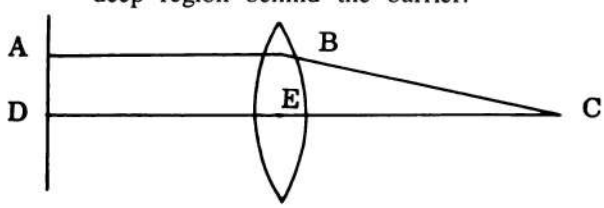
(b) Of what optical arrangement is this a model?

This is an important problem which is well worth class discussion. It affords the opportunity of reviewing lenses and refraction from the wave point of view.





(a) The straight wave fronts are bent in such a way that they converge to a "focus" in the deep region behind the barrier.



(b) This is a model of a converging lens. Notice that the wave travels more slowly in the shallow region and that the wave "comes together," i.e., focuses, at a point C whose position is such that the time for each part of any wave front to get to C is the same as that for any other. The time to go from A to B to C is the same as to go from D to E to C. Clearly ABC is the longer path, but this is just compensated by the fact that DEC goes through a greater length of shallow water where the wave travels slowly.

You may want to compare this with Problem 7 of this chapter where it was seen that focusing came about, in the reflection from an elliptical barrier, because it took the same time for all segments of the wave front to reach the focus. In this problem, though the lengths of the paths traveled by the various segments of the wave front differ, the important idea is the same as before; that all parts of the wave front arrive at the focus at the same time.

6–26

In Fig. 6–28 we see a photograph taken under the same conditions as those in Problem 25 except that here the waves have a shorter wavelength.

(a) How do the two photographs differ?

(b) What can you conclude from this difference?

(a) In Figure 6–28, the wavelength is shorter and the focal length is longer than in Figure 6–27.

(b) The photograph of Figure 6–28 shows that the waves have a shorter wavelength than those of Figure 6–27. Therefore, we conclude that the wave generator is oscillating with a higher frequency in this case.

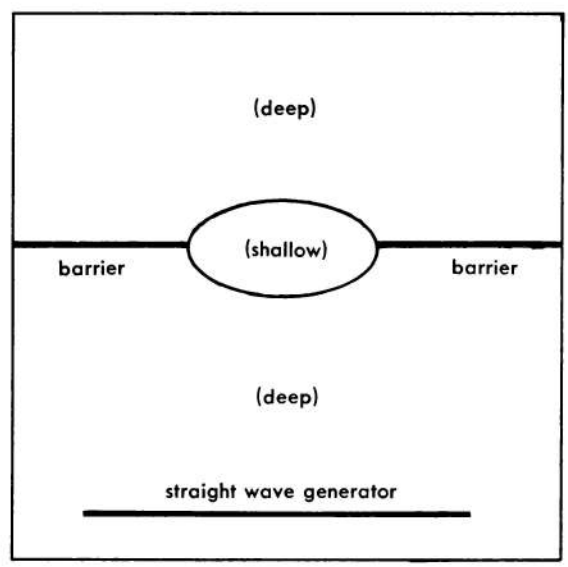
We can also see that the effective focal length of the lens has increased for the higher frequency and shorter wavelength. This indicates that the ratio of deep to shallow water speeds has decreased for the higher frequency.

6–27

Suppose the oval region in Fig. 6–26 is deep and the surrounding region is shallow.

(a) What will happen to the straight waves?

(b) Of what optical arrangement is this a model?



(a) A wave will travel faster in the deep water than in the shallow water. Thus, since the middle part of a straight wave travels over more deep water than the ends, the middle part will get ahead. The wave, after it passes the deep water, will be curved outward as if coming from a point source. The pattern will be that of segments of outgoing circular wave fronts.

(b) This arrangement is a model of a convex air lens inside glass or water.

6–28

How do we know that the wavelength of light must be very much less than a centimeter?

This problem concerns the order of magnitude of diffraction effects.

Light can pass through a slit 1 cm wide and cast sharp shadows of the edges of the slit. We can conclude from the dependence of diffraction on wavelength that the wavelength of light must be much less than 1 cm.

6–29

Imagine our eyes were sensitive only to light of wavelength 0.1 mm. How would this affect our ability to see? Could you thread a needle?

If the wavelength of light were 0.1 mm, we should see only objects larger than 0.1 mm and hence could not see clearly the eye of a needle. Because of the size of our pupils, we could only distinguish between points having an angular separation greater than a few degrees. (We would not even be able to see the needle clearly.)

6–30

If sound is a wave phenomenon, how would you explain the common experience of hearing sounds from around corners?

This exercise concerns the qualitative idea of diffraction of sound waves. The diffraction of sound waves allows us to hear around corners even without reflecting surfaces to help "turn the corner." Short wavelengths are diffracted less than long wavelengths. This can be very qualitatively tested, but beware of *reflections*. Hi-fi fans will know that a tweeter (the small high frequency or short wavelength speaker) must "point at" the listener. The (low frequency or long wavelength) woofer can be pointed in any direction. Because diffraction effects are so important, even with large-scale objects, we can conclude that the wavelength of sound waves must be of the order of feet.

6–31

Sound waves in air usually travel at about 330 meters per second. Audible sounds have a frequency range of about 30 to 15,000 cycles per second. What is the range of wavelengths of these sound waves?

Since $v = f\lambda$, $\lambda = \frac{v}{f}$.

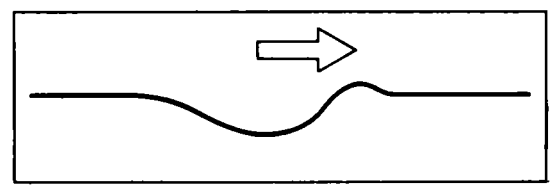
$$\lambda_1 = \frac{330 \text{ m/sec}}{30 \text{ cycles/sec}} = \underline{11 \text{ m.}}$$

$$\lambda_2 = \frac{330 \text{ m/sec}}{15{,}000 \text{ cycles/sec}} = \underline{0.022 \text{ m.}}$$

CHAPTER 7

7–1*

If a periodic wave whose pulses each had the shape shown in Fig. 7–18 were traveling along a spring,



and you wanted to produce nodes by sending appropriate periodic pulses in the opposite direction, what shape pulses would you use? (Section 1.)

7–2*

In Fig. 7–3 how far from the wall will pulse *c* meet pulse *a* to make the next node? (Section 1.)

A distance λ from the wall.

7–3†

(See the Appendix in the text.)

7–4*

In Fig. 7–19 the circles represent the crests of waves produced by sources S_1 and S_2. At which of the points *A*, *B*, and *C* is there a "double crest," a "double trough," or a nodal point? (Section 2.)

"Double crest": *A*.
"Double trough": *C*.
Nodal point: *B*.

7–5

Draw the sets of concentric circles and the interference pattern from two sources with $d = 5\lambda$ at the time:

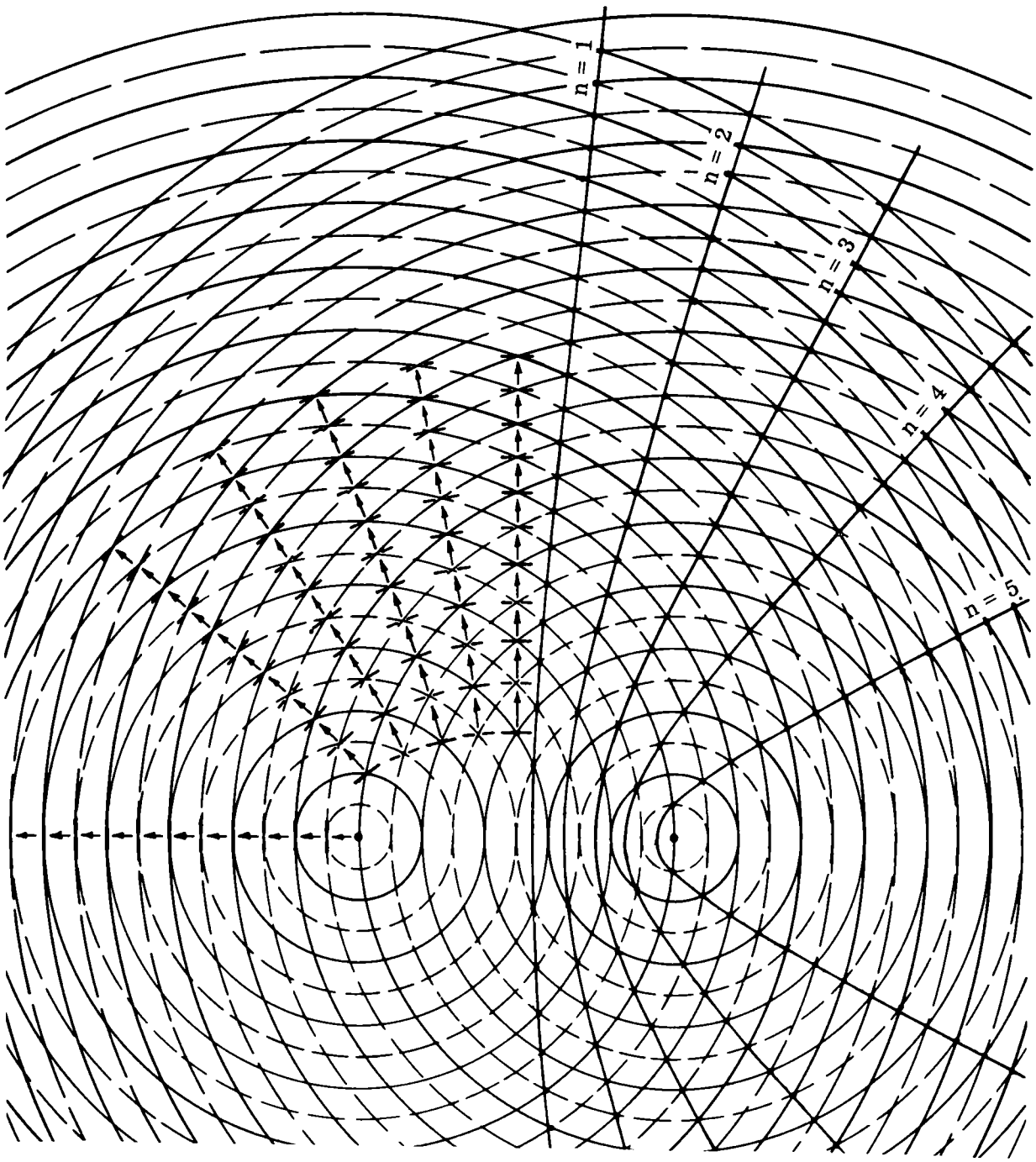
(a) when the generators have just produced crests,

(b) when they have just produced the following troughs.

How have the reinforced crests moved during the time interval between these drawings?

The solution of this problem should help students understand Sections 2 and 3. Encourage students to draw carefully; without careful drawing the regularity of the pattern at the more distant points will be lost.

(a) The diagram has the scale of $\lambda = 1$ cm. The crests at time (a) are represented by solid lines. The troughs for the same time are



drawn with broken lines. Cancellation occurs where a crest and a trough cross each other. Points of cancellation have been shown with black dots and these points have been connected on the right side of the diagram to show the nodal pattern on that side.

(b) For part (b), a new drawing is not necessary. The same drawing can be used if we now let solid lines represent troughs and broken lines crests. Keeping in mind this change of representation we can see what changes have occurred between time (a) and time (b), one-half period later.

On the left side of the drawing the short full lines locate the double crests at the time (a), and the short broken lines locate the double crests at time (b). The connecting arrows indicate how the double crests have moved in the time between (a) and (b). It is interesting to note that as your eye moves from one radial line of double crests to an adjacent nodal line and then to the next line of double crests, you see that the two neighboring lines of double crests are out of phase. Some students may prefer to use colored pencils to indicate the different parts of this diagram.

7–6

Draw the interference pattern for the case $d = 5\lambda$ on a piece of paper large enough so that you can see the nodal lines become straight at a great distance from the sources. Continue these straight lines back toward the sources and show that they all pass close to the midpoint of the line joining the sources.

If the construction for Problem 5 was done carefully, it may be used in the solution of this problem. In order to obtain straight nodal lines, the larger circles should have radii of about 9λ to 13λ.

7–7*

In Fig. 7–4, is there a nodal line along the perpendicular bisector of the line joining S_1 and S_2? (Section 2.)

No. Along this line are double crests separated by double troughs.

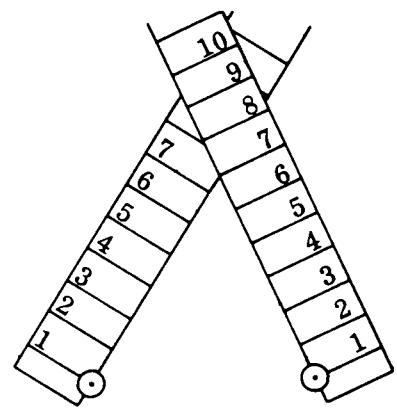
7–8

Fold two pieces of ruled paper into long strips about 2 cm wide and hold them as in Fig. 7–20. Imagine that the lines are wave crests. Your fingers then represent the sources of the waves. Notice how the crests from both sources add together. Now by sliding the free ends sidewise, locate nodal lines and moving wave regions.

This home project provides another way of visualizing the construction of an interference pattern from a two-point source. The location of nodal lines with this device will be inexact unless each strip is rotated around a point on its inner edge. Thumb tacks or pins, used as points about which to rotate the strips, will hold the "sources" in fixed positions, making the device easier to handle. To simulate sources in phase, the two strips should be tacked at comparable points (e.g., both tacked at lines).

As well as being a home project, the use of this device might make a good classroom demonstration if the strips (including spacing of the lines) were scaled up to a size adequate for class viewing.

Cancellation, reinforcement, or something in between can be determined for any point on the surface surrounding the sources by bringing the strips together at that point. If comparable positions on the strips (lines, midpoints, etc.) come together, reinforcement occurs and the point so located is in a moving wave region. Where a line on one strip meets a midpoint on the other strip, cancellation occurs and the point so located is on a nodal line.



The strips can be used to search out the nodal lines. If the lines on the strips are numbered as in the diagram at the right, we can trace out the path of nodal lines by locating several points that lie along the nodal line. For example, if we wish to trace the first nodal line, move the strips so that 3.5 on one strip touches 4 on the other strip. Mark the point. Then bring 4.5 opposite 5; mark, etc. The second nodal line can be located similarly by bringing 3.5 opposite 5, 4.5 opposite 6, etc.

7–9*

Which nodal line in Fig. 7–10 is made up of all points P such that $PS_2 - PS_1 = 3\lambda/2$? (Section 3.)

The second nodal line to the left of the central line.

7–10*

You know the distances from a point on a nodal line to the two point sources in a ripple tank. What else do you have to know to calculate the wavelength of the waves? (Section 3.)

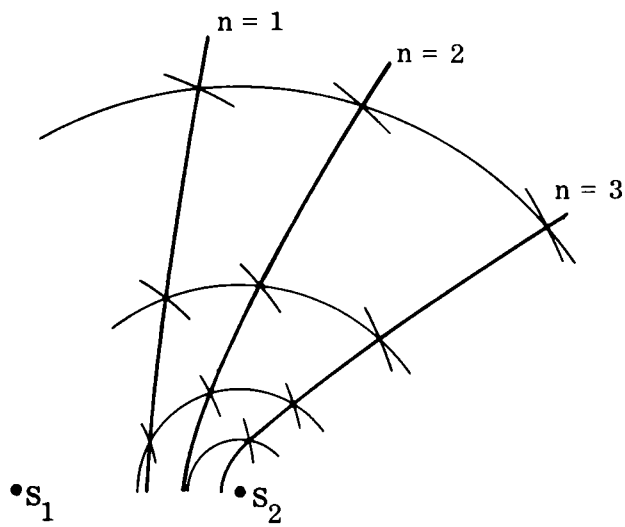
On which nodal line the point lies, i.e., the value of n.

7–11

Construct the nodal lines for two point sources with $\lambda/d = \frac{1}{3}$ by the method of Fig. 7–11. Is this really a different method from that used in Problem 6?

This graphical exercise illustrates the path-difference characteristic of the nodal lines.

The accompanying figure, drawn on the scale $\lambda = 1$ cm, was made with a view to minimizing the number of compass settings required. A large arc of a circle is drawn about S_2, with any convenient radius r. The compass is then set successively to $r + \frac{1}{2}\lambda$, $r + \frac{3}{2}\lambda$, $r + \frac{5}{2}\lambda$; and the intersections of these circles, centered on S_1, with the original arc are drawn. After a range of values which covers all of the desired space has been tried, the intersections on the line joining S_1 and S_2 are calculated and marked. The appropriate intersections are then linked up to form the nodal lines.



The advantage of this method is that convenient values of r can be chosen, with as many or as few as are needed. There is no need to draw a whole series of circles spaced λ apart, as was done in Problems 5 and 6. One physical interpretation of the path-difference method is that we fix our attention on a particular part of the wave from S_2 (a crest, a trough, or some intermediate point) and plot its intersection with that part of the wave from S_1 which has just the opposite displacement. The intersection is a point on the nodal line, and we plot the motion of the intersection as both waves move outward from their sources. In a sense, this viewpoint is complementary to the approach of Problems 5 and 6 of watching the whole surface at one instant, and observing the lines of zero displacement. The two methods are simply different ways of finding path differences.

7–12

What would happen to the nodal lines if one of the two sources made weaker and weaker waves and then quit?

The nodal lines would remain in the same positions but become less and less distinct. There would no longer be complete cancellation and hence the water along these lines would have an increasing amount of wave motion propagating along them. When the second source finally stopped altogether, there would of course be simply waves propagating outward from the operating source.

7–13*

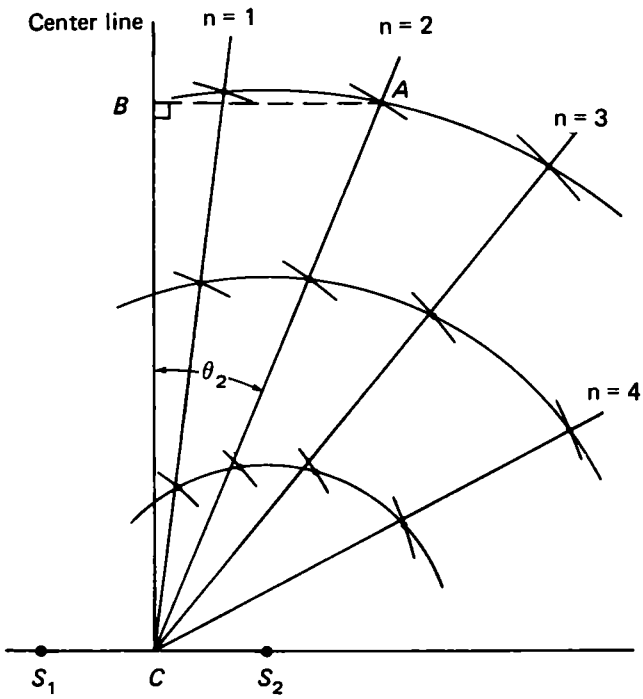
For two point sources separated by a distance d is there a range of wavelengths λ which will produce an interference pattern with no nodal lines? (Section 4.)

Yes, all λ for which $\lambda > 2d$.

7–14

Two sources 6.0 cm apart operating in phase produce water waves with a wavelength of 1.5 cm. Draw the nodal lines far from the sources. Determine the position of each line by means of intersecting arcs of circles drawn from the two sources. Measure the angle between the second nodal line and the center line of the pattern. Compare the sine of this angle with $(n - \frac{1}{2})\lambda/d$.

This graphical exercise confirms the analytical definition in Section 4 of the angular position of the nodal lines. A careful drawing will be convincing.



The accompanying drawing was made using the same method as in Problem 11. It has been scaled down; the original drawing had a radius = 15 cm. Only the points on the nodal lines "far" from the sources are necessary; the other points were included in our drawing to show the amount of deviation from a straight line.

Numerical results on $\sin \theta_2$ were as follows:

$AB = 6.05$ cm, $AC = 15.84$ cm,

$$\sin \theta_2 = \frac{6.05\text{ cm}}{15.84\text{ cm}} = 0.382 \text{ (graphically).}$$

By the path difference formula,

$$\sin \theta_2 = \frac{(n - \frac{1}{2})\lambda}{d} = \frac{(2 - \frac{1}{2})(1.5\text{ cm})}{6.0\text{ cm}}$$

$$= 0.375 \text{ (theoretically).}$$

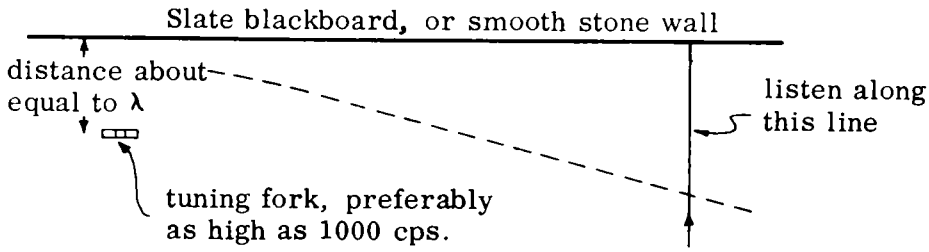
We have a relative error of

$$\frac{0.382 - 0.375}{0.375} = 1.9\%.$$

7–15

Suggest an interference experiment to prove that sound is a wave phenomenon. How could you use such an experiment to determine the wavelength of sound?

This is a good problem illustrating the application of the techniques of this chapter to a different, but familiar, phenomenon. This exercise should be kept theoretical and speculative, since actual experiments are difficult to perform. If you are concerned with the dimensions of their suggested experiments, students can refer to the range of wavelengths for sound found in Problem 31, Chapter 6. The following is a simple demonstration to observe interference of sound waves. A tuning fork of reasonably high frequency held about a distance λ from a slate blackboard will work quite well. If a student moves his head along the line indicated, he should hear quite distinct regions of sound and regions of quiet. The position of the nodes (quiet zone) will be enough for an order of magnitude estimate of λ. (Try to strike the tuning fork so as to excite only its fundamental frequency of oscillation. The presence of harmonics can be somewhat confusing.)


Slate blackboard, or smooth stone wall
distance about equal to λ
listen along this line
tuning fork, preferably as high as 1000 cps.

7–16

(a) From Figs. 7–6 and 7–9 find the ratio λ/d by using the equation $\sin \theta_n = (n - \frac{1}{2})\lambda/d$.

(b) The dimensions of the photographs are one-fourth the actual size. By measuring d on the photographs estimate the actual value of λ.

You should point out that variations in measurement may lead to slightly different values.

(a) Any of the four nodal lines shown in Figure 7–6 or any of the three nodal lines in Figure 7–9 can be used. Since a center line is hard to locate accurately, the angles were measured by taking one half of the angle between the nth nodal line on the right and the nth nodal line on the left.

Typical measured values for Figure 7–6 and Figure 7–9 are:

Figure 7–6:

Nodal line number	Angle, θ	λ/d
1	7°	0.25
2	21°	0.23
3	37°	0.24
4	53°	0.23

A fair average is $\lambda/d = \underline{0.24}$.

Figure 7–9:

Nodal line number	Angle, θ	λ/d
1	10°	0.35
2	32°	0.35
3 (very crude)	55°	0.33

A fair conclusion is that $\lambda/d = \underline{0.35}$ in this case.

(b) For Figure 7–9 the measured source separation is about 1.6 cm which implies a 6.4-cm separation of the actual sources. Thus the actual wavelengths were:

Figure 7–6 $\lambda = 0.24 \times 6.4 = \underline{1.5 \text{ cm}}$

Figure 7–9 $\lambda = 0.35 \times 6.4 = \underline{2.2 \text{ cm}}$.

7–17

Look up the definition of "hyperbola" and show that nodal lines are hyperbolas.

We hope that the definition of a hyperbola which students will find is just the one we have found for the nodal lines: the locus of points whose distances to two fixed points have a constant difference. From this they will know that nodal lines are hyperbolas, which from a physics point of view adds nothing to their understanding of interference, but enables them to talk to a math student.

There are other definitions of a hyperbola which are equivalent to the path-difference definition, but it would be best to limit class discussion to the definition given above.

7–18

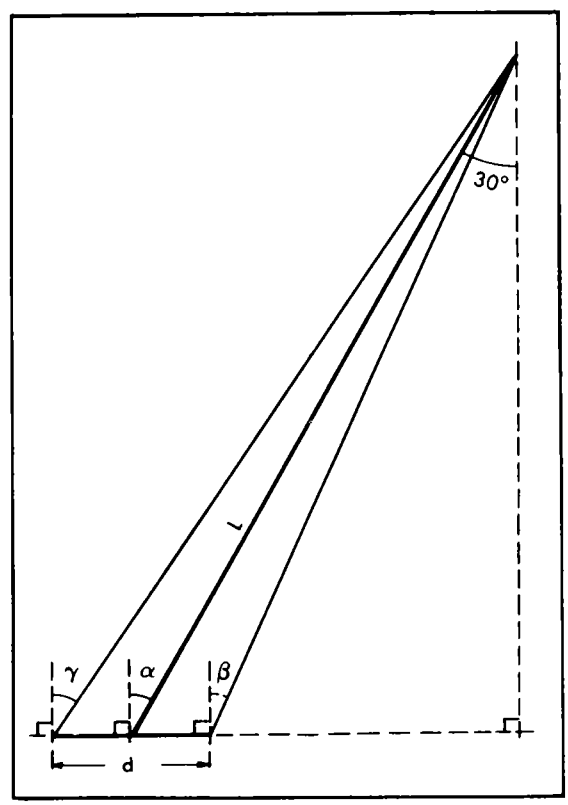
In Fig. 7–21, $L = 50$ cm, $d = 10$ cm, $\alpha = 30°$. What are γ and β? Find γ and β when $L = 500$ cm. Does this convince you that it is a good approximation to set $\gamma \approx \beta \approx \alpha$ when L is much greater than d?

Let b be the base of the right triangle with the vertex angle $= \alpha = 30°$. Then $b = L \sin \alpha$.

When $L = 50$ cm, $b = L \sin 30° = 50 \text{ cm} \times 0.5 \text{ cm} = 25$ cm.

$$\text{Sin } \beta = \frac{25 \text{ cm} - d/2}{50 \text{ cm}} = \frac{25 \text{ cm} - 5 \text{ cm}}{50 \text{ cm}}$$
$$= \tfrac{20}{50} = 0.40; \quad \underline{\beta = 23.6°.}$$
$$\text{Sin } \gamma = \frac{25 \text{ cm} + d/2}{50 \text{ cm}} = \frac{25 \text{ cm} + 5 \text{ cm}}{50 \text{ cm}}$$
$$= \tfrac{30}{50} = 0.60; \quad \underline{\gamma = 36.9°.}$$

When $L = 500$ cm, $b = 500 \text{ cm} \times \sin 30° = 250$ cm.



$$\text{Sin } \beta = \frac{250\text{ cm} - 5\text{ cm}}{500\text{ cm}} = \frac{245}{500} = 0.490;$$

$$\underline{\beta = 29.3°.}$$

$$\text{Sin } \gamma = \frac{250\text{ cm} + 5\text{ cm}}{500\text{ cm}} = \frac{255}{500} = 0.510;$$

$$\underline{\gamma = 30.7°.}$$

Since $29.3° \approx 30° \approx 30.7°$, students should be convinced that as L becomes very much larger than d, as is true when performing Young's experiment, $\gamma \approx \alpha \approx \beta$.

7–19

One red and one blue car are going around a circular race track 5.0 km in circumference. They move at constant speed. Each car takes 2.5 minutes for each lap. The blue car always comes around 0.50 minute behind the red.

(a) What is the phase delay p of the blue car with respect to the red car?
(b) What is the speed of each car?
(c) If the track were only 4.0 km long, would this change the answers to (a) and (b)?

(a) Since each car takes 2.5 minutes to complete the lap, the period, T, is 2.5 minutes. A delay of 0.50 minute is $\frac{1}{5}$ of a period. Therefore the phase delay, p, is $\underline{\frac{1}{5}}$.

(b) The speed of each car is the circumference, 5 km, divided by the time required to go around the track, 2.5 minutes. The speed is 5 km/2.5 min = $\underline{2\text{ km/min.}}$

(c) If the track were 4 km in circumference, the phase delay, p, would be unchanged ($p = \frac{1}{5}$) since the phase delay depends only on the timing of the cars, and that is unchanged. The speed, however, is changed: speed = 4 km/2.5 min = $\underline{1.6\text{ km/min.}}$

The *phase* is independent of the amplitude and velocity of the motion. This result also applies to the ripple tank. The pattern we see depends only on p, and not on the velocity of the waves or their amplitude.

7–20*

In Fig. 7–14, $l = 0.25$ cm and $\lambda = 0.70$ cm. What is the phase delay? (Section 5.)

$p = 0.36$.

7–21*

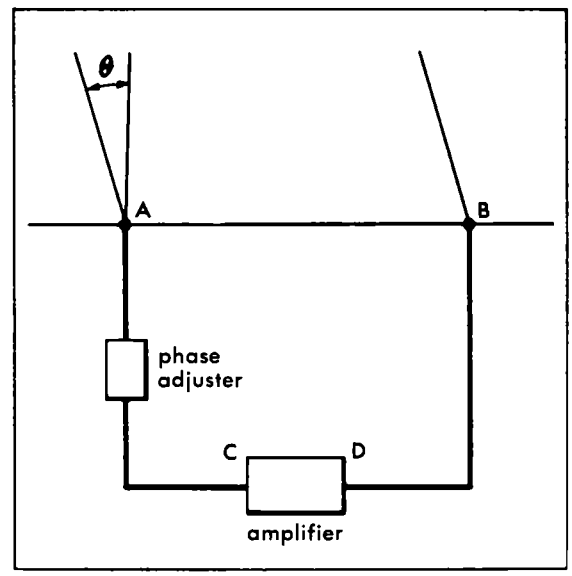
Is a phase delay of 0 the same as a phase delay of 1? (Section 5.)

Yes, in both cases the two sources dip simultaneously.

7–22

In radioastronomy, one method for "looking" at different directions in the heavens is rather like using an adjustable-phase two point source rippler in reverse.

Antennas A and B, tuned to the same wavelength, are set up a certain distance apart, and the signals they receive are mixed in an amplifier after the signal from one of them has been fed through a phase adjuster (Fig. 7–22). Maximum signal is obtained if the signals entering the receiver at C and D are in phase.



The phase adjuster changes the phase by changing the length of the transmission cable between A and C. When the cable AC is equal in length to the cable BD, the signals from a point directly overhead will be in phase at the receiver. To "look" in the direction θ, the cable AC must be lengthened. If the distance AB is 100 m, and if the frequency to which the receiver is tuned is 20 megacycles/second, what phase difference must be introduced to make the system look in a direction 5° away from the vertical?

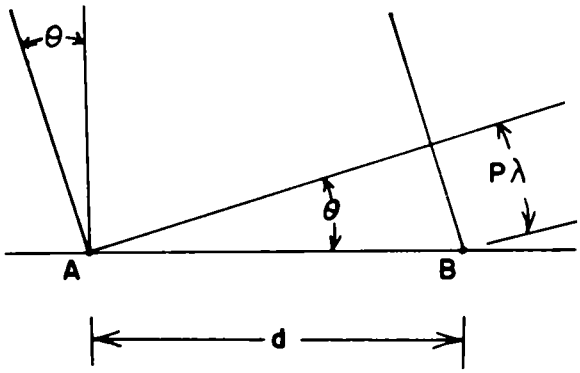
The signal that reaches B must be a fraction of a wavelength p, the phase difference, behind the wave reaching A. From the diagram below,

$$\sin\theta = p\lambda/d$$

$$p = d\sin\theta/\lambda$$

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8\text{ m/sec}}{2 \times 10^7\text{ cycles/sec}} = 15\text{ m}$$

$$p = \frac{100\text{ m} \times 0.087}{15\text{ m}} = \underline{0.58.}$$

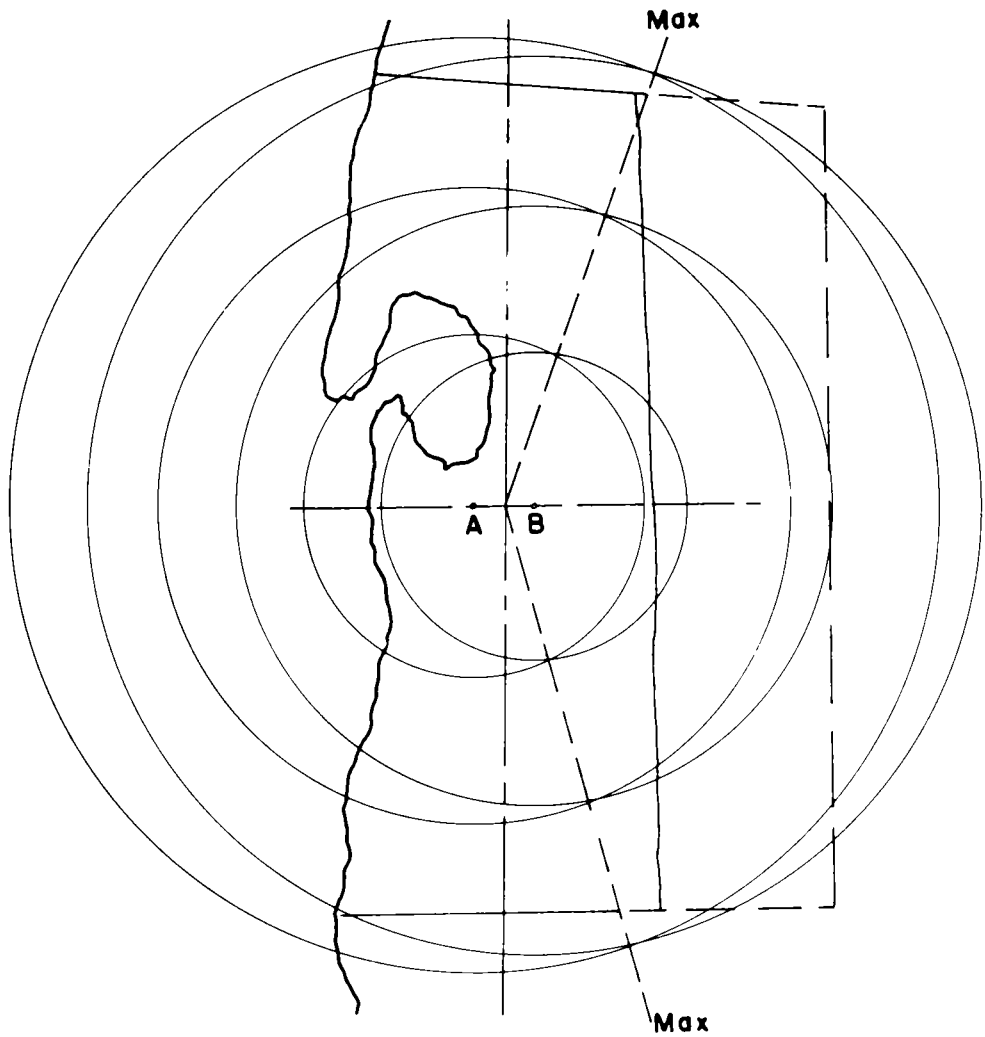


7–23

Standard radio broadcasting stations operate on wavelengths of between 200 and 600 meters. The usual antennas are vertical towers that behave like point sources of waves in the plane of the earth's surface. (a) How could you arrange two point source radio antennas to give good coverage of a city built in a strip along the seacoast and waste a minimum of power over the water or in unpopulated areas?

(b) If the city expanded inland, how would the station be able to provide better signal strength to the newly populated areas?

(a) Essentially, the problem reduces to finding an arrangement in which the signal reinforcement is a maximum in the direction of the coast and a minimum perpendicular to it. We do not want to have any actual nodal lines, because people living along a nodal line would not be able to receive any signal at all. For any two sources in phase, there is a line of maximum reinforcement along their perpendicular bisector. But as we saw in Problem 13, when the source separation d is less than a half wavelength, there are no nodal



lines. Therefore the two antennas should be placed slightly less than a half wavelength apart at positions A and B in the diagram.

(b) Relocating the station would be one solution. However, the line of maximum reinforcement can be made to swing inland by decreasing the length of the transmission line feeding antenna A so that its signal is a fraction of a wavelength ahead of the signal from antenna B (that is, the sources are slightly out of phase). The new lines of maximum reinforcement are marked on the diagram.

7–24†
(See the Appendix in the text.)

7–25
What will be the appearance of the interference pattern of two point sources if the phase delay between them is altered suddenly?

A kink or jog will be produced in the entire nodal pattern. This kink will travel outward at approximately the wave velocity.

CHAPTER 8

8–1*
In a ripple tank the size of a lake, with long waves on it, how would you search for nodal lines with a boat in the dark? (Section 1.)

Try to find places where the boat does not bob up and down. Along nodal lines water does not move; on either side of them, the boat will bob.

8–2
Why can't we see interference from two incandescent filaments?

The question is discussed and answered in Sections 1 and 3 of the text. Briefly: the position of the dark and bright bars depends on the phase delay between sources, as well as on the path difference. If the phase delay varies with time, the interference pattern shifts. A rapidly shifting pattern results in uniform illumination of the screen. There is no reason to believe that the phase delay between two incandescent filaments should stay constant.

8–3
In an interference pattern produced by white light passing through two narrow slits, the distance between the black bars is 0.32 cm. The distance between slits is about 0.02 cm, and the distance to the screen on which the bars are observed is 130 cm. Find the average wavelength of white light.

This problem may be solved using the formula for the spacing between dark bars, $\Delta x = L\frac{\lambda}{d}$. However, mechanically substituting into the formula is unsatisfactory; be sure to bring out the basic reasoning involved in obtaining the formula.

$$\Delta x = L\frac{\lambda}{d}$$

$$\lambda = \Delta x\frac{d}{L}$$

$$\lambda = 0.32\text{ cm} \times 0.02\text{ cm}/130\text{ cm} = \underline{5 \times 10^{-5}\text{ cm}}.$$

8–4
A source of red light produces interference through two narrow slits spaced a distance $d = 0.01$ cm apart. At what distance from the slits should we place a screen so that the first few interference bars are spaced one centimeter apart? What will be the spacing of the bars if we then use violet light?

$$\Delta x_{\text{red}} = L\frac{\lambda_{\text{red}}}{d}$$

$$L = d\frac{\Delta x_{\text{red}}}{\lambda_{\text{red}}} = \frac{0.01\text{ cm} \times 1\text{ cm}}{6.5 \times 10^{-5}\text{ cm}}$$

$$= 1.5 \times 10^{2}\text{ cm} = \underline{1.5\text{ m}}.$$

If we use a different wavelength, the spacing varies directly with the wavelength (since the path difference in wavelengths must remain the same).

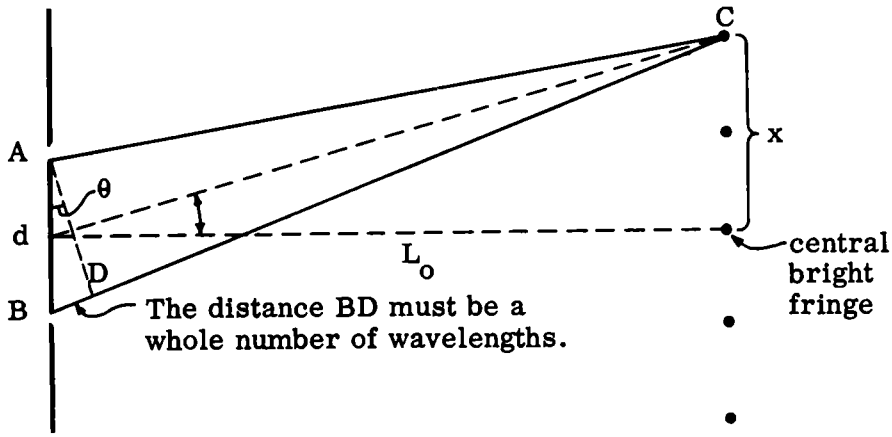
$$\frac{\Delta x_{\text{violet}}}{\Delta x_{\text{red}}} = \frac{\lambda_{\text{violet}}}{\lambda_{\text{red}}}$$

$$\Delta x_{\text{violet}} = \frac{\lambda_{\text{violet}}}{\lambda_{\text{red}}} \times \Delta x_{\text{red}}$$

$$= \frac{4.5 \times 10^{-7}\text{ meter}}{6.5 \times 10^{-7}\text{meter}} \times 1\text{ cm} = \underline{0.7\text{ cm}}.$$

8–5
When a source of light of wavelength λ is used in a two-slit experiment with narrow slits separated a distance d, at what angles do you expect to find the *maxima* in the light intensity in the interference pattern?

The central maximum in the interference pattern occurs when the path lengths from the two slits to the screen are equal—assuming the slits are at



equal distances from the source. Maxima also occur when the paths from the two slits to the screen differ by whole numbers of wavelengths.

Consider the figure. The light going from slit B to the maximum at C must have gone a whole number of wavelengths farther than the light from the slit A to the maximum at C. But BC is longer than AC by the distance BD. Hence, $BD = n\lambda$, where $n = 0, 1, 2, 3, \ldots$.

It can be shown that if C is far away (AC is large compared to AB), $BD = AB \sin \theta$. But $AB = d$, the distance between the slits. Finally then, $d \sin \theta = n\lambda$, and $\underline{\sin \theta = n\lambda/d}$ where $n = 0, 1, 2, 3, \ldots$.

8–6†
(See the Appendix in the text.)

8–7*
How far would light travel in air in 3×10^{-9} sec? (Section 3.)

0.9 meter.

8–8
Calculate the period of yellow light. About how many wavelengths are included in a light wave during emission of a light burst by a single atom?

The period, T, of a wave is the inverse of the frequency, $T = \frac{1}{f}$. Letting c = speed of a light wave, we have

$$c = \lambda f = \frac{\lambda}{T} \text{ and } T = \frac{\lambda}{c}$$

$$\lambda \text{ yellow} \approx 5.8 \times 10^{-7} \text{ m}$$

$$T \approx \frac{5.8 \times 10^{-7} \text{ m}}{3 \times 10^{8} \text{ m/sec}} = \underline{1.9 \times 10^{-15} \text{ sec.}}$$

The burst of light from a single atom lasts for a time t_B of the order of 10^{-9} sec. In this time the number of periods is of the order of:

$$\frac{t_B}{T} \approx \frac{10^{-9} \text{ second}}{10^{-15} \text{ second/period}} = 10^6 \text{ periods.}$$

A single atom, therefore, emits a wave train of the order of $\underline{10^6 \text{ wavelengths}}$ long.

8–9*
A double-slit interference pattern was produced with the yellow light of sodium, and a spacing of 1.0 mm between nodes was obtained. What would be the spacing if violet light of potassium were used in the same system? (Section 4.)

$\Delta x = 0.68$ mm.

8–10*
What is the ratio of the spacing of the nodes in Fig. 8–H (facing page 147) between red and blue light? (Section 5.)

The nodes for red light are 1.4 times as far apart as those for blue light.

8–11
Suggest an optical method for measuring the width of a narrow slit.

One straightforward method: set up a source (slit or narrow filament) in front of the slit to be measured and measure the distance x between the central maximum and the first dark space in the pattern appearing on a screen. We now know x and can measure L. To find the slit width w: $x/L \approx \sin \theta = \lambda/w$, and $\underline{w = \lambda L/x}$.

8–12*
Why is the partial reinforcement referred to in Fig. 8–10 for a single-slit interference pattern so much

less than the central maximum for the same slit? (Section 6.)

All sources are in phase in producing central maximum. In Figure 8–10, two thirds of sources cancel each other and remaining one third are separated by phase delays of less than $\frac{1}{2}$.

8–13

(a) When yellow light passes through a slit 1 mm wide, at what angles are the first three nodes in the diffraction pattern?
(b) If the slit is 10 times as wide?
(c) If it is $\frac{1}{10}$ as wide?

The nodes in the diffraction pattern occur at angles such that the path difference $w \sin \theta$ from the two edges of the slit to the screen is equal to an integral number of wavelengths:

$$w \sin \theta_n = n\lambda, \text{ and } \sin \theta_n = n\lambda/w.$$

For $\lambda = 5.8 \times 10^{-7}$ meter, and $w = 10^{-3}$ meter, 10^{-2} meter, 10^{-4} meter, respectively, we get, for the first node with a 1-mm slit,

$$\sin \theta = \frac{\lambda}{w} = \frac{5.8 \times 10^{-5}\,\text{cm}}{10^{-1}\,\text{cm}} = 5.8 \times 10^{-4}.$$

Since our trig tables do not go this small a value, to get the angle it is necessary to realize that for small angles the angle in radians is equal to the sine of the angle. Angle in radians $= 5.8 \times 10^{-4}$. Since angles in radians and degrees are proportional,

$$\frac{\theta}{5.8 \times 10^{-4}} = \frac{180 \text{ degrees}}{\pi \text{ radians}}, \quad \text{or} \quad \underline{\theta = 0.033°.}$$

n/w	1 mm	10 mm	0.1 mm
1	$\theta = 0.033°$	0.0033°	0.33°
2	$\theta = 0.066°$	0.0066°	0.66°
3	$\theta = 0.100°$	0.0100°	1.00°

8–14

In Fig. 8–7, how will the diffraction pattern be affected
(a) if you cover the right-hand half of the sources?
(b) if you cover the extreme left quarter and the extreme right quarter?

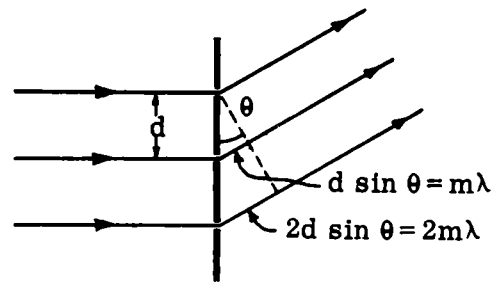
(a) If the right-hand half of the sources is covered, the overall intensity of the interference pattern will be cut in half, the center line of the pattern will shift $\frac{1}{4}w$ to the left, and the pattern will be approximately twice as wide as it was.

(b) The center line will remain at the original position but the intensity and width of the pattern will change as in (a).

8–15

When a source of wavelength λ is used with three slits, each separated from its neighbor by the distance d, show that you get maximum intensity at the same angles as for two slits only.

The condition required to obtain complete reinforcement between any number of waves coming from a set of sources all in phase is clearly that the path difference from any two sources be an integral number of wavelengths. If the sources are equally spaced, this condition is satisfied when the path difference from adjoining sources is an integral number of wavelengths.

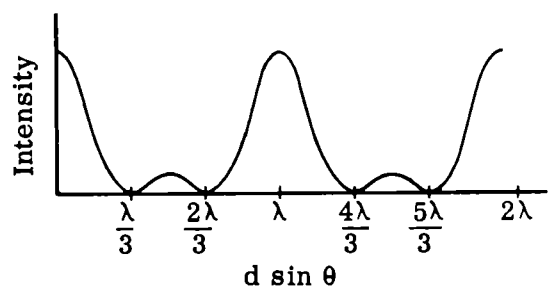


It seems likely that there are secondary maxima where partial reinforcement occurs. In fact, the intensity pattern from three slits looks as shown below.

The existence of minima when

$$Nd \sin \theta = (\text{integer}) \times \lambda \ (N = \text{number of slits}),$$

but when $d \sin \theta/\lambda$ is not an integer, is suggested by the discussion in the text (Figures 8–8 and 8–9). It is true in general, but can be proved by the trick of pairing off slits only when N is a power of 2. With more than 2 slits the principal maxima occur at the same angles as with two slits but they are narrower.



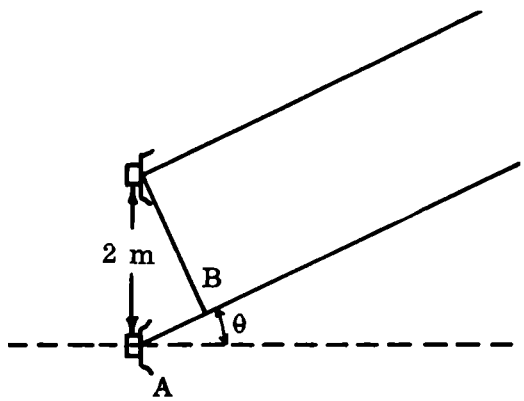
8–16

Two 3-inch loudspeakers emit a steady pitch whose frequency is 1,000 vibrations per second. These sources are in phase and are 2 meters apart.

(a) At what angles would you expect to hear no sound? (The speed of sound is about 300 m/sec.)

(b) What do you think would happen if you tried this experiment in a room with hard-surfaced walls?

This problem requires an extension of the students' knowledge of interference of light into the realm of sound.



(a) No (or very little) sound will be heard at an angle θ such that the distance AB is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc. because the sound from the speakers will cancel.

For this problem $\lambda = \frac{v}{f} = \frac{300\text{ m/sec}}{1000/\text{sec}} =$ 0.3 meter, $AB = 2 \sin \theta$ meters. No sound will be heard at

$$2 \sin \theta = \frac{0.3}{2}, \frac{0.9}{2}, \frac{1.5}{2}, \frac{2.1}{2}, \text{ etc.}$$

$$\sin \theta = 0.075, 0.225, 0.375, 0.525, 0.675, 0.825, 0.975.$$

$$\theta = 4.3°, 13.0°, 22.0°, 31.7°, 42.5°, 55.6°, 77.0°.$$

(b) Because of multiple reflections the sound would travel over many different paths of different lengths in reaching the ear or a microphone. Since it would arrive in many different phases, a nodal pattern could not be observed.

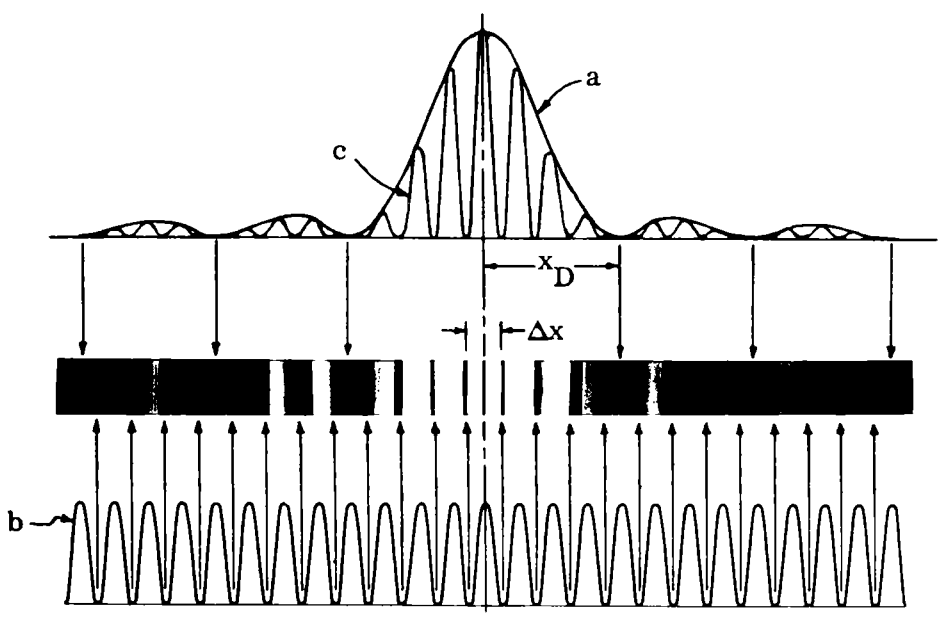
8–17

In Fig. 8–12, identify the dark regions arising from the diffraction pattern of each slit and the dark bars arising from interference between the slits.

Curve (a) represents the intensity distribution to be expected from diffraction by a slit.

Curve (b) represents the intensity distribution to be expected from the interference of the light from two point (or line) sources in phase if they emit light uniformly in all directions.

Curve (c) obtained by multiplying (a) and (b) gives the intensity distribution to be expected when we have two line sources which emit light according to the diffraction pattern of a slit rather than uniformly in all directions.



Arrows show dark regions due to diffraction by each slit.

Arrows show dark bars due to interference between slits.

8–18*

How does the magnification in the top picture in Fig. 8–14 compare with that in the bottom picture? (Section 8.)

Magnification is the same. Check distances between centers of images.

8–19

Microscopes in which the object is illuminated by ultraviolet light can give higher magnifications than microscopes that use visible light.

(a) How do you explain this?

(b) How are the images seen if no visible light is used?

(c) Since glass is opaque to ultraviolet light, how can such a microscope be made?

(a) For a microscope to give high magnification, the lens diameter (aperture) must be very small so as to permit a small radius of curvature for the lens surface. But, magnification will not be helpful if, because of diffraction effects from the small aperture, two adjacent points cannot be distinguished.

The central maximum of the diffraction pattern produced by S_2 is in the direction from S_2 through the center of the slit. If the first node in the diffraction pattern of S_1 occurs in the same direction, we must have $w \sin \theta = \lambda$. (See drawing.)

Therefore, $\sin \theta = \frac{x}{L} = \frac{\lambda}{w}$.

However, to distinguish between two points, the angular separation of the points must be greater than θ, where $\sin \theta$ is about equal to λ/w, and w is the diameter of the lens. Because w is more or less fixed by the magnification sought, it follows that, to get as small a θ (angular separation of two points that can be distinguished) as possible, we must make λ as small as possible. Hence the use of ultraviolet light.

(b) The images are displayed on a fluorescent screen or on photographic film.

(c) If lenses are used, they must be transparent to the wavelength used. Quartz is transparent down to about 2100 A, fluorite down to about 1200 A; however, air absorbs strongly below about 1800 A. In principle, one could build a microscope using mirrors instead of lenses.

8–20

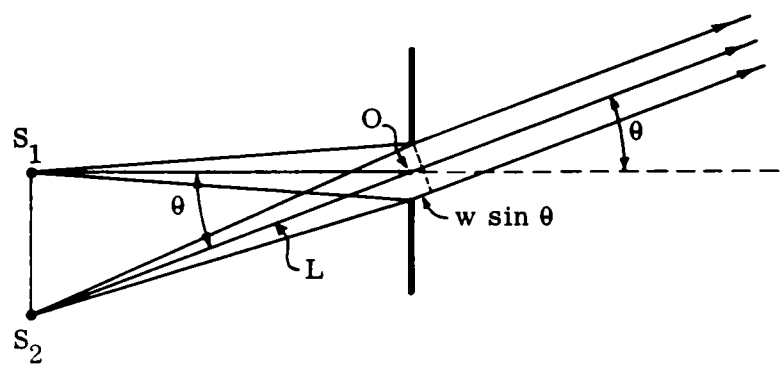
Two images can just be resolved when the central maximum of one falls on the first node of the other.

(a) Show that the resolution of a narrow slit depends upon λ/w where w is the slit width.

(b) About how close together can two line sources be placed and still be resolved if they are viewed through a 0.01-cm slit (about the smallest size you can make easily) 3 meters away from the sources?

(a) Since the phase delay between the two sources varies rapidly, there is no interference between the light waves from the two sources. Each produces its own separate diffraction pattern, and the two patterns are simply superposed.

(b) For $w = 10^{-2}$ cm, and $L = 3 \times 10^2$ cm, $x = L\frac{\lambda}{w} = 3 \times 10^4\lambda$. If we take λ as, say, 5.7×10^{-5} cm, $w \approx 1.7$ cm.



8–21
Stars are often photographed through a blue filter. What is the advantage of this?

The angular resolution of a telescope depends on the ratio λ/D of the wavelength of light to the diameter of the telescope's objective (lens or mirror). If light of a shorter wavelength is used, the angular resolution is improved, so that stars which subtend a smaller angle at the earth can be resolved.

8–22*
What would happen to the distance between the lines in Fig. 8–G (facing page 147) if red light were used instead of green? (Section 9.)

It would increase.

8–23*
What is the wavelength in water of green light whose wavelength in vacuum is 5600 A? (The speed of light in water is $\frac{3}{4}c$.) (Section 9.)

$\lambda = 4200$ A.

8–24*
Several different materials are used to make $\lambda/4$ thick films for red light at 6100 A. Will these films all have the same thickness? (Section 9.)

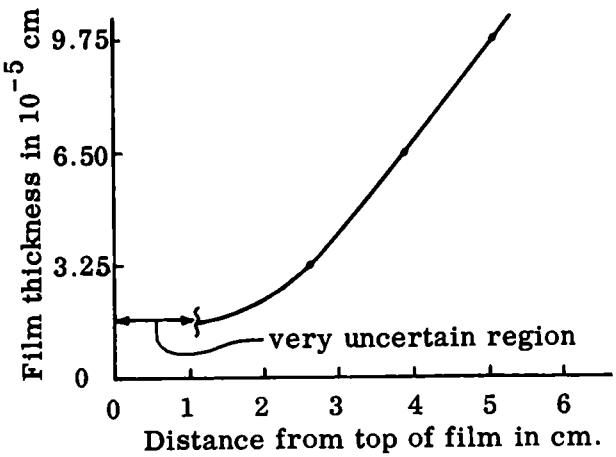
No; the wavelength within each material depends on the index of refraction of the material.

8–25*
In Fig. 8–G (facing page 147), if at the left-hand side you put 2 sheets between the glass plates instead of one, how will it affect the pattern? (Section 9.)

The bars would be thinner and only half as far apart.

8–26
From the photograph of a soap film at the top in Fig. 8–F, plot a graph of film thickness against vertical distance down the film.

The photograph in Figure 8–F shows alternate bright and dark fringes. The upper dark fringe shows that the thickness in that region is less than $\frac{1}{4}$ wavelength. The next dark fringe is in the region where the film is $\frac{1}{2}$ wavelength thick. If we take 6.5×10^{-5} cm as the wavelength of red light, the graph of thickness against vertical distance down the film is as shown below, left. Distances from the top were measured along a vertical line through the center of the film.



8–27
Two pieces of plate glass 10 cm long make an air wedge like that shown in Fig. 8–G. The plates are separated at one end by a human hair with a diameter of 0.09 mm. The reflected interference pattern is observed by looking in a direction perpendicular to the surfaces of the plates.
(a) What is the spacing of the bars if the incident light is blue?
(b) How many bright bars are seen per centimeter if the incident light is red?
(c) Is the light reflected from the end where the plates are in contact a maximum or a minimum?
(d) Can you tell from this experiment which reflected waves are turned upside down?

(c) One way to handle this problem is to begin with part (c), noting that the light reflected from the region near the end where the plates are in contact must be a minimum. Where there is *no* air gap, there is *no* reflection. This implies that the two reflected waves cancel. Therefore one of them must have been turned upside down. For a very small air gap the waves reflected from the two sides of the gap would still nearly cancel and there would be very little reflected light.

(a) From (c) we see that there will be a black bar near the end where the plates touch. Now start counting bars. The next black bar will be where the air wedge is $\lambda/2$ thick (a total path difference of λ); the next black bar at λ. When we come to the end, we will have counted n bars and the thickness of the wedge of air is $n/2$ wavelengths. However, it is also 0.09 mm thick. If we take the wavelength of blue light as 4750 A, the thick-

ness in wavelengths is 0.09 mm ÷ 4750 × 10^{-7} = 190λ. We will have counted 380 bars in 10 cm; so the spacing between bars is $\frac{10}{380}$ = 0.026 cm.

(b) For red light (λ = 6500 A) the hair is fewer wavelengths thick. There are fewer bars, and the spacing is greater:

$$\frac{6500\text{ A}}{4750\text{ A}} \times 0.026 = 0.036\text{ cm}.$$

The number of bars seen per cm is 1/spacing = 28 bars/cm.

(d) We cannot tell from this experiment which waves are turned upside down.

8–28

Suppose that in Figure 8–16 the thickness of the film is $\frac{1}{2}\lambda$.

(a) What is the path difference between rays *a* and *b*?
(b) How many times are the waves inverted?
(c) What is the over-all phase delay between rays *a* and *b*?

(a) The twice-reflected ray *b* is delayed a whole wavelength with respect to the unreflected wave *a* because of its extra journey back and forth across the film.
(b) There is no inversion of the waves upon reflection in this case since both reflections occur at a glass-to-air interface. The inversion occurs at the air-to-glass interface.
(c) A crest in the twice-reflected wave will come out of the bottom of the thin film exactly in phase with a crest in the unreflected wave.

8–29

Lenses are often coated with a thin film to reduce the intensity of reflected light.

(a) If the index of refraction of the coating is 1.3, what is the smallest thickness that will give a minimum reflection of yellow light?
(b) Such lenses often show a faint purple color by reflected light. Why?

(a) Since the reflections from air to the film of $n = 1.3$, and from the film ($n = 1.3$) to glass ($n = 1.5$) are of the same type and both are inverted, a film thickness of λ/4 will give a total path difference of λ/2 and the two reflected waves will interfere destructively. Yellow light has a wavelength of 5800 A in air. The wavelengths in two different media are related by $n = \lambda_{air}/\lambda_{film}$:

$$\lambda_{film} = \frac{\lambda_{air}}{n} = \frac{5800\text{ A}}{1.3} = 4460\text{ A}.$$

The film should be λ/4 = 1100 A thick.

(b) If the film is of the proper thickness to cancel yellow light, red light and violet light will be somewhat reflected. The reflected light will therefore have a reddish-violet or purple tinge.

CHAPTER 9

9–1*

In the table below, which displacements are equal? (Section 1.)

Table

	x_1(m)	x_2(m)
(1)	5	8
(2)	7	−2
(3)	−5	−2
(4)	15	12
(5)	0	2
(6)	−5	−8
(7)	−5	0

In each case, find the displacement $\Delta x = x_2 - x_1$:

(1) 3 (3) 3 (5) 2 (7) 5
(2) −9 (4) −3 (6) −3

Evidently, displacements 1 and 3 are the same, as are displacements 4 and 6.

9–2*

Does the graph of a car trip in Fig. 9–30 represent a real situation? Explain. (Section 1.)

No. The graph indicates that between $t = 2$ and $t = 3$ hours, the car is in two different positions at the same time.

9–3

Express the following velocities in mi/hour. Give examples of objects that move with such velocities.

(a) 1 m/sec
(b) 10 m/sec
(c) 30 m/sec
(d) 250 m/sec
(e) 8000 m/sec

Students unfamiliar with English and metric equivalents will have to look one up; a dictionary

will usually say that 1 meter is the equivalent of 39.37 inches.

	Velocity (m/sec)	Velocity (mi/hr)	Object
(a)	1	2.24	A person taking a leisurely walk.
(b)	10	22.4	An auto in a restricted speed zone.
(c)	30	67.2	An auto on a turnpike.
(d)	250	560	A jet airplane.
(e)	8000	17,900	A space vehicle.

9–4

Identify the parts of the graph in Fig. 9–4 where the motion is steady, and determine the velocity of the object in those regions.

On a position-time graph, motion is steady when the graph is a straight line, since in such conditions the position changes steadily with the time.

The velocity can be found under these conditions by dividing the displacement during the interval by the time interval in which the displacement occurs. The following table summarizes the information called for.

Time Interval	Size of Interval (Δt)	Displacement (Δx)	Velocity
0-3 sec	3 sec	0 cm	0 cm/sec
3-10	7	3.5	0.5
12-13	1	0	0
13-19	6	−6	−1.0
19-22	3	0	0
22-23	1	1	1.0

9–5

Find the slopes of the graphs in Fig. 9–31. State the units in each case.

This problem gives practice in finding slopes of straight lines and, in particular, emphasizes units.

(a) 60 mi/hr.
(b) $\frac{2}{3}$.
(c) 15 miles per gallon.
(d) −4 pounds per week.

9–6

Identify all points on the graph of Fig. 9–4 at which the instantaneous velocity is zero.

The instantaneous velocity is zero wherever the tangent to the graph is horizontal; that is, whenever the slope of the position-time graph is zero.

In the intervals from 0 sec to 3 sec and from 19 sec to 22 sec, the instantaneous velocity and the average velocity are both zero. In addition, at $t = 10$ the instantaneous velocity is zero.

9–7

The position-time graph of a car is shown in Fig. 9–32.

(a) At what time is the car going at the greatest velocity?
(b) How fast is it traveling at that time?
(c) How fast is the car going at 0.70 hour?
(d) What is the average velocity for the first 0.70 hour?

This problem requires the interpretation of a position vs. time graph where the velocity is constantly changing. Some will find this difficult.

(a) The point (a) is found by inspection as that point having the steepest slope. The time at which the car is going at the greatest velocity is about 0.45 hour.
(b) The sloping line is drawn tangent to the curve at point (a). The slope is about 60 miles in 0.4 hour, or about 150 mi/hr. Accuracy is poor here.
(c) At 0.70 hour the slope of the graph shows the velocity as about 16 mi/hr.
(d) During the first 0.70 hr the car traveled 35 miles. It therefore had an average velocity of 50 mi/hr. Note that this is *not* half its final speed. That holds *only* for the case of uniformly accelerated motion, which is not true here.

9–8*

What is the average velocity over the first 90 sec in the graph of Fig. 9–9? (Section 3.)

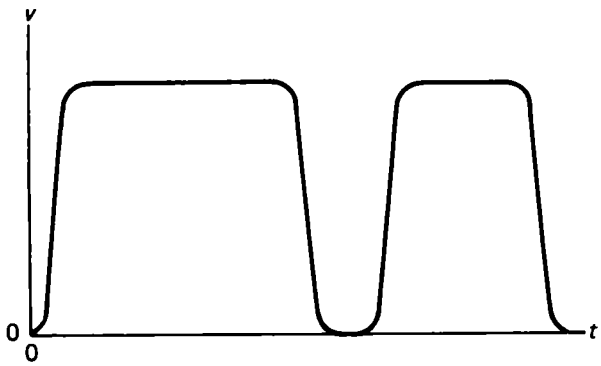
$v_{ave} = 0.48$ m/sec.

9–9

A train travels 60 mi/hr for 0.52 hr, 30 mi/hr for the next 0.24 hr, and then 70 mi/hr for the next 0.71 hr. What is its average velocity?

A train which travels 60 mi/hr for 0.52 hour covers 31.2 miles. In the next time period it covers 7.2 miles, then 49.7 miles. The total distance traveled is 88.1 miles in 1.47 hours. The average velocity is 88.1 miles ÷ 1.47 hours = 60 mi/hr.

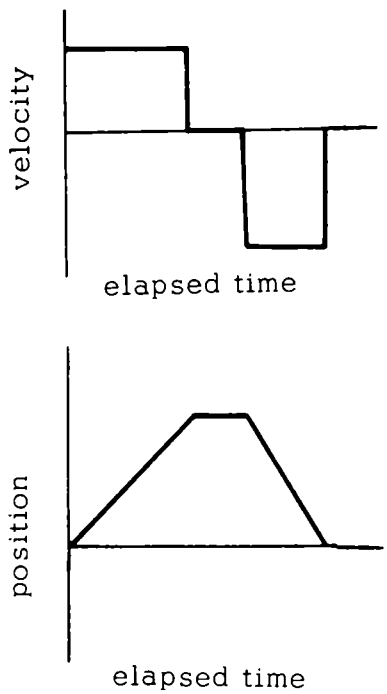
9–10
Sketch a velocity-time graph for a shuttle train which runs between cities A and C with an intermediate stop at city B. All cities are on a straight line.



9–11*
What is the displacement of a car which travels at a steady velocity of 40 mi/hr (a) for 3 hr, (b) for $\frac{1}{2}$ hr? (Section 4.)

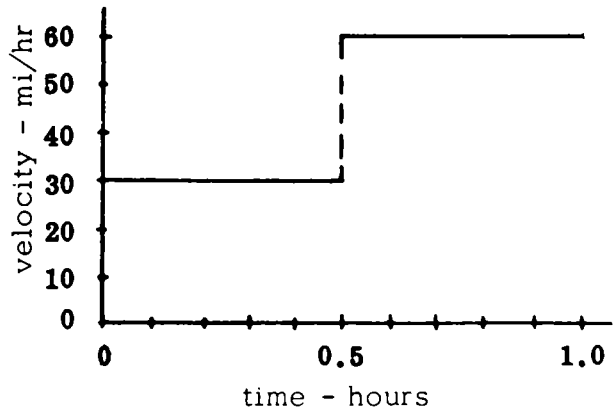
(a) 120 miles.
(b) 20 miles.

9–12
A man walks to the corner to mail a letter and comes back. Sketch graphs showing his velocity and position plotted against time.



Students' graphs may vary from those given here. It is most important that their position-time graph corresponds to their velocity-time graph.

9–13
The position-time graph of a car traveling along a road is shown in Fig. 9–33. Make a graph of its velocity versus time.



For the first half hour the graph has a constant slope indicating a constant speed. Since the car went 15 miles in 0.5 hour, it was going 30 miles per hour. For the second half hour it went from 15 to 45 miles, or a distance of 30 miles. Its speed was therefore 60 mi/hr.

9–14
John rode his bicycle as fast as he could from his house to Tom's house. After a short time he rode back as fast as he could. Figure 9–34 shows a position-time graph of his trip. Plot the velocity-time graph of John's trip. From the information given and your graph, what would you give as a plausible description of the road between John's and Tom's houses?

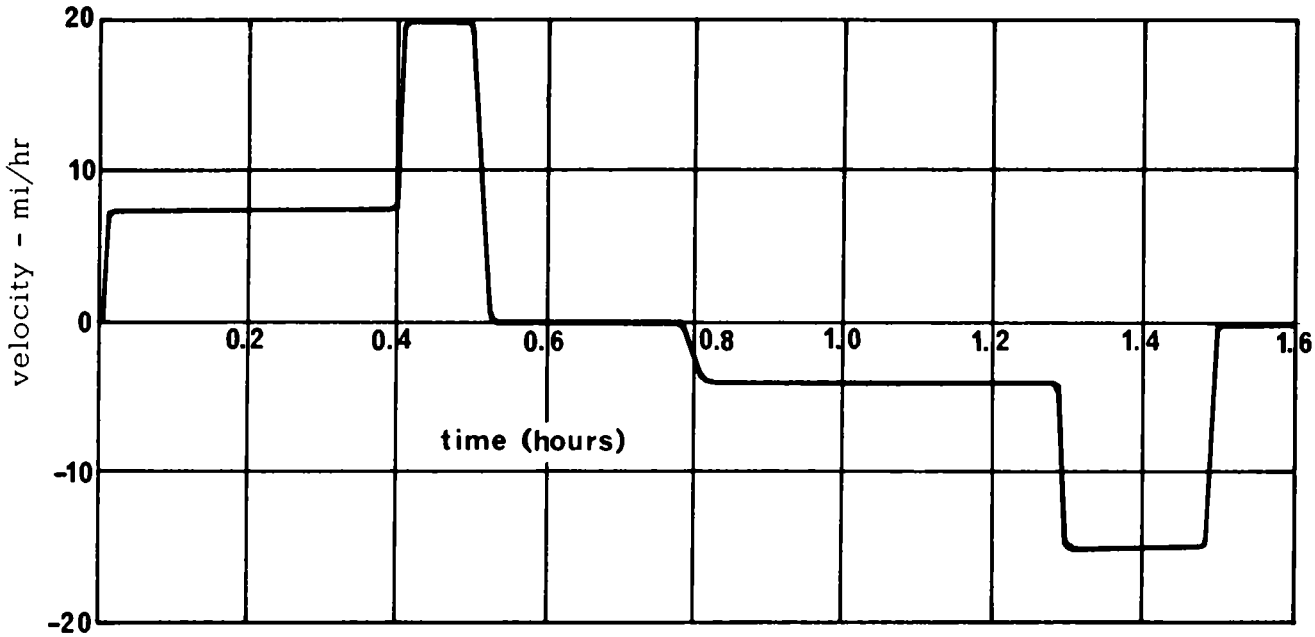
The slopes of the straight portions of the graph are 7.5, 20, 4, and 15 mi/hr.

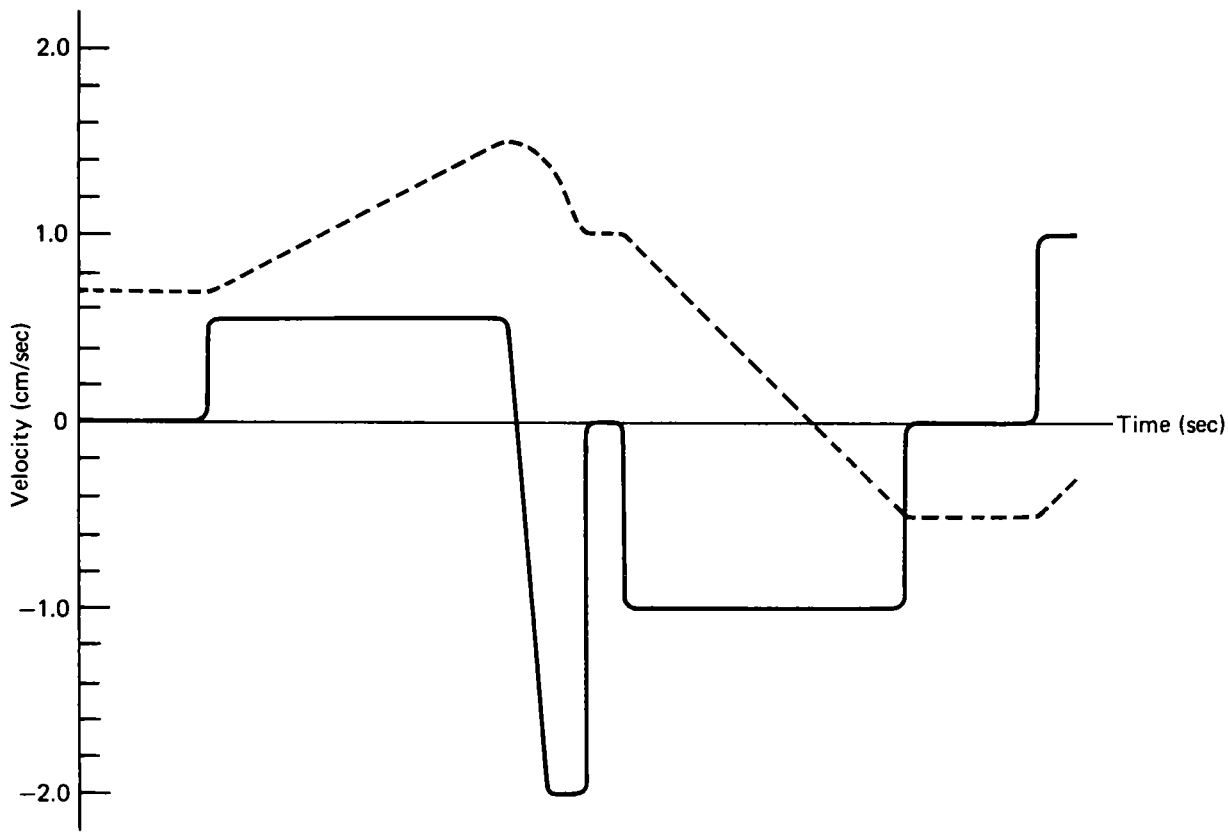
The road from John's house had a slight upgrade for 3 miles, then a steeper downgrade of 2 miles to Tom's house.

9–15
Sketch a velocity-time graph for the object whose displacement-time graph is given in Fig. 9–4.

See graph at lower right.

9–16†
(See the Appendix in the text.)


velocity - mi/hr
20
10
0
-10
-20
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
time (hours)


Velocity (cm/sec)
2.0
1.0
0
-1.0
-2.0
Time (sec)

9–17

The accompanying data show the instantaneous velocity of a car at intervals of 1 second. Plot the velocity versus time, and use your graph to answer the following questions.

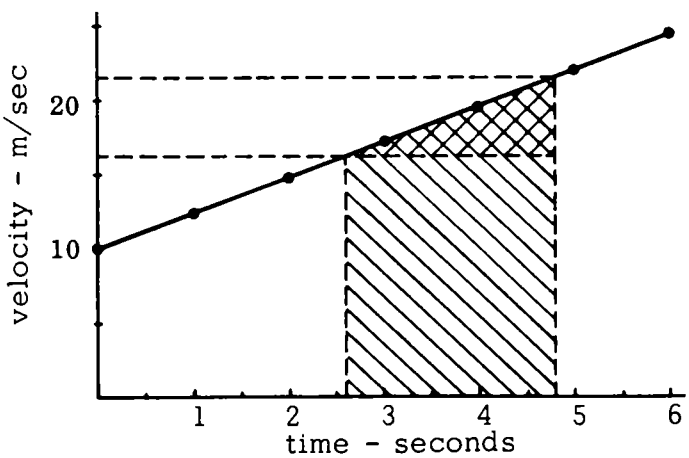
(a) How fast is the car going at 2.6 sec? At 4.8 sec?

(b) How far did the car travel between the two instants in part (a)?

TIME (SEC)	VELOCITY (M/SEC)
0.0	10.0
1.0	12.4
2.0	14.8
3.0	17.2
4.0	19.6
5.0	22.0
6.0	24.4

This problem gives practice in plotting graphs of velocity vs. time and finding a distance traveled by the area method.

(a) From a careful graph, the car's velocity at 2.6 sec can be read as 16.2 m/sec, and 21.5 m/sec at 4.8 sec.



(b) By breaking up the area as shown, the total distance traveled (shaded area) between the two instants is found to be $16.2 \times (4.8 - 2.6) = 35.6$ meters (the rectangle) plus $\frac{1}{2} \times (4.8 - 2.6) \times (21.5 - 16.2) = 5.8$ meters (the triangle), or 41.4 meters in all.

9–18

A train speeds up according to the velocity-time graph shown in Fig. 9–36. How far does it travel in the first six minutes?

This problem forces the student to perform a square-counting calculation in a problem in which he must compute a distance from a velocity vs. time graph.

Six minutes = 0.10 hour. The area under the graph from 0 to 0.10 hour is 4.53 miles.

Perhaps the most straightforward way to count these squares is to count the number of large ones (38) and then the left-over small ones. The area of each large square is 0.1 mile, which gives 3.8 miles. There are then 161 small squares completely under the curve and an estimated 21 more in partial small squares. Since each small square is 0.004 mi this is 0.73 mi. Then $3.8 + 0.73 = 4.53$ miles in all. This counting procedure seems to be good to better than 10 little squares or 0.04 mile, about 1%.

Some students may have learned Simpson's rule for finding areas. It is fine if they know it, but don't teach it.

9–19*

A car going at 20 mi/hr accelerates to 60 mi/hr in 6.0 seconds. What is the average acceleration? (Section 6.)

$$a = \frac{\Delta v}{\Delta t} = \frac{40 \text{ mi/hr}}{6.0 \text{ sec}}$$

$a =$ 6.7 miles per hour per second.

9–20*

Express an acceleration of 1.0 $\frac{\text{mi}}{\text{hour-sec}}$ in units of $\frac{\text{m}}{\text{sec}^2}$. (Section 6.)

From the solution to problem 3, 1 m/sec = 2.24 mi/hr; hence,

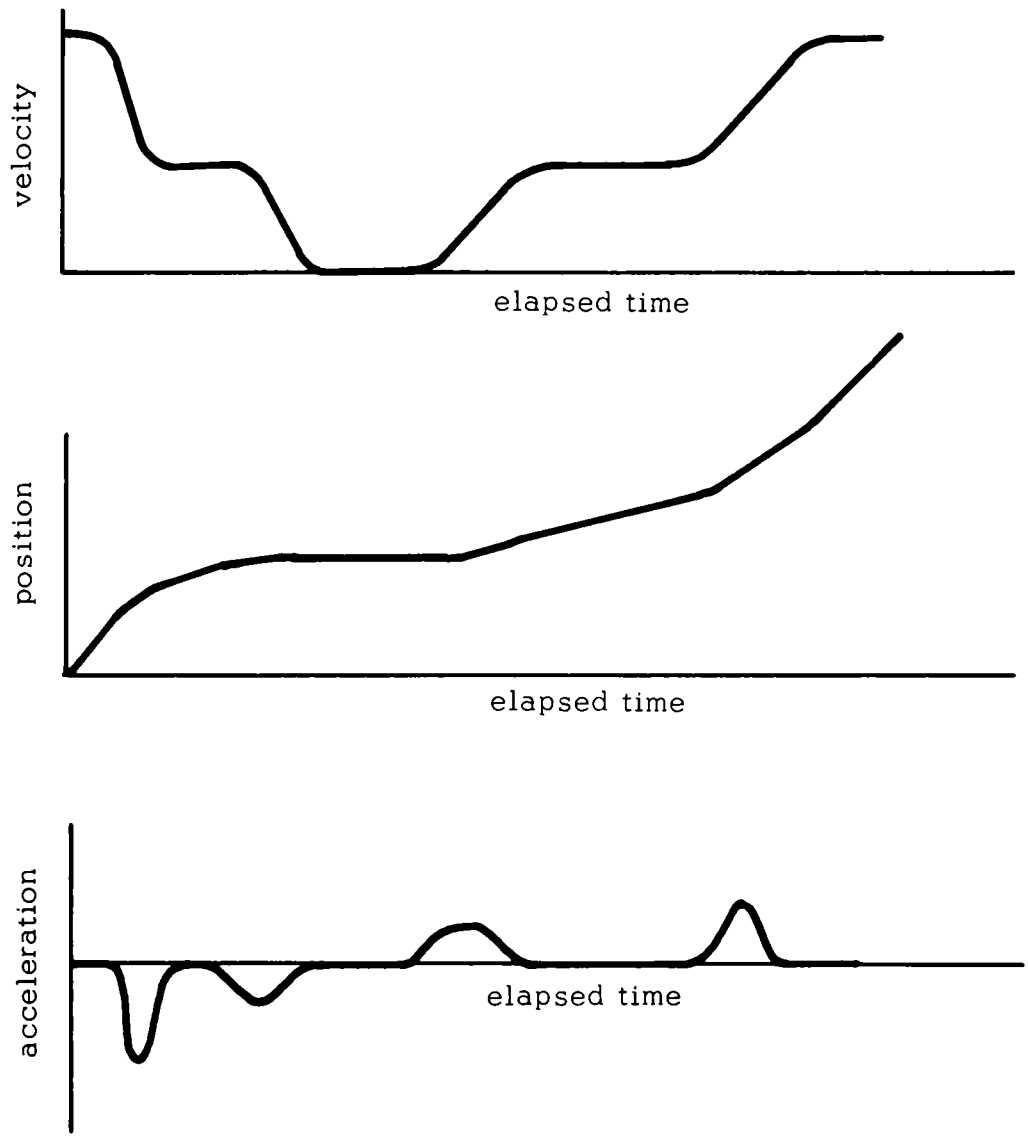
$$1 \text{ mi/hr} = \frac{1}{2.24} \text{ m/sec} = 0.446 \text{ m/sec}.$$

Then

$$1 \frac{\text{mi/hr}}{\text{sec}} = 0.446 \frac{\text{m/sec}}{\text{sec}} = 0.446 \text{ m/sec}^2.$$

9–21

A car driving along a highway slows down as it enters a small village. It is stopped by a traffic light in the center, goes on to the edge of the village, and speeds up again. Sketch graphs, one above the other, showing its velocity, its position, and its acceleration plotted against time.



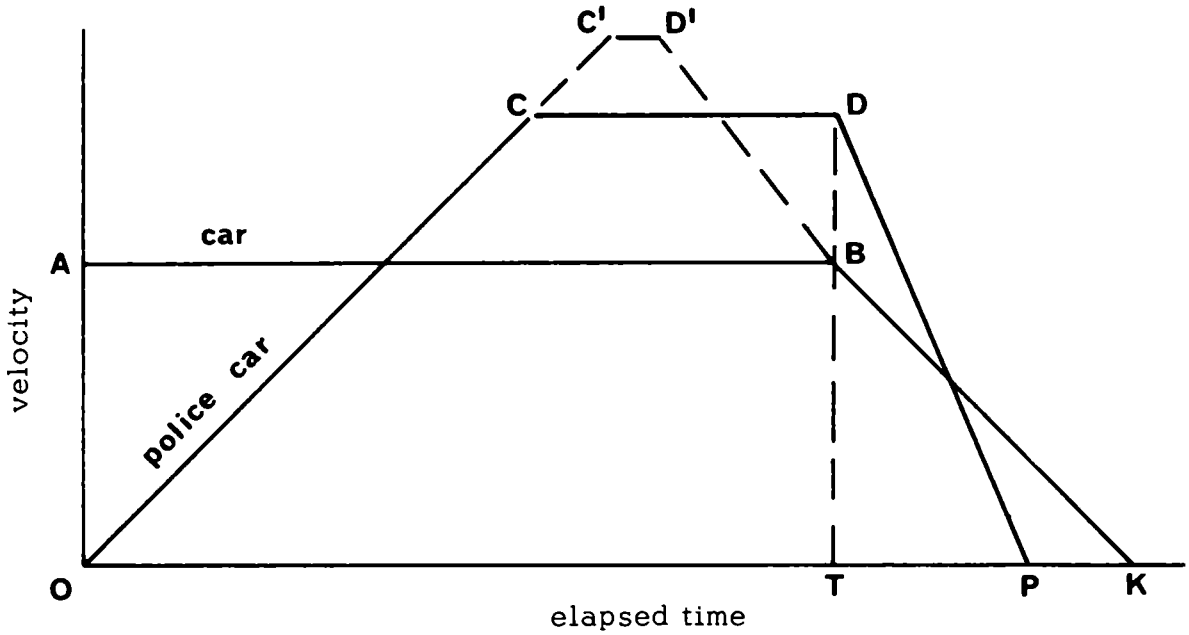
(It is helpful if the three graphs are drawn directly under each other and vertical lines are drawn through all three graphs to show which time intervals correspond.)

9–22

A car is traveling along a road at a steady velocity. It passes an unmarked police car parked beside the road. The police car accelerates, overtakes the speeding car, passes it, and signals it to a stop. Sketch a graph showing the velocities of the two cars plotted against time.

The police car must travel the same total distance as the other car in order to pass it and must therefore accelerate to a higher speed. The area $OCDT$, which represents the distance traveled by the police car when it has reached the point at which it passes the speeder, equals the area $OABT$, which represents the distance traveled by the speeder. After the police car has signaled, the two cars come to zero speed at the same distance. The areas PDT and KBT must be equal.

There are other solutions, such as: $OABT =$



OC'D'BT, after which both cars stop at the same time *K*.

This problem should be used after Problem 16÷.

9–23

Figure 9–37 is a multiple-flash photograph of a moving ball taken at $\frac{1}{30}$ sec intervals. The ball is moving from left to right, and the zero point on the scale lines up with the right-hand edge of the ball's initial position.

(a) Plot a graph of position against time to describe this motion.

(b) From your graph in (a), construct a velocity-time graph.

(c) What does your velocity-time graph tell you about the acceleration of the moving ball?

(a) See position vs. time graph, upper right.

(b) The velocity vs. time graph can be constructed in at least two ways. The first way is to measure the slope of the position vs. time graph at various times. A sample measurement at $t = \frac{5}{30}$ sec is shown. This gives a slope of

$$1.05 \text{ m} \div \tfrac{6}{30} \text{ sec} = 5.25 \text{ m sec}.$$

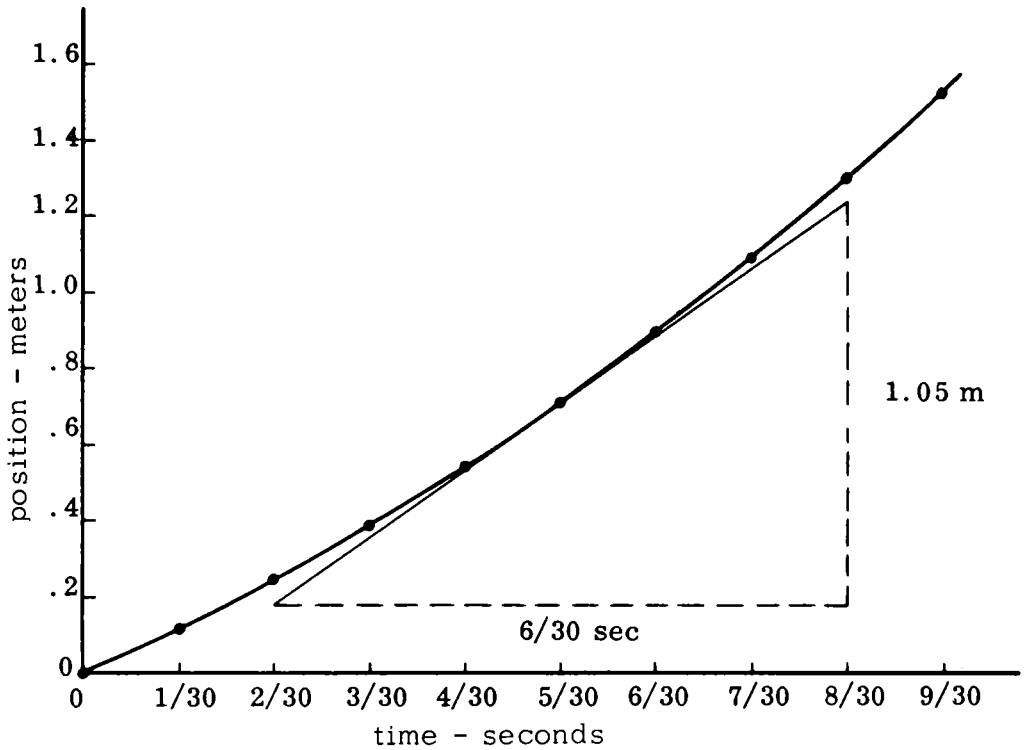
This value is plotted on the graph. Note that the slope of the tangent line is very nearly the slope of the line drawn through the $\frac{4}{30}$ sec and $\frac{6}{30}$ sec points. This affords a second method for finding the speed at $\frac{5}{30}$ sec:

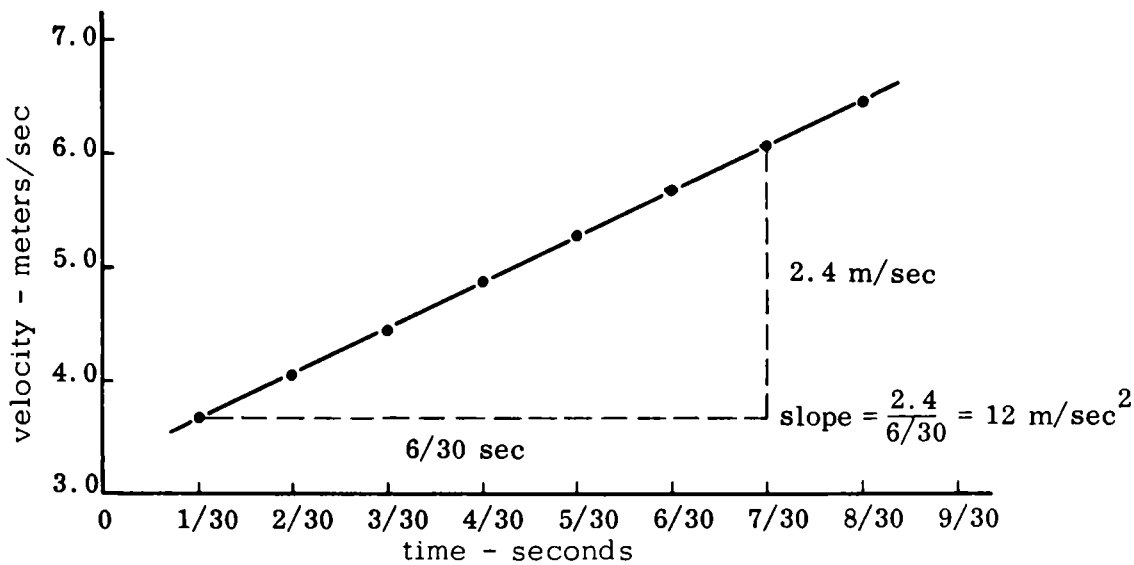
$$\text{velocity at } \tfrac{5}{30} \text{ sec} = \frac{\text{change of position from } \frac{4}{30} \text{ to } \frac{6}{30} \text{ sec}}{\frac{2}{30} \text{ sec}}$$

$$= \frac{0.89 \text{ m} - 0.54 \text{ m}}{\frac{2}{30} \text{ sec}} = 5.25 \text{ m/sec}.$$

The entire velocity vs. time graph, found by this second method, is shown below.

(c) The points of the velocity-time graph scatter slightly as shown. However, the scatter is probably due to measuring difficulties and appears to be random. We therefore conclude the velocity increases uniformly with time. The acceleration is constant and is $\underline{12 \text{ m/sec}^2}$ from the measured slope.


1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
position - meters
1.05 m
6/30 sec
0
1/30
2/30
3/30
4/30
5/30
6/30
7/30
8/30
9/30
time - seconds


7.0
6.0
5.0
4.0
3.0
velocity - meters/sec
2.4 m/sec
slope = $\frac{2.4}{6/30}$ = 12 m/sec^2
6/30 sec
0
1/30
2/30
3/30
4/30
5/30
6/30
7/30
8/30
9/30
time - seconds

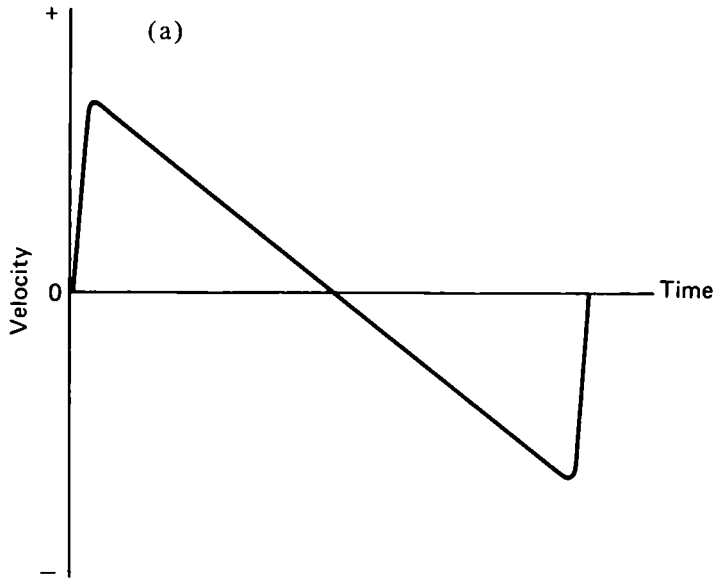
9–24

(a) Sketch a velocity-time graph for a ball thrown straight up from the time it is still at rest in the hand till it is momentarily at rest on the ground.

(b) Should the area between the curve and the time axis above the time axis be equal to the corresponding area below the time axis?

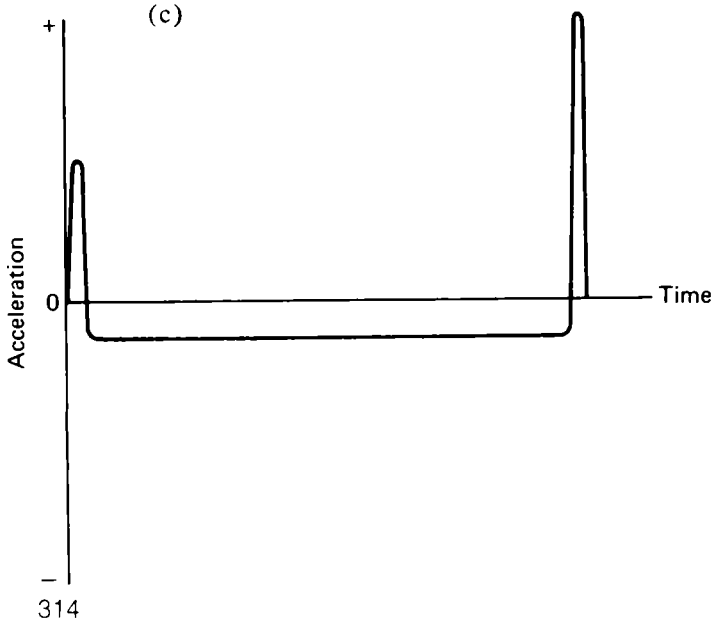
(c) Use the graph in (a) to sketch the acceleration-time graph for the same motion.

(a)



(b) Almost; the distance the ball rises must equal the distance the ball has fallen as it passes the thrower's hand, and it will fall the additional distance representing the height of the thrower's hand above ground.

(c)



9–25

From the graph of velocity versus time for a car shown in Fig. 9–38, deduce the graph of acceleration versus time.

To construct the acceleration vs. time graph, you must take the slope of the velocity vs. time graph at several places. This would be a good place to review the interval-on-the-graph method as compared to the tangent-line method for calculating slopes. Thus the method at (a) might be used, then the method at (b).

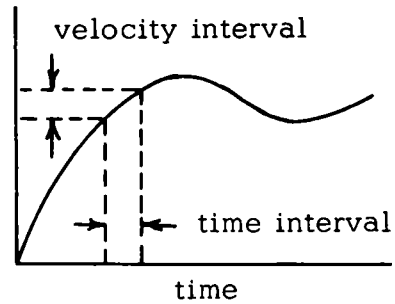


Figure (a)

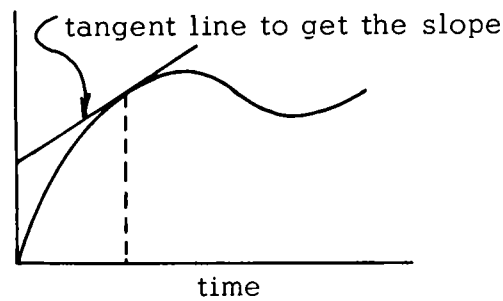


Figure (b)

The resulting acceleration vs. time graph is shown on the next page. Note the negative acceleration as the car is slowing down.

9–26

A bobsled has a constant acceleration of 2.0 m/sec^2 starting from rest.

(a) How fast is it going after 5.0 seconds?

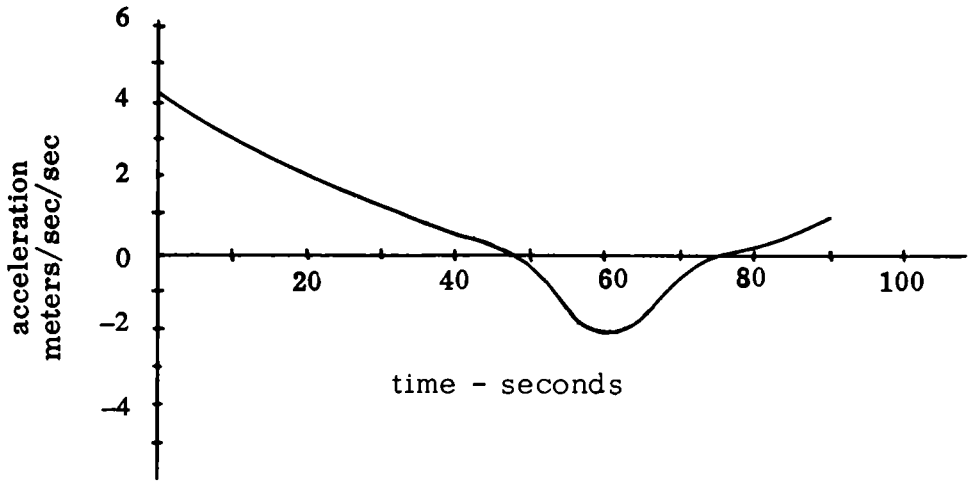
(b) How far has it traveled in 5.0 seconds?

(c) What is its average velocity in the first 5.0 seconds?

(d) How far has it traveled by the time its velocity has reached 40 m/sec?

(a) $v = at$

$$v = 2.0\,\frac{\text{m}}{\text{sec}^2} \times 5.0\,\text{sec} = 10\,\underline{\text{m/sec}}$$



(b) $x = \frac{1}{2}at^2$

$$x = \tfrac{1}{2} \times 2.0 \frac{\text{m}}{\text{sec}^2} \times (5.0 \text{ sec})^2 = \underline{25 \text{ m.}}$$

(c) Its average speed is the distance traveled divided by the time, or $\frac{25}{5} = \underline{5 \text{ meters/sec.}}$ Note that this is just half the velocity at the end of a 5-second interval—a general result for constant acceleration from rest.

(d) $v = at$; $t = \dfrac{v}{a}$

$$t = \frac{40 \text{ m/sec}}{2.0 \text{ m/sec}^2} = 20 \text{ sec.}$$

$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2} \times 2.0 \frac{\text{m}}{\text{sec}^2} \times (20 \text{ sec})^2$$

$$= \underline{400 \text{ m.}}$$

It should be pointed out that these equations may be used only because the problem involves constant acceleration starting from rest. If conditions were otherwise, the general forms would have to be used, as in the following problem.

9–27

A car, initially traveling at uniform velocity, accelerates at the rate of 1.0 m/sec² for a period of 12 seconds. If the car traveled 190 meters during this 12-second period, what was the velocity of the car when it started to accelerate?

$$x' = x + v(t' - t) + \tfrac{1}{2}a(t' - t)^2.$$

Since x and $t = 0$,

$$x' = vt' + \tfrac{1}{2}at'^2.$$

Solving for v,

$$v = \frac{x' - \frac{1}{2}at'^2}{t'}$$

$$v = \frac{190 \text{ m} - \frac{1}{2}\left(1.0 \frac{\text{m}}{\text{sec}^2}\right)(12 \text{ sec})^2}{12 \text{ sec}}$$

$$= \frac{190 \text{ m} - \frac{1}{2}\left(1.0 \frac{\text{m}}{\text{sec}^2}\right)(144 \text{ sec}^2)}{12 \text{ sec}}$$

$$v = 9.8 \underline{\text{ m/sec}}$$

This is a good place to remind students that one check on their problem solution is having the correct units with their answer.

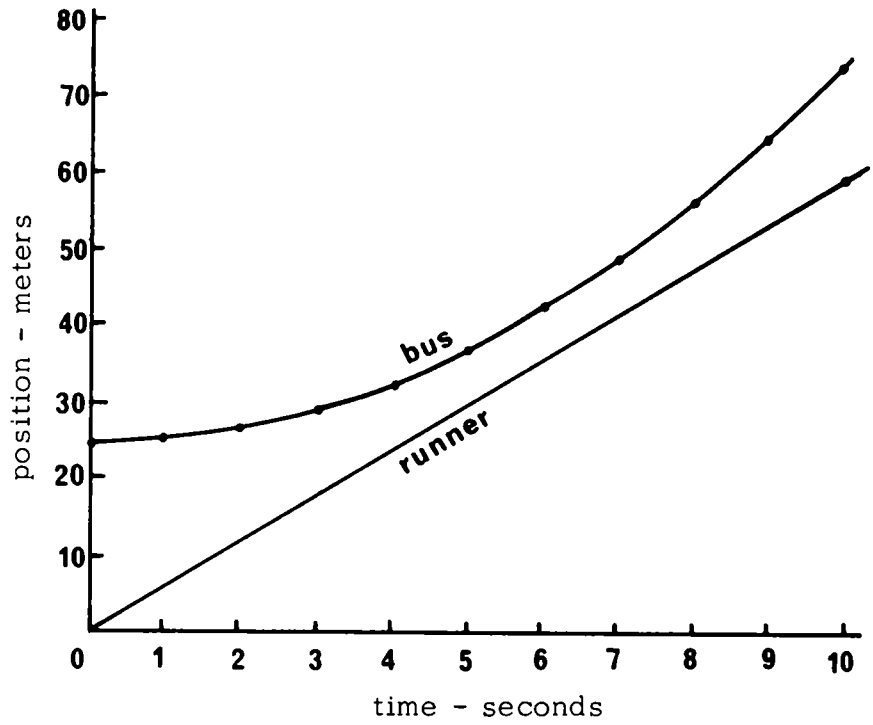
9–28†

(See the Appendix in the text.)

9–29

A pedestrian is running at his maximum speed of 6.0 m/sec to catch a bus stopped by a traffic light. When he is 25 meters from the bus the light changes and the bus accelerates uniformly at 1.0 m/sec². Find either (a) how far he has to run to catch the bus or (b) his frustration distance (closest approach). Do the problem either by use of a graph or by solving the appropriate equations.

From the graph you can see that his closest approach is about 7 meters. (*Note:* A bus actually does not have a constant acceleration.) To solve the problem algebraically, you would use the general form of the equation to find displacement,



$$x' = x + v(t' - t) + \tfrac{1}{2}a(t' - t)^2.$$

For the pedestrian,

$$x' = vt'$$
$$x' = 6.0\,\frac{\text{m}}{\text{sec}}\,t'.$$

For the bus,

$$x' = x + \tfrac{1}{2}a(t')^2$$
$$x' = 25\text{ m} + \frac{1}{2}\left(1.0\,\frac{\text{m}}{\text{sec}^2}\right)(t')^2.$$

If the pedestrian catches the bus, then x' for the bus and x' for the pedestrian are equal.

$$6t' = 25 + \tfrac{1}{2}t'^2$$
$$\tfrac{1}{2}t'^2 - 6t' + 25 = 0.$$

However, by applying the general equation for solving quadratics, $t' = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we see that there is no real solution since the quantity $(b^2 - 4ac)$ is negative. Therefore, the pedestrian never catches the bus.

His frustration distance would be the smallest value of $(x'_{\text{bus}} - x'_{\text{pedestrian}}) = 25 + \dfrac{1}{2}t^2 - 6t$. Rearranging,

$$x'_{\text{bus}} - x'_{\text{pedestrian}} = \tfrac{1}{2}t^2 - 6t + 25$$
$$= \tfrac{1}{2}(t^2 - 12t + 50)$$
$$= \tfrac{1}{2}[(t - 6)^2 + 14].$$

The smallest value then for $(x'_{\text{bus}} - x'_{\text{pedestrian}})$ would be when $t = 6$,

$$x'_{\text{bus}} - x'_{\text{pedestrian}} = \tfrac{1}{2}[(6 - 6)^2 + 14] = \underline{7\text{ m}}$$
$$= \text{frustration distance.}$$

9–30

A rocket which placed a satellite in orbit attained a velocity of 2.90×10^4 km/hr in 2.05 minutes.

(a) What was the average acceleration in km/hr-sec? In m/sec^2?

(b) If the rocket had enough fuel to maintain this same acceleration for an hour, what velocity would it have at the end of the hour if it started from rest?

(c) How far would it travel during this hour?

(a) $a_{\text{ave}} = \dfrac{\Delta v}{\Delta t} = \dfrac{2.90 \times 10^4\text{ km/hr}}{2.05\text{ min} \times 60\text{ sec/min}}$

$a_{\text{ave}} = \underline{236\text{ km/hr/sec.}}$

$$236\,\frac{\text{km}}{\text{hr/sec}} = 236\,\frac{\text{km}}{\text{hr/sec}} \times \frac{1000\text{ m/km}}{3600\text{ sec/hr}}$$
$$= \underline{65.5\text{ m/sec}^2.}$$

(b) $v = at = 236\,\frac{\text{km}}{\text{hr-sec}} \times 3600\,\text{sec}$

$= \underline{8.5 \times 10^5\,\text{km/hr}}$

or, $236\,\frac{\text{km}}{\text{hr-sec}} \times 1\,\text{hr} = \underline{236\,\text{km/sec.}}$

(c) $x = \frac{1}{2}at^2 = \frac{1}{2} \times 65.5\,\frac{\text{m}}{\text{sec}^2} \times (3600\,\text{sec})^2$

$= \underline{4.24 \times 10^8\,\text{m}}$

or $\frac{1}{2} \times 236\,\frac{\text{km}}{\text{hr-sec}} \times (1\,\text{hr})^2 \times 3600\,\frac{\text{sec}}{\text{hr}}$

$= \underline{4.24 \times 10^5\,\text{km.}}$

This is just about the distance to the moon.

This whole problem "neglects" the effect of gravity. If a student asks, "What about gravity?" this would be a good time to emphasize the difference between kinematics and dynamics. Kinematics is concerned only with the *description* of the motion of the rocket, not with *why* the rocket accelerates as it does. It would take fuel to overcome gravity as well as to accelerate the rocket. It would be very difficult to make a rocket which would actually accelerate uniformly regardless of whether gravity was acting on it or not. Here we must just *assume* the rocket to be uniformly accelerated!

CHAPTER 10

10–1*

What are the length and the direction of a displacement from (1, 2) to (3, 4)? (Section 1.)

A horizontal line through the point (1, 2) and a vertical line through the point (3, 4) form a right triangle whose hypotenuse is the given displacement and whose legs are 2 units each. Then the length of the displacement is $\sqrt{2^2 + 2^2} = \underline{2\sqrt{2}}$; evidently the direction of the displacement is $\underline{45°}$ up from the horizontal.

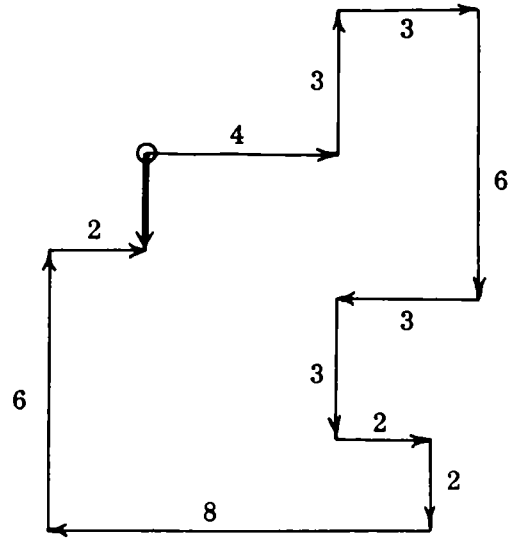
10–2†

(See the Appendix in the text.)

10–3

A man follows this route: From his house he travels four blocks east, three blocks north, three blocks east, six blocks south, three blocks west, three blocks south, two blocks east, two blocks south, eight blocks west, six blocks north, and two blocks east. How far and in what direction will he be from home?

Starting to the right (east) from O, the route is as indicated in the diagram.



The man is now two blocks south of his starting point. *His total displacement is two blocks south.*

Finding the net E–W and N–S displacements is far simpler than adding the successive displacements. The mental addition would go something like this: East = 4 + 3 + 2 + 2 = 11, West = 3 + 8 = 11, so he is neither east nor west of his starting point. N = 3 + 6 = 9, S = 6 + 3 + 2 = 11, so he is 2 blocks south of his starting point. This is a fine solution and should be mentioned when you discuss components, since it is essentially the component method of adding vectors.

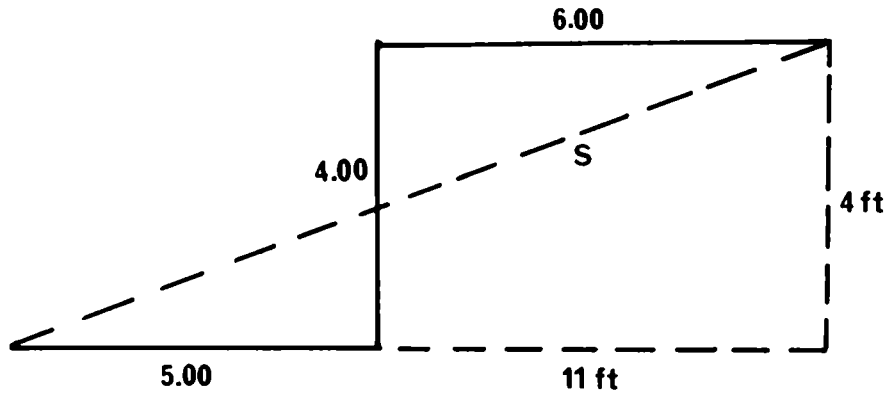
10–4

Suppose you have three sticks 4.00, 5.00, and 6.00 ft long which can be joined by connecting the ends at a 90° angle.

(a) What is the largest displacement possible between the unconnected ends of the sticks?

(b) Displacement vectors can be added in any order and the result is the same. Why do you get different lengths between the ends of the sticks in this problem when you change the order of the sticks?

(a) The longest displacement is attained with the 4.00-ft stick in the center, as shown in the diagram, and all the sticks in the same plane. The displacement equals $S = \sqrt{11^2 + 4^2} = \underline{11.7\,\text{feet.}}$



(b) Vectors have both length and direction, and add as vectors. Here the directions of the sticks are changed, making them different vectors from one configuration to the next. It is not surprising that different vectors should have different sums.

10–5

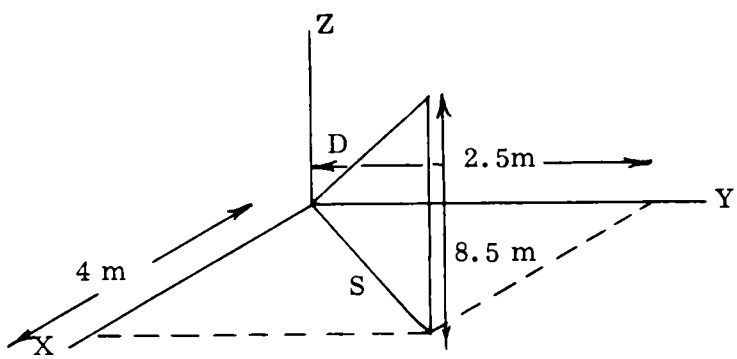
What is the magnitude of a displacement whose components along the perpendicular x, y, and z axes are respectively 4.00 m, 2.50 m, and 8.50 m?

If the students know the three-dimensional Pythagorean theorem the answer is simply:

Magnitude of displacement

$$= \sqrt{4.00^2 + 2.50^2 + 8.50^2}$$
$$= \underline{9.72 \text{ m.}}$$

If this requires proof, class discussion of this problem may include it as shown in the drawing.

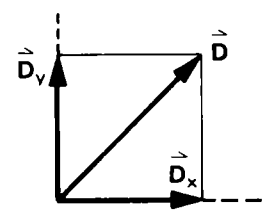


This problem may be approached with the question: If X and Y are horizontal, and a ball is dropped from the displaced object, how far out from the origin would the ball hit?

By the familiar two-dimensional Pythagorean theorem, the answer is clearly $S = \sqrt{4.00^2 + 2.50^2}$. But $D^2 = S^2 + 8.50^2 = 4.00^2 + 2.50^2 + 8.50^2$, and so to the answer.

10–6

In Fig. 10.5, what are the x and y vector components of the short displacement in both (a) and (b)?



10–7

A ship sails 10 miles in a northeast direction. What are the components of its displacement (a) to the north? (b) to the south?

(a) 7.1 miles
(b) −7.1 miles

10–8*

What are the north and east components of a velocity vector of magnitude 100 mi/hr and direction 30° west of south? (Section 3.)

−86.6 mi/hr north and −50.0 mi/hr east.

10–9*

What is the component in the y direction of a vector of length 2 cm in the x direction? (Section 3.)

Zero.

10–10

(a) Find the result of adding a vector 2 cm east to one 3 cm northwest.

(b) Find the result of adding a vector 8 cm east to one 12 cm northwest.

(c) Compare the results of parts (a) and (b), and state a theorem about adding a pair of vectors which are multiples of another pair. Can you prove the theorem in general?

This problem leads to the idea that if $\vec{A} + \vec{B} = \vec{C}$, then $k\vec{A} + k\vec{B} = k\vec{C}$. It may also help the students understand scaling of vector drawings.

(a) 2 cm east + 3 cm northwest = a vector 2.12 cm long, 3.3° west of north.

(b) 8 cm east + 12 cm northwest = a vector 8.48 cm long, 3.3° west of north.

(c) Notice the vectors of part (b)—including the sum—are four times the vectors of part (a). In general then, if $\vec{A} + \vec{B} = \vec{C}$, $k\vec{A} + k\vec{B} = k\vec{C}$. Both sides of a vector equation may be multiplied by the same constant—or scalar. A geometrical proof is started by noting that the vector sum $\vec{A} + \vec{B} = \vec{C}$ is represented by a triangle, and that the vector sum $k\vec{A} + k\vec{B} = \vec{D}$ is also represented by a triangle. Two of the sides of one of the triangles are proportional to two sides of the other triangle. The included angles are equal. It then follows that the triangles are similar and that the length of $\vec{D}$ is k times the length of $\vec{C}$ and makes the same angles with $k\vec{A}$ and $k\vec{B}$ as does $\vec{C}$ with $\vec{A}$ and $\vec{B}$. Hence $\vec{D} = k\vec{C}$.

10–11

Suppose the ball in Fig. 10–12 had an initial horizontal velocity of 1.5 m/sec. instead of 2.00 m/sec. When it crosses the lowest white line, what is (a) its displacement along the x axis? (b) along the y axis (c) its velocity?

From the analysis of Fig. 10–12 given in the text, we expect that the horizontal velocity of the ball will be a constant 1.5 m/sec to the right, and that the downward motion will start from rest and have a constant acceleration of 9.8 m/sec^2 downward. Let us answer part (b) first.

(b) The displacement along the y axis is 6×0.15 m $= 0.90$ m, downward.

(a) The motion along the y axis provides the time interval:
$0.90 \text{ m} = \frac{1}{2} \times 9.8 \text{ m/sec}^2 \times t^2$ and $t = 0.43$ second.
In this time, the displacement along the x axis is 1.5 m/sec $\times$ 0.43 sec = 0.64 meter, to the right.

(c) The vector component of the velocity in the horizontal direction is 1.5 m/sec to the right. The vector component of the velocity in the vertical direction is 9.8 m/sec^2 $\times$ 0.43 sec = 4.2 m/sec, downward. $\vec{V} = \vec{V}_x + \vec{V}_y$; its magnitude is $\sqrt{1.5^2 + 4.2^2} =$ 4.5 m/sec, and its direction is $\phi = \tan^{-1} 4.2/1.5 =$ 81.5°, below the horizontal.

10–12*

Suppose you were told that Fig. 10–12 represented a ball thrown *upward*. In what way would Fig. 10–14 have to be changed in order to correspond to the motion of the ball? (Section 5.)

Each velocity vector would have to be turned around to point in the opposite direction, but its tail would remain in the same place. Its magnitude would be unchanged.

10–13*

How can a motion with constant speed be an accelerated motion? (Section 7.)

If the motion is along a *curved* path, the *direction* of the velocity changes, requiring that there be an acceleration.

10–14†

(See the Appendix in the text.)

10–15

A watch has a second hand 2.0 cm long.

(a) Compute the speed of the tip of the second hand.

(b) What is the velocity of the tip of the second hand at 0.0 seconds? at 15 seconds?

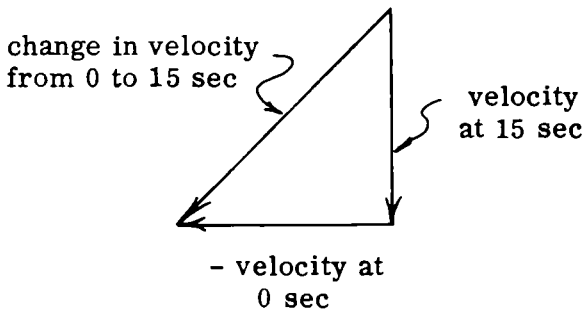
(c) Compute its change in velocity between 0.0 and 15 seconds.

(a) In 60 seconds the tip of the second hand travels a distance equal to the circumference. Therefore

$$v = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{\Delta t}$$

$$v = \frac{2\pi \times 2.0\,\text{cm}}{60\,\text{sec}} = \underline{0.21\,\text{cm/sec}}.$$

(b) Considering the watch is held vertically, the velocity at 0.0 sec is a vector 0.21 cm/sec, horizontally to the right; at the 15-second point, it is a vector 0.21 cm/sec vertically down.



(c) The change in velocity is the vector difference between the velocities at 15 and 0 seconds. It is a vector 0.21 cm/sec $\times \sqrt{2} =$ 0.30 cm/sec directed at 45° down and to the left.

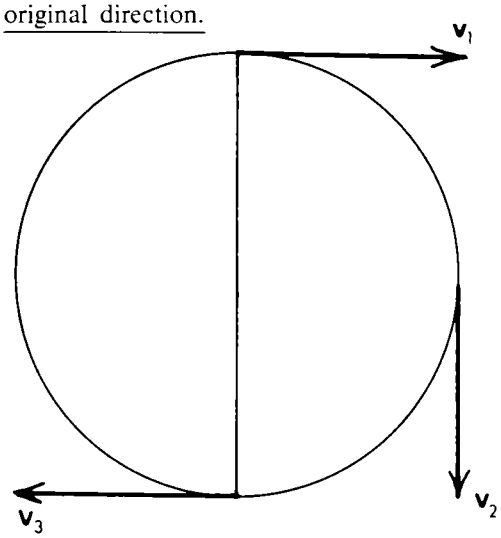
10–16

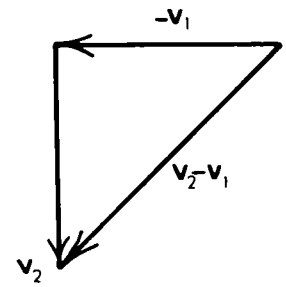
Suppose an airplane flies in a circle of circumference 10 miles at a constant speed of 100 mph.

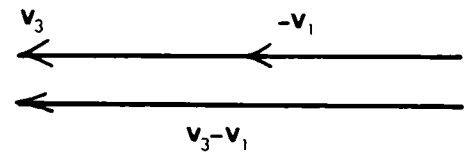
(a) What is the change in velocity in one fourth of a revolution?

(b) What is the change in velocity in one half of a revolution?

(a) The velocity has changed from $\vec{v}_1$ to $\vec{v}_2$, so the change in velocity is $\vec{v}_2 - \vec{v}_1$ as shown in the diagram; 141 mi/hr at 135° from the original direction.







(b) The velocity after one half of a revolution is opposite to the original velocity, so the change in velocity is $\vec{v}_3 - \vec{v}_1 = 2\vec{v}_3$ or 200 mi/hr at 180° from the original direction.

10–17

Show that the expression $\frac{v^2}{R}$ for centripetal acceleration has the units of acceleration.

An exercise in dimensional analysis related to circular motion. The units of velocity are (length/time), so the units of v^2/R are (length/time)$^2 \times$ 1/length = length/time^2, the same as those of acceleration.

10–18

When you ride a bicycle at 10 mi/hr your speed is about 5 m/sec.

(a) At this speed what is the centripetal acceleration of a point on the rim of a wheel (radius, 0.30 m)?

(b) How does this acceleration compare with the acceleration of the ball in Fig. 10–12?

The rim speed of the wheel with respect to the hub must be 5 m/sec.

(a) $a_c = \frac{v^2}{R} = \frac{(5\,\text{m/sec})^2}{0.30\text{m}} = \underline{83\,\text{m/sec}^2}$

(b) The acceleration of the ball in Fig. 10–12 is g, or 9.8 m/sec^2; the centripetal acceleration of the point on the rim of the wheel is 83/9.8, or about 8.5 times larger.

10–19

The outermost tip of the eccentric screw on a motor used to generate waves in a ripple tank is 0.75 cm from the center of the motor's shaft. The period of the rotating shaft is about 0.025 sec when the motor is running at its top speed.

(a) What is the acceleration of the tip of the screw?

(b) What would the acceleration be at half the above speed?

(a) $a_c = \dfrac{4\pi^2 R}{T^2} = \dfrac{4 \times (3.14)^2 \times 0.75}{(2.5 \times 10^{-2})^2}$

$= \underline{4.73 \times 10^4 \text{ cm/sec}^2}$

(b) Half the speed means that the period is doubled; this in turn means that the acceleration will be $\frac{1}{4}$ as much, or $\underline{1.2 \times 10^4 \text{ cm/sec}^2}$.

10–20

The planet mercury has a period of 7.6×10^6 sec and it is 2.6×10^6 m from the sun. What is its acceleration toward the center of the sun?

$$a_c = \frac{4\pi^2 R}{T^2} = \frac{4 \times (3.14)^2 \times 2.6 \times 10^6}{(7.6 \times 10^6)^2}$$

$$= \underline{1.8 \times 10^{-6} \text{ m/sec}^2}$$

10–21

The astronauts in an Apollo spacecraft circled the moon once every 6.5×10^3 sec. They were about 1.7×10^6 m from the center of the moon. What was their centripetal acceleration?

$$a_c = \frac{4\pi^2 R}{T^2} = \frac{4 \times (3.14)^2 \times 1.7 \times 10^6}{(6.5 \times 10^3)^2}$$

$$= \underline{1.7 \text{ m/sec}^2}$$

10–22*

If you are in a free balloon being carried along by the wind at a constant velocity, and you hold a lightweight cloth flag in your hand, which way will the flag wave? (Section 8.)

Evidently there is no motion of the balloon with respect to the air, and the flag will not wave at all.

10–23*

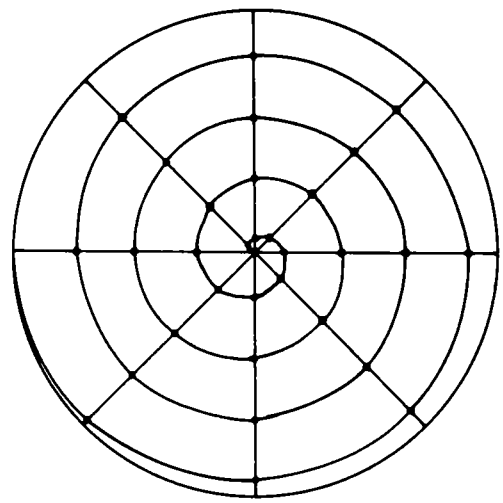
Describe the motion of a swing as it is seen from another swing exactly like it, when both swings started together in the same direction with the same push. (Section 8.)

The swing will appear to be at rest.

10–24

A bug placed at the center of a phonograph turntable crawls at a constant speed directly toward a point on the edge of the turntable. Describe the motion of the bug as seen by you from above when the turntable has a speed of 16 revolutions per minute and it takes the bug 15 seconds to crawl from the center to the edge of the turntable.

You will see the bug move in a 4-turn spiral, as shown in the diagram.



10–25

A plane flying north at 320 km/hr passes directly under another plane flying east at 260 km/hr.

(a) What is the horizontal component of the displacement of the second plane relative to the first 20 minutes after they pass each other? 50 minutes after they pass?

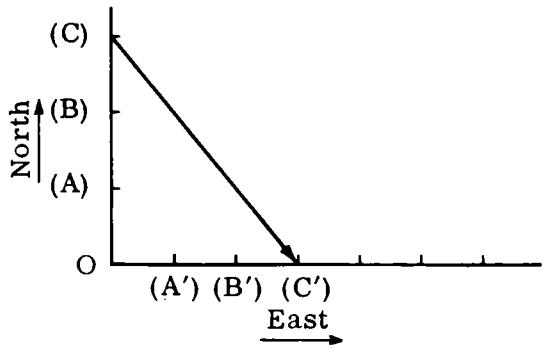
(b) What is the horizontal component of the velocity of the plane flying east relative to the plane flying north?

(c) Does the direction of this velocity vector (relative to the earth) change?

This problem involves components of a three-dimensional velocity vector which will probably be difficult for students to visualize. This type of problem is useful in developing such spatial visualization.

"Horizontal component" is a slightly ambiguous term. It means, in this case, the vector formed by dropping a perpendicular from a vector to a *plane* rather than a line.

(a) This problem is best understood by drawing a diagram which shows the airplanes at different times:



The airplanes are at A and A', B and B', etc. When the airplanes are at C and C', the displacement vector is as shown. Note that this is only the horizontal component. The real vector tilts up because the airplanes are at different heights. After 20 minutes, the distance OC is 320 km/hr $\times \frac{1}{3}$ hr $= 320/3$ km; OC' is $260 \times \frac{1}{3} = 260/3$ km. The horizontal component of the displacement is:

$$\sqrt{(320/3)^2 + (260/3)^2} = \sqrt{102400/9 + 67600/9}$$
$$= \sqrt{\frac{1.7 \times 10^5}{9}}$$
$$= \sqrt{\frac{17 \times 10^4}{9}} = \frac{10^2}{3}$$
$$\times \sqrt{17} = \frac{4.12 \times 10^2}{3}$$
$$= \underline{137 \text{ km.}}$$

Similarly, after 50 minutes the horizontal component of the displacement is 344 km.

(b) The horizontal component of the velocity vector is 137 km $\div \frac{1}{3}$ hr $=$ 411 km/hr. It is at 141° clockwise from north.

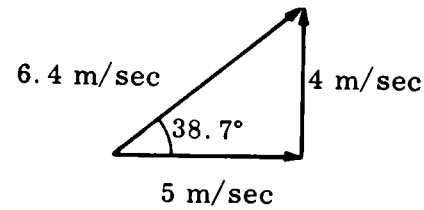
(c) In the diagram above, the vectors AA', BB', and CC' are all in the same direction. The relative velocity vector must therefore be in this same direction and does not change.

Notice that in this problem, the actual three-dimensional displacement vector of one airplane from the other is a very complicated thing indeed, changing its magnitude and direction in a very peculiar way. The horizontal component, however, is quite simple.

10–26

An ocean liner is traveling at 18 km/hr. A passenger on deck walks toward the rear of the ship at a rate of 4.0 m/sec. After walking 30 meters he turns right and walks at the same rate to the rail, which is 12 meters from his turning point.

(a) What is his velocity relative to the water surface while walking to the rear? While walking toward the rail?



(b) Draw the displacement vectors relative to the water surface for his stroll. What was the total displacement from his starting point?

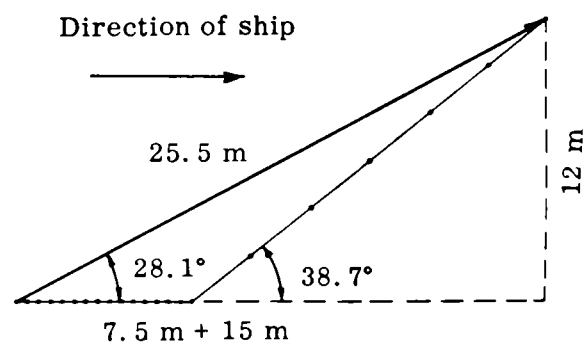
(a) For the first part of his walk, the passenger is going back at 4 m/sec while the ship is going forward at 18 km/hr = 5 m/sec. Relative to the water he is traveling in the same direction as the ship at 1 m/sec.

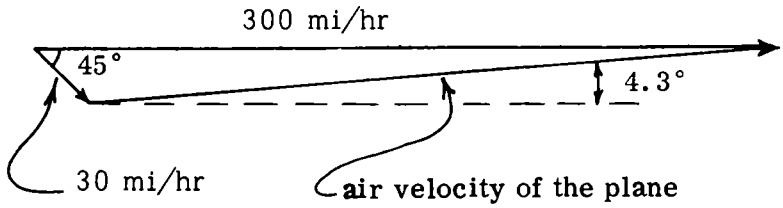
He then walks 4 m/sec toward the side while, at the same time, the ship is carrying him forward 5 m/sec. He moves $\sqrt{4^2 + 5^2} =$ 6.4 m/sec, 38.7° away from the ship's direction of travel.

(b) He walks 7.5 seconds on the first part of his stroll, 3 seconds on the second. His positions relative to the water at 0.5-second intervals are shown in the diagram below. His total displacement of 25.5 m at an angle of 28.1° with the direction of the ship can be measured from the diagram. An analytical solution is the displacement =

$$\sqrt{(7.5 \text{ m} + 15 \text{ m})^2 + 12 \text{ m}^2} = 25.5 \text{ m}$$

at an angle with direction of ship = Arc tan 12 m/(7.5 m + 15 m) = 28.1°.





10–27

An airplane is flying toward a destination 200 miles due east of its starting point, and the wind is from the northwest at 30 mi/hr. The pilot wishes to make the trip in 40 min.

(a) What should be the heading?

(b) At what air speed should he fly?

It is often helpful in such problems to assume the answer is known and roughly sketch a preliminary diagram. Since the plane's destination is directly east, its velocity relative to the ground must be a vector due east. Since it must travel 200 miles in $\frac{2}{3}$ hour, its ground speed must be 300 mi/hr. The wind (velocity of the air relative to the ground) is 30 mi/hr from the northwest. The plane must obviously head somewhat north of east. The resulting diagram is drawn as shown above.

The final step is to make a careful drawing and find that the plane must fly 280 mi/hr, 4.3° north of east.

10–28

A man rows a boat "across" a river at 4.0 mi/hr (i.e., the boat is always kept headed at right angles to the stream). The river is flowing at 6.0 mi/hr and is 0.20 mi across.

(a) In what direction does his boat actually go relative to the shore?

(b) How long does it take him to cross the river?

(c) How far is his landing point downstream from his starting point?

(d) How long would it take him to cross the river if there were no current?

(a) The man heads directly across the river, i.e., perpendicular to the banks. At the same time, the river carries him downstream. His velocity relative to the ground (shore) is his velocity relative to the water, plus the velocity of the water relative to the shore. The addition is done graphically.

(b) No matter how fast he is carried downstream, he goes across the river at 4 mi/hr. Therefore the time required to cross the river is 0.2 mile ÷ 4 mi/hr = 0.05 hour.

(c) In 0.05 hour he is swept (at 6 mi/hr) a distance of 0.3 mile downstream.

(d) If there were no current he would still cross the river in 0.2 mile ÷ 4 mi/hr = 0.05 hour since his motion across the river is unaffected by the perpendicular motion downstream.

10–29

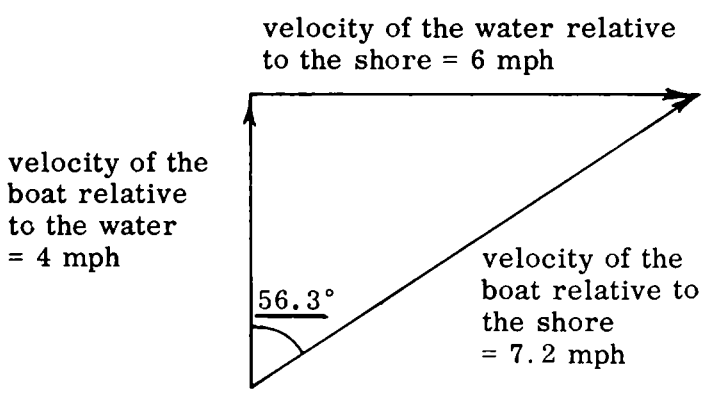
An airplane maintains a heading of due south at an air speed of 540 mi/hr. It is flying through a jet stream which is moving east at 250 mi/hr.

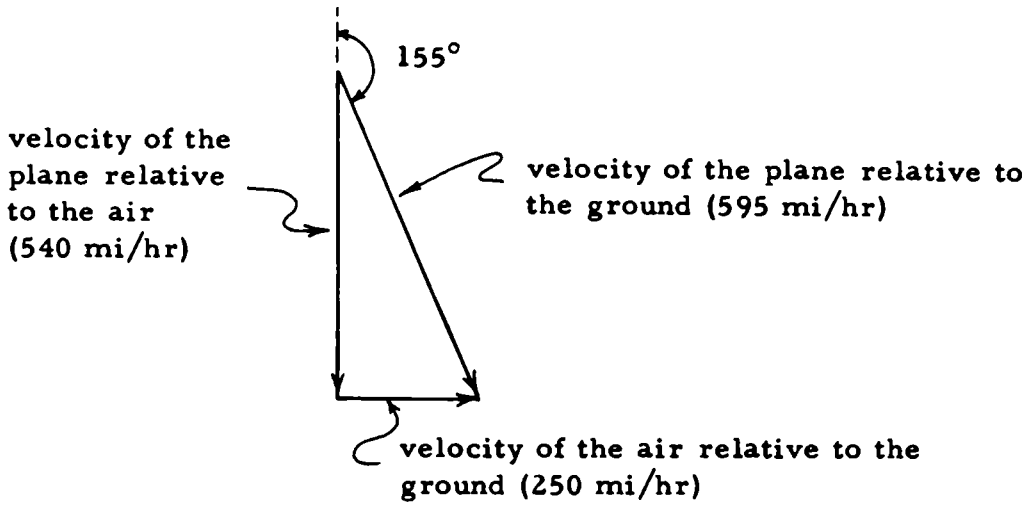
(a) In what direction is the plane moving with respect to the ground?

(b) What is the plane's speed with respect to the ground?

(c) What distance over the ground does the plane travel in 15 min?

This is a basic problem in the vector addition of velocities. Beginning students sometimes have difficulty in visualizing correctly the physical char-


velocity of the water relative to the shore = 6 mph
velocity of the boat relative to the water = 4 mph
56.3°
velocity of the boat relative to the shore = 7.2 mph


155°
velocity of the plane relative to the air (540 mi/hr)
velocity of the plane relative to the ground (595 mi/hr)
velocity of the air relative to the ground (250 mi/hr)

acteristics of this kind of problem. The effect of wind on the motion of an aircraft is dramatically apparent if you watch a slow plane on a windy day. If some of your students haven't noticed this, suggest they watch a small plane flying in a crosswind.

(a) and (b) The plane is flying south at 540 mi/hr, but is being blown east at 250 mi/hr. A graphical solution shows that these two motions cause the plane to travel at a heading 155.2° clockwise from north, at 595 mi/hr relative to the ground.

(c) At 595 mi/hr the plane will cover 149 miles in 15 minutes.

CHAPTER 11

11–1*

What is the difference between *kinematics* and *dynamics?* (Section 1.)

Kinematics is the study of motion of objects, without considering the cause of the motion; dynamics is the study of how forces cause motion.

11–2*

Was it necessary for Galileo to measure time in order to get the extrapolation which led him to the law of inertia? (Section 2.)

No. Time was not involved in his reasoning.

11–3

A ball is released from rest on the left-hand incline of Fig. 11–4 at a height of 10 cm above the lowest point.

(a) If there is no friction, how high vertically will it rise on the right-hand incline?

(b) If the right-hand incline rises 1 cm for every 10 cm of horizontal distance, how far will the ball travel horizontally on it?

(c) If the incline rises only ½ cm for every 10 cm of horizontal distance, how far will the ball go?

(a) The ball will rise to the same height, i.e., 10 cm.

(b) Since there is 1 cm rise for 10 cm horizontal travel, in rising 10 cm the ball will travel a horizontal distance of 100 cm.

(c) The slope is halved. The vertical height reached is still 10 cm, so the horizontal distance is now 200 cm.

We are progressing toward Galileo's ideal case of zero slope, when the ball will travel an indefinitely large distance.

11–4*

If you roll a putty ball around the inside of a bowl, it soon comes to rest in the center. A steel ball will orbit quite a few times before coming to rest in the center. What would be the "ideal" motion of a ball in a bowl? (Section 2.)

The ball would orbit forever.

11–5

Why is it particularly dangerous to drive on an icy highway?

A force is required to produce the acceleration associated with turning, slowing, and speeding up. This force is developed between a car and the earth through the friction between tires and road

surface. The force of friction between tires and ice can be less than one tenth what it might be between tires and a dry road. The relatively small frictional force that is developed on an icy road makes it difficult to change appreciably either the magnitude or direction of a car's speed. Thus, on an icy road, a car often perversely maintains its speed and direction despite brakes, throttle, or steering. In such cases, eventual contact with a telephone pole, a roadside ditch, or another car may supply a decelerating force.

11–6*

If a ball is rolling with a velocity of 20 cm/sec and no force acts on it, what will be its velocity after 5 sec? (Section 2.)

$v = 20$ cm/sec.

11–7*

In Table 1, what would you predict for the average velocity during the eighth time interval if the puck continued to be pulled by the stretched loop? (Section 3.)

$v = 20.9$ cm/flash.

11–8*

Suppose a Dry Ice puck increases its velocity so that it travels first at 10 cm/sec, at 12 cm/sec at the end of the next second, 14 cm/sec at the end of the second after that, and so on. What can you tell about the force acting on the puck? (Section 3.)

The force is constant.

11–9

A body is pulled across a smooth horizontal surface by a spring loop that is kept stretched by a constant amount. It is found that the body is accelerated at 15 cm/sec^2. What will be the acceleration of the body if it is pulled by two loops, each just like the first loop, side by side and stretched by that same amount? (See Fig. 11–10.)

Both springs pull on the body. Since the springs are stretched by the same amount, they pull with equal force. Therefore, $F_{\text{total}} = F_1 + F_2 = 2F_1$. Since the force has been doubled, the resultant acceleration must also be doubled:

If

$$\frac{F_1}{m} = 15 \text{ cm/sec}^2,$$

$$\frac{2F_1}{m} = 2 \times 15 \text{ cm/sec}^2 = \underline{30 \text{ cm/sec}^2}.$$

11–10

An object sliding on a low-friction bearing is pulled with a constant force. In a time interval of 0.3 sec the velocity changes from 0.2 m/sec to 0.4 m/sec. In a second trial, the object is pulled with another force. In the same length of time the velocity now changes from 0.5 m/sec to 0.8 m/sec.

(a) What is the ratio of the second force to the first?

(b) If the body is pulled with the second force for 0.9 sec, what change in velocity results?

(a) Because the masses and the time intervals are the same in the two trials, the ratio of the forces is equal to the ratio of the *changes* of velocity (which equals, for straight-line motion in one direction, the changes in speed):

$$\frac{F_2}{F_1} = \frac{m_2 \dfrac{\Delta v_2}{\Delta t_2}}{m_1 \dfrac{\Delta v_1}{\Delta t_1}} = \frac{\Delta v_2}{\Delta v_1} = \frac{0.8 \text{ m/sec} - 0.5 \text{ m/sec}}{0.4 \text{ m/sec} - 0.2 \text{ m/sec}}$$

$$= \underline{1.5}.$$

(b) In the second trial, since the time is three times longer, the change in velocity will be three times greater, *i.e.*, $9 \dfrac{\text{m}}{\text{sec}}$.

11–11

A block is pulled along a horizontal surface by 2, 4, 6, and 8 parallel bands of rubber. All bands are alike and each is stretched the same length in each experiment. The graph (Fig. 11–18) shows the resulting accelerations plotted versus the number of bands.

(a) What can you conclude from the fact that the plot yields a straight line?

(b) What does the intercept of the graph with the horizontal axis measure?

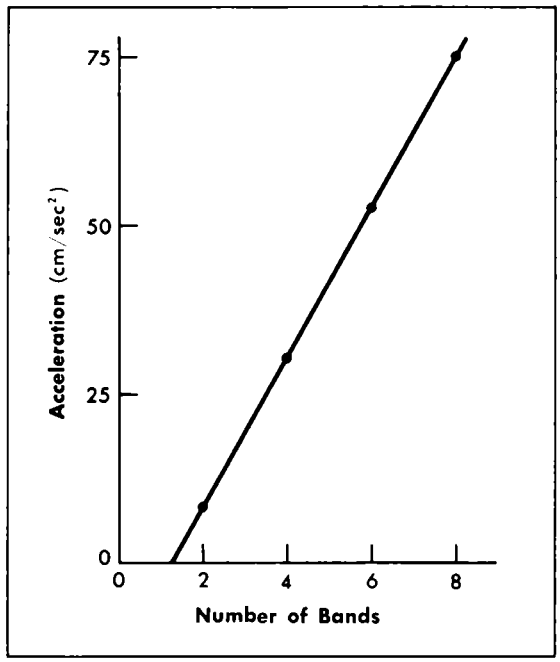
(c) Can you use the graph to predict the acceleration of the block produced by stretching one rubber band by the standard amount?

(d) Suppose you repeat the experiment, changing only the surface on which the block is pulled. How will the new graph relate to the old one?

The purpose of this problem is to show that we can get to Newton's law of motion also without reducing friction to an insignificant amount, provided the frictional force is independent of velocity. This is the case only over a very limited range.

(a) The increase in acceleration is proportional to the increase of force applied by the rubber bands.

(b) The intercept corresponds to the smallest force to be applied by the rubber bands to keep the block moving at a constant velocity.



This force is equal in magnitude to the frictional force opposing the force supplied by the rubber bands when the block is moving.

(c) We certainly do not expect the block to accelerate in the opposite direction to which we pull it. Apparently, the frictional force can equal any force applied by the bands up to that given by the intercept. The extrapolation of the graph below the horizontal axis is useless.

(d) A different surface means most likely a different frictional force; hence the intercept will be at a different point. But we expect the slope of the graph to be the same as before, because the increase of the acceleration is proportional to the applied force.

11–12*

In Fig. 11–9, suppose the photograph represented a body moving from right to left. In which direction would the force be acting in this case? (Section 4.)

Toward the right.

11–13*

A car can accelerate by 3 m/sec². What is its acceleration if it is towing another car like itself? (Section 5.)

$a = 1.5$ m/sec².

11–14

Why is the flask in Fig. 11–12 closed? Be prepared to explain in class.

The flask is closed so that no product of the chemical reaction can escape from the system. (Actually no gas is generated in this reaction.) Thus the mass of the flask plus contents remains constant.

Note: Rigorously, if heat, light, or sound energy are given off during the reaction, there will be a corresponding decrease in mass as dictated by the mass equivalence of energy. However, this change is negligibly small, and it is probably best not to discuss the point unless the question is raised by a student. A change of mass Δm corresponds to a change of energy:

$$E = \Delta mc^2 = \Delta m(3 \times 10^8 \text{ m/sec})^2$$

$$= \Delta m(9 \times 10^{16} \text{ m}^2/\text{sec}^2)$$

$$= \Delta m(9 \times 10^{16} \text{ joules/kg}).$$

Very good heating coal gives about 3×10^7 joules of heat energy per kg. This corresponds to a mass change of only $3 \times 10^7/9 \times 10^{16} = 3 \times 10^{-10}$ kg for each kg burned; this constitutes a change of only one part in 3×10^{10} parts.

11–15

Why do we make such careful distinctions between gravitational and inertial mass, rather than talking about one mass only, since they are equivalent?

The two definitions of mass differ because they arise from very different physical situations: weighing in one case and accelerating in the other. Their equivalence has to be shown experimentally.

11–16*

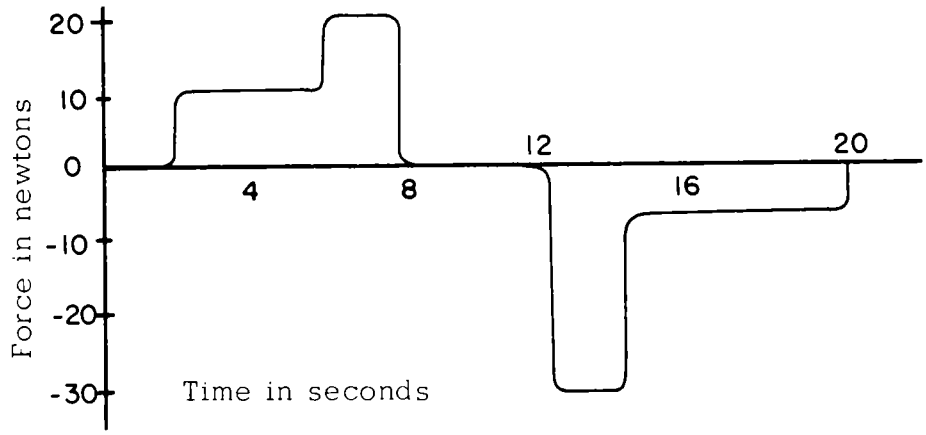
Two objects are accelerated separately by the same force. Object A accelerates at 20 cm/sec² and B at 60 cm/sec². What is the ratio of (a) their inertial masses, (b) their gravitational masses? (Section 6.)

(a) 3.0.
(b) 3.0.

11–17*

A carton is balanced by two identical cans on a balance. A certain force accelerates the carton by 2 m/sec². How much will the same force accelerate one of the cans? (Section 6.)

$a = 4$ m/sec².



11–18*
A body with a mass of 0.5 kg is accelerated at 4 m/sec^2. How large a force is acting? (Section 7.)

$F = 2$ newtons.

11–19*
A boy gives a 0.5-kg ball a speed of 6 m/sec from rest in 0.2 sec. What average force did he apply to the ball? (Section 7.)

$F = 15$ newtons.

11–20
A force of 5 newtons gives a mass m_1 an acceleration of 8 m/sec^2, and a mass m_2 an acceleration of 24 m/sec^2. What acceleration would it give the two when they are fastened together?

This problem may be done in several ways. If it is handled step by step, we can note that

$$m_1 = \frac{F}{a_1} = \tfrac{5}{8}\text{ kg, and } m_2 = \frac{F}{a_2} = \tfrac{5}{24}\text{ kg.}$$

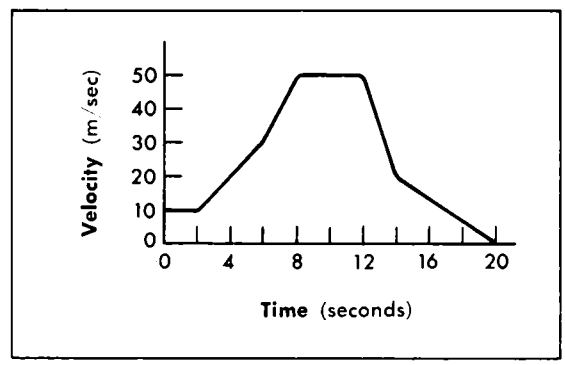
Hence, $m_1 + m_2 = \frac{5}{8} + \frac{5}{24} = \frac{5}{6}$ kg, and

$$a = \frac{F}{m_1 + m_2} = \frac{5}{\frac{5}{6}} = \underline{6\text{ m/sec}^2.}$$

In following these steps, it is apparent that F drops out. This might have been anticipated by remembering that force is proportional to both mass and acceleration. We could have written

$$a = \frac{F}{m_1 + m_2} = \frac{F}{F/a_1 + F/a_2}$$
$$= \frac{F}{F/8 + F/24} = \frac{F}{F/6} = \underline{6\text{ m/sec}^2.}$$

11–21
The graph in Fig. 11–19 shows the velocity, along a straight line, of an object of mass 2 kg, as a function of time. Plot a graph of the force as a function of time.



11–22†
(See the Appendix in the text.)

11–23†
(See the Appendix in the text.)

11–24
You observe an object covering distance in direct proportion to t^3, where t is the time elapsed.
(a) What conclusion might you draw about the acceleration? Is it constant? Increasing? Decreasing? Zero?
(b) What might you conclude about the forces? Be prepared to discuss in class.

(a) If an object moves with *constant velocity* from $x = vt$, we know that $x \propto t$. If an object

moves with *constant acceleration,* from $x = \frac{1}{2}at^2$, we know that $x \propto t^2$. Hence our given object cannot have zero acceleration or constant acceleration. Further, when $x \propto t^3$, x is increasing with t more rapidly than when $x \propto t^2$. Therefore the acceleration must be increasing with t.

What relation between a and t must hold for $x \propto t^3$? Compare the case of constant acceleration with our given case. If instead of $x \propto t^3$, we write $x \propto t \times t^2$, it may be a bit easier to see the relation.

$$\text{Given case: } x \propto t \times t^2.$$
$$\text{Constant accel: } x \propto a \times t^2.$$

For the case of $x \propto t \times t^2$, if the acceleration is proportional to t instead of being constant, x will be proportional to t^3. Hence, if $x \propto t^3$, $\underline{a \propto t}$. The acceleration is increasing uniformly with time.

(b) From $F = ma$, $F \propto a$. If $a \propto t$, $F \propto t$. The accelerating force must be increasing uniformly with time.

11–25*

If several forces of different magnitudes and directions act on an object, in what direction will the object accelerate? (Section 9.)

Along the direction of the net force.

11–26*

In Fig. 11–13, what would be the acceleration of the puck if its mass were 0.50 kg, $F_2 = 10$ nt, and $F_1 = 6$ nt? (Section 9.)

$a = 8$ m/sec^2 (to the right).

11–27*

In Fig. 11–15, suppose $F_1 = F_2 = 2.0$ nt. What would be the net force on the puck? (Section 9.)

$\vec{\mathbf{F}} = 3.7$ newtons to the right along the dotted line.

11–28

A block of mass 8.0 kg, starting from rest, is pulled along a horizontal tabletop by a constant force of 2.0 newtons. It is found that this body moves a distance of 3.0 m in 6.0 sec.

(a) What is the acceleration of the body?

(b) What is the ratio of the applied force to the mass?

(c) Since your answer to part (b) is not equal to that to part (a) (at least, it shouldn't be), what conclusions can you draw about this motion? Give numerical results, if possible.

(a) The block starts from rest and is acted upon by a constant force. Assuming that the *net* force acting upon the block was constant, the acceleration will also be constant. Then the distance traveled will be:

$$x = \tfrac{1}{2}at^2, \text{ and } a = \frac{2x}{t^2}$$
$$a = \frac{2 \times 3.0 \text{ m}}{(6 \text{ sec})^2} = \tfrac{1}{6} \text{ m/sec}^2.$$

(b) $\dfrac{F}{m} = \dfrac{2.0 \text{ newtons}}{8.0 \text{ kg}} = \dfrac{1}{4}\dfrac{\text{nt}}{\text{kg}} = \tfrac{1}{4} \text{ m/sec}^2.$

(c) The observed acceleration, $\frac{1}{6}$ m/sec^2, was less than $\frac{1}{4}$ m/sec^2, the acceleration computed from the ratio F/m. Therefore, some other force (retarding) must also have acted. It could be assumed to have been a friction force. In order to evaluate this force, we must assume that it also was constant. Then we can solve for the net effective force which must have been acting to produce the observed acceleration:

$$F_{\text{applied}} - F_{\text{friction}} = ma,$$

and

$$F_{\text{friction}} = F_{\text{applied}} - ma$$
$$F_{\text{friction}} = 2 \text{ nt} - (8 \text{ kg} \times \tfrac{1}{6} \text{ m/sec}^2)$$
$$= \underline{\tfrac{2}{3} \text{ newton.}}$$

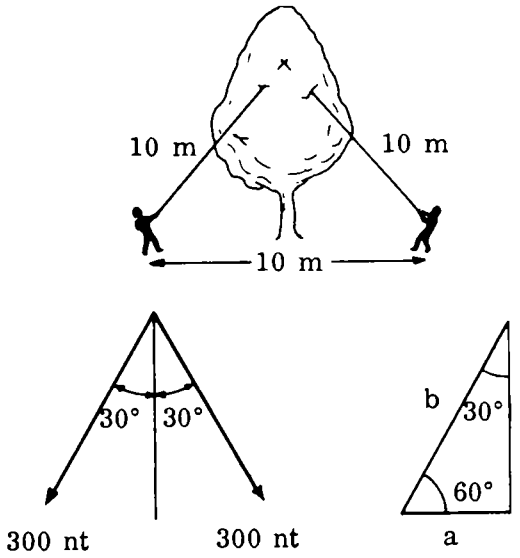
The force of friction is opposed to the motion of the block.

11–29

Two men wish to pull down a tree by means of a rope fastened near the top. If they use only one rope, the tree will come down on top of them. To prevent this, they tie two ropes 10.0 meters long to the same point and stand on the ground 10.0 meters apart when they pull. If each pulls with a force of 300 newtons, what is the force exerted by the ropes on the tree?

Since each rope is 10.0 m long and the two men stand 10.0 m apart, the three lines form an equilateral triangle, and the angle between the ropes is equal to 60°.

The two forces are equal; so the resultant must be along the line which bisects the 60° angle. To find the component of each of the forces along this line, note that we have a 30°, 60°, and 90° triangle. The ratio of the sides $a{:}b{:}c = 1{:}2{:}\sqrt{3}$. Therefore, $\dfrac{b}{c} = \dfrac{300}{F_r}$, $F_r = \dfrac{300c}{b} = \dfrac{300 \times \sqrt{3}}{2} =$



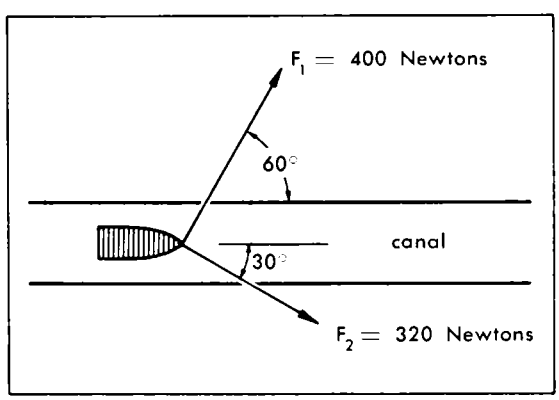
260 newtons.

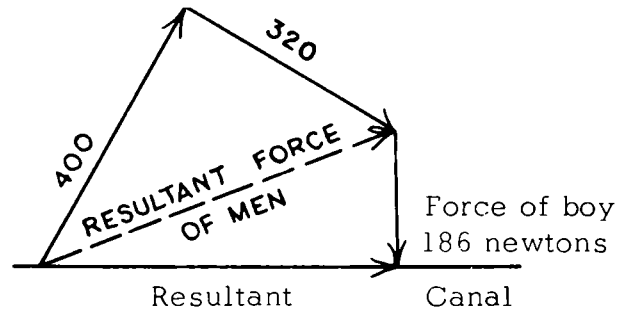
Each of the ropes pulling on the tree exerts a force which has a component of 260 newtons in the resultant direction. Thus, the force exerted by the ropes on the tree is 520 newtons.

11–30

Two men and a boy are pulling a boat along a canal. The two men pull with forces $\vec{F}_1$ and $\vec{F}_2$ whose magnitudes and directions are indicated in Fig. 11–21. Find the magnitude and direction of the smallest force which the boy could exert to keep the boat in the middle of the canal.

To move in the direction of the canal, the resultant must be in the direction of the canal. We see from the vector diagram that the boy must pull with a force of 186 newtons perpendicular to the canal and on the same side as F_2.





11–31*

You have to push with a force of 200 nt to slide a refrigerator across a floor at constant velocity. What is the force of friction acting on the refrigerator? (Section 9.)

$\vec{F} = 200$ newtons, in the direction opposite to that of your push.

11–32

The retarding force of air resistance on a balloon is proportional to the square of the velocity. For a certain balloon, inflated a certain amount, this force is given in newtons by $F_R = 0.2v^2$ where v is the velocity in m/sec. The balloon and the air inside have a combined mass of 10 gm.

(a) Draw graphs of the balloon's acceleration as a function of velocity when you pull it with a 1.8-newton force and with a 7.2-newton force.

(b) What is the maximum velocity that the balloon will reach in each case?

(c) If the mass were 5.0 gm, how would this affect the maximum velocity?

(d) What do you think would be the effect on the maximum velocity if you inflated the balloon to a larger volume?

(a) $$a = \frac{F_{\text{net}}}{m} = \frac{F_{\text{applied}} - F_{\text{resistance}}}{m}$$
$$= \frac{F_{\text{applied}} - 0.2v^2}{m}.$$

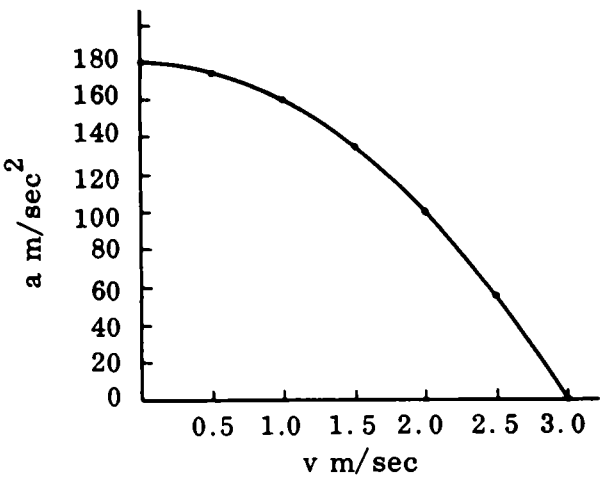
For an applied force of 1.8 newtons,

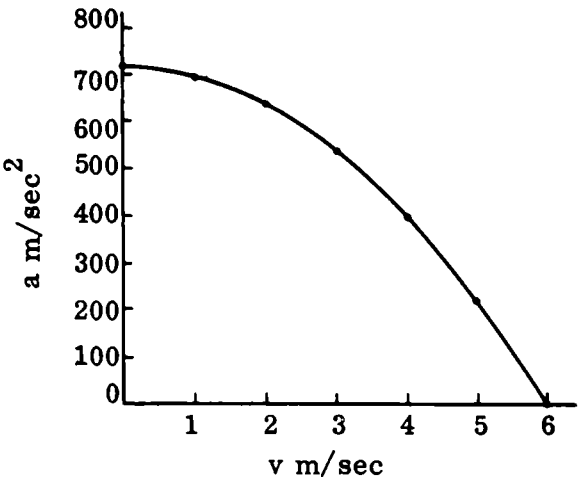
$$a = \frac{1.8 - 0.2v^2}{10^{-2}} = 180 - 20v^2 \text{ m/sec}^2.$$

For a 7.2-newton force,

$$a = \frac{7.2 - 0.2v^2}{10^{-2}} = 720 - 20v^2 \text{ m/sec}^2.$$

(b) The maximum velocity is that velocity for which $a = 0$. In the first case, the balloon





reaches a terminal velocity of 3.0 m/sec. In the second case, it reaches a velocity of 6.0 m/sec.

(c) A change in balloon mass would not affect the velocity. The resistive force of the air depends on the velocity only, not on the mass. Thus the resistive force will equal the applied force at some particular velocity, independent of the mass. This can be demonstrated analytically as follows:

$$a = \frac{F_{\text{applied}} - F_{\text{resistance}}}{m}$$

$$ma = F_{\text{applied}} - F_{\text{resistance}}$$

$$= F_{\text{applied}} - 0.2v^2.$$

To satisfy the maximum velocity condition we must set $a = 0$. Thus,

$$0 = F_{\text{applied}} - 0.2v^2_{\text{max}}$$

$$v^2_{\text{max}} = 5F_{\text{applied}}$$

$$v_{\text{max}} = \sqrt{5F_{\text{applied}}} \text{ (independent of } m),$$

where v is in m/sec and F is in newtons.

(d) If the balloon were larger, it would present a greater area to the air, and the force of air resistance would be larger at any given velocity. (This means that the coefficient of v^2 would be larger.) The result would be that the resistive force would become equal to the applied force at a lower velocity than before. Therefore, v_{max} would be decreased.

11-33

Aristotle thought that a constant force was required to produce a constant velocity and from this he concluded that, in the absence of force, bodies would come to rest.

(a) Name several situations where a constant force seems to produce a constant velocity.

(b) How do you explain each of the situations in (a) in the light of Newton's law of motion?

(a) Situations in which a constant force seems to produce a constant velocity:

(1) A horse pulls a wagon with a constant force at constant velocity;

(2) A car attains a constant speed for any given gas pedal position (constant force);

(3) An airplane flies at a constant air speed when its engine speed and propeller pitch remain constant;

(4) The balloon of Problem 32 would move with a constant velocity if a constant force were applied to it.

(b) In each example of this kind, careful inspection shows that, in addition to the applied force, there are other forces which oppose the applied force. In all cases of constant velocity, the applied force is balanced by opposing forces. [Usually these opposing forces are not constant but are proportional (but not directly) to v.] When the applied force exceeds the opposing forces at any given v, positive acceleration occurs. If the applied force is smaller than the opposing forces, negative acceleration results.

In the examples given in part (a), when the object is moving with constant velocity, the sources of the forces which oppose and balance the applied force are:

(1) For the horse and wagon, the friction of wheels on bearings and of wheels on the road surface;

(2) For the car, road and bearing friction and (especially at high speeds) air resistance;

(3) For the airplane, air resistance;

(4) For the balloon, air resistance.

11–34
How would you define a unit of mass if people had placed a standard spring at Sèvres instead of a standard mass?

Since this is a chapter on dynamics, our first thought might be of a dynamical method of defining a mass unit.

A direct way to establish a unit of mass would be to perform an acceleration experiment. The unit of mass would be the mass of an object which would be accelerated at a specified rate by application of the standard force (and that force only). While the condition that no other force act on the body is a very difficult one to satisfy, the "frictionless" pucks described and used in the films and text could be employed in such a way that the net force is given by the spring and nothing else.

A practical experimentalist, knowing the equivalence of gravitational and inertial mass, might simply hang a mass from the spring. He would have to recognize that the result would be sensitive to his position relative to other massive objects. Consequently, his position on the earth would have to be standardized. For great accuracy, his position relative to the sun and moon also would have to be standardized. He would also have to recognize that an appreciable change in the configuration of the earth would destroy his standard. Still, this would be a more practical standard than that determined by an acceleration experiment.

A more sophisticated procedure would be to define the mass standard in terms of a frequency of oscillation of that standard when set into motion hanging from the standard spring, in a vacuum. The frequency of an oscillating mass varies inversely as the square root of the oscillating mass. This method would be by far the most practical. Furthermore, it is a direct measure of inertial mass.

While most students will not be familiar with simple harmonic motion at this point in the course, this type of answer may be suggested by the students' experience with the inertial balance (Experiment 22). A brief discussion of this idea can make it seem reasonable without going into an analysis of simple harmonic motion.

CHAPTER 12

12–1*
If the gravitational force on an object is 49.0 newtons where the gravitational field is 9.80 nt/kg, what is the object's mass? (Section 1.)

$mg = 5.00$ kg.

12–2*
The gravitational field at the surface of the moon is one sixth of that at the surface of the earth.
(a) How much would a 70-kg man weigh on the moon? On the earth?
(b) What would be his mass on the earth? On the moon? (Section 1.)

(a) Weight on moon: 1.1×10^2 newtons.
Weight on earth: 6.9×10^2 newtons.
(b) Mass on earth: 70 kg.
Mass on moon: 70 kg.

12–3
Which two locations would you pick from Table 1 to investigate the change in g with altitude?

You would want two places which are at about the same latitude, so that you would know that the difference in gravitational field strength was due only to the difference in altitude. Good choices would be Brussels and Banff or, perhaps, New York and Denver.

It can be seen that latitude does affect the value of g by comparing the values for Java and New Zealand, which are at approximately the same altitude.

12–4*
What is the magnitude of the gravitational field of the earth at a point where the acceleration of gravity is 9.81 m/sec^2? (Section 2.)

$g = 9.81$ newtons/kg.

12–5
What happens if you throw a Ping-Pong ball down faster than its terminal velocity?

The resistance of the air will be larger than the gravitational force on the ball; the ball will slow down until it reaches the terminal velocity, which it will maintain from then on.

12–6

If different objects had different ratios of their inertial and gravitational masses, would they all accelerate at the same rate in the gravitational field of the earth? Explain.

This problem calls attention to the simplest implication of the relation between gravitational and inertial mass.

Since $\vec{\mathbf{a}} = \frac{m_g}{m_i}\vec{\mathbf{g}}$, a variable ratio m_g/m_i implies a variable acceleration. Thus, for instance, if objects that are made of different materials had different ratios m_g/m_i, they would accelerate differently in the earth's gravitational field.

Note that the constancy of m_g/m_i is not a derived fact; it is based on experiments which demonstrate the universality of the acceleration due to gravity. If m_g/m_i were not constant, the only changes to be expected are in those phenomena directly related to the force of gravity. Other aspects of dynamics would be quite unchanged.

Inasmuch as the constancy of m_g/m_i is founded directly on experiment, an experimental physicist might well design experiments to test this constancy more carefully. Direct acceleration experiments cannot be done with extreme precision or with a large range of masses, but every such experiment that has been performed has been found to be consistent with a constant m_g/m_i to within the limits of accuracy of the experiment. The most precise experiments yet performed essentially involve the comparison of the periods of pendulums made of different materials.

These experiments show that, to about 1 part in 10^8, the ratio m_g/m_i is the same independent of whether the pendulum is made of a low-density material (such as aluminum) or a high-density material (such as lead). Artificial earth satellite experiments, currently being planned, will extend the precision of related experiments to about 1 part in 10^{11}.

12–7*

In Fig. 12–2, how long would you make your scale in order to photograph the ball over 30 time intervals, if the ball started from rest at the beginning of the first interval? (Section 2.)

The scale should be 4.9 meters long.

12–8

What is the acceleration of a falling 0.2-kg ball if the gravitational field is 9.80 nt/kg and the air resistance acting on the ball is 0.5 nt?

Let F_a = force of air resistance. Choose the downward direction to be positive. Then

$$\mathbf{a} = \frac{F_a + mg}{m} = \frac{F_a}{m} + g$$
$$= -\frac{0.5\text{ nt}}{0.2\text{ kg}} + 9.8\frac{\text{nt}}{\text{kg}}$$
$$= \underline{7.3\text{ m/sec}^2.}$$

The direction of the acceleration is downward.

12–9

A ball is thrown straight up with a velocity of 15 m/sec. (Neglect air resistance.)

(a) How fast will it be going after 1.2 sec?
(b) How far above the ground will it be at that time?
(c) How fast will it be going after 2.3 sec?
(d) How far above the ground will it be at that time?
(e) What is the ball's acceleration at the top of its rise?

This problem may be solved using the kinematic equation for constant acceleration, not beginning at rest. Let $a = -9.8\frac{\text{m}}{\text{sec}^2}$ and $v = 15\frac{\text{m}}{\text{sec}}$.

(a) $v' = v + a\,\Delta t$
$$= 15\frac{\text{m}}{\text{sec}} + \left(-9.8\frac{\text{m}}{\text{sec}^2}\right)(1.2\text{ sec})$$
$$v' = 3.2\frac{\text{m}}{\text{sec}}.$$

(b) $\Delta x = v_{av}\,\Delta t$
$$\Delta x = \left(\frac{15 + 3.2}{2}\frac{\text{m}}{\text{sec}}\right)(1.2\text{ sec})$$
$$\Delta x = 10.9\text{ m},$$

(c) $v' = v + a\,\Delta t$
$$= 15\frac{\text{m}}{\text{sec}} + \left(-9.8\frac{\text{m}}{\text{sec}^2}\right)(2.3\text{ sec})$$
$$v' = -7.5\frac{\text{m}}{\text{sec}}.$$

(d) $\Delta x = v_{av}\,\Delta t$
$$\Delta x = \left(\frac{15 - 7.5}{2}\frac{\text{m}}{\text{sec}}\right)(2.3\text{ sec})$$
$$\Delta x = 8.6\text{ m}.$$

(e) The ball's acceleration at the top of its rise, and at all other times in its flight, is $\underline{-9.8\text{ m/sec}^2}$.

12–10†
(See the Appendix in the text.)

12–11
Two 0.5-kg Dry Ice pucks on a flat table are tied together by a string.
(a) What is the total gravitational force on these pucks?
(b) At what rate will they speed up if pulled across the table by a horizontal force equal to the gravitational force found in (a)?
(c) If one puck remains on the table and the other hangs over the edge, what will be the acceleration of the pucks?

This problem deals with free fall, a special case of Newton's law. It also introduces a new kind of problem in which gravitational force on part of a system can accelerate the entire system.

(a) The total gravitational force is approximately 10 nt, because each 0.5-kg mass experiences about a 5-nt force (using 10 nt/kg for g). Actually, $F = 2 \times 0.5 \times 9.8 = \underline{9.8 \text{ nt.}}$
(Note that since the pucks will not accelerate in this case, the table must push up on each disc with a force equal to the pucks' weight.)

(b) If a single object is pulled with a net force equal to the gravitational force, it must have the same acceleration it would in free fall, $\underline{9.8 \text{ m/sec}^2}$ (or $\approx 10 \text{ m/sec}^2$).

Mathematically: $F = \text{mass} \times a$, but $F = 2mg$ and mass $= 2m$

$$F = 2mg = 2ma$$

$$a = g.$$

(c) Some students will see directly that, with one 0.5-kg mass on the table, and the other hanging over the edge, we have a total of 1 kg of mass and that the only accelerating (unbalanced) force is the force of gravity on a 0.5-kg mass. This force is 0.5 kg × 9.8 newtons/kg = 4.9 newtons. A force of 4.9 newtons acting on 1 kg of mass results in an acceleration of 4.9 m/sec². A bit more formally:

$$a = \frac{F}{m} = \frac{F}{m_1 + m_2} = \frac{m_2 g}{m_1 + m_2}$$

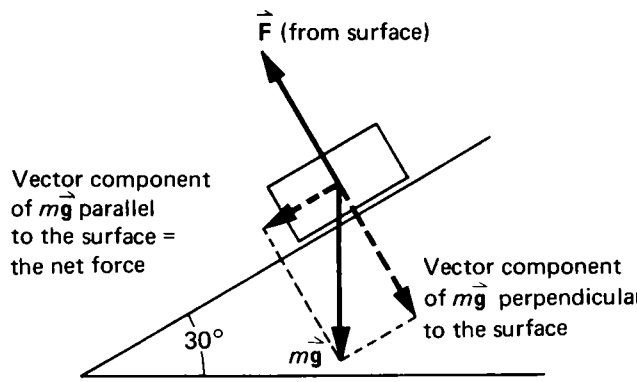
$$= \frac{0.5 \text{ kg} \times g}{1 \text{ kg}} = \tfrac{1}{2}g$$

$$= \underline{4.9 \text{ m/sec}^2} \text{ (or } \approx 5 \text{ m/sec}^2\text{)}.$$

12–12
An object of mass m is moving on a frictionless inclined plane that makes an angle of 30 degrees with the horizontal.
(a) What is the net force on the object?
(b) What is its acceleration down the plane?

(a) We can resolve the force $m\vec{g}$ into two vector components, one along the inclined plane and one perpendicular to it. Since the object doesn't move into the surface, there is no *net* force perpendicular to the surface. The perpendicular vector component of $m\vec{g}$ is balanced by the force from the surface.

Since there is no friction, the force from the surface of the plane can have no vector component parallel to the plane. The only force component which is parallel to the plane is the parallel vector component of $m\vec{g}$. This parallel component of $m\vec{g}$ is the net force. Finding this component requires either trigonometry or a scale drawing. For either method it may be clearer to draw a separate force diagram.


F (from surface)
Vector component of mg parallel to the surface = the net force
Vector component of mg perpendicular to the surface
30°
mg

By geometry it may be shown that the angle between $m\vec{g}$ and $m\vec{g}_{\text{perpendicular}}$ is also 30°. Then the sin 30° = $m\vec{g}_{\text{parallel}}$, and the magnitude of the net force, $m\vec{g}_{\text{parallel}}$, = sin 30° $m\vec{g} = \frac{1}{2}\ mg$. The direction of the net force is down, along the slope of the inclined plane.

(b) The acceleration down the plane is given by

$$F = ma = \tfrac{1}{2}mg, \text{ and}$$

$$a = \underline{\tfrac{1}{2}g, \text{ down along plane.}}$$

12–13*

How can you tell that the horizontal component of velocity of the ball projected horizontally in Fig. 12–4 is constant and its vertical component is not? (Section 3.)

Horizontal component of displacement is constant over equal time intervals; vertical component is not.

12–14

A baseball is thrown from center field to home plate. If the ball is in the air 3.00 seconds and we neglect air resistance, to what vertical height must it have risen?

The assumption must be made that the ball is caught at the same height from which it was thrown. In this case, the symmetry of the trajectory makes it clear that the ball rises for 1.50 seconds and falls for 1.50 seconds.

The height y that the ball can reach is then

$$y = \tfrac{1}{2}gt^2 = \frac{1}{2}\left(9.8\ \frac{\text{m}}{\text{sec}^2}\right)(1.50\ \text{sec})^2$$

$$y = 11.0\ \text{m}.$$

12–15

Plot the trajectory of an object that is thrown into the air with an initial velocity of 10 m/sec at an angle of 45 degrees with the horizontal.

Since the horizontal and vertical components of the initial velocity are both $\frac{10\sqrt{2}}{2} = 7.07$ m/sec, the kinematic equation $x' = x + v\ (t' - t) + \frac{1}{2}a\ (t' - t)^2$, letting $(t' - t) = t$, gives x and y as functions of t:

$$x = 7.07t$$
$$y = 7.07t - 4.9t^2.$$

We could calculate x and y for various values of t. It is a little simpler, however, to eliminate t from the above equations, and to obtain one equation relating x and y:

$$t = x/7.07$$
$$y = 7.07(x/7.07) - 4.9(x/7.07)^2 = x - 0.098x^2.$$

We can now plot y against x. The curve is a parabola, and we can calculate its salient features. The curve crosses the x-axis both at $x = 0$ and at $x = 1/0.098 = 10.2$ m. The maximum value of y is halfway between at $x = 5.1$; here $y = 2.55$ m. In tabular form:

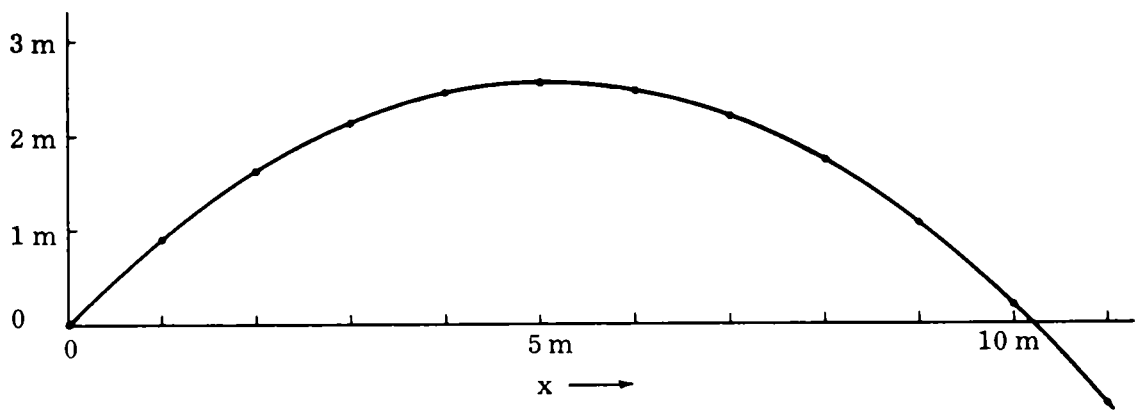
(a) $y_{max} = 2.55$ m; see below.

(b) 10.2 m; see below.

These results can be obtained in several other ways, such as working with the original equations involving t. For example, $y = 0$ for $t = 0$ or for $t = 7.07/4.9$. y_{max} corresponds to $t = 7.07/2(4.9)$. The total distance $x = 7.07t = 7.07(7.07/4.9) = 10.2$ m.

(c) When the object hits the ground again its vertical velocity is equal in magnitude (but opposite in direction) to the initial vertical velocity; $v_{fy} = -7.07$ m/sec. The x velocity is unchanged; $v_x = 7.07$ m/sec. The magnitude of the velocity is therefore 10 m/sec as before. This can be seen from the symmetry of the trajectory that has been plotted. But remember that velocity is a vector; although the speed is unchanged the direction is ↘ instead of ↗.

0	1	2	3	4	5	6	7	8	9	10	11
0	0.90	1.61	2.12	2.43	2.55	2.48	2.20	1.73	1.06	0.20	-0.86



12–16*
What is the centripetal force needed to keep a 3.0-kg object moving in a circle of 2.0 m radius at a speed of 4.0 m/sec? (Section 5.)

$F = 24$ newtons.

12–17*
What is the effect on the speed and direction of motion of an object when a force acts on it in a direction perpendicular to its path? (Section 5.)

The speed does not change but the direction of motion changes toward the direction of the force.

12–18*
What is the velocity of the puck in Fig. 12–10 when it is at the bottom of the photograph? (Section 5.)

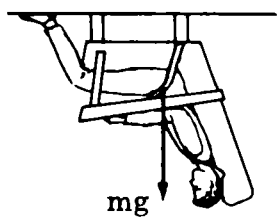
$\vec{v} = 0.52$ m/sec toward the left if puck is moving clockwise as in Figure 12–11.

12–19*
What is the acceleration of the puck in Fig. 12–10 when it is at the bottom of the photograph? (Section 5.)

$\vec{a} = 0.62$ m/sec^2 toward center of circle.

12–20
How fast must a plane fly in a loop-the-loop of radius 1.00 km if the pilot experiences no force from either the seat or the safety belt when he is at the top of the loop? In such circumstances the pilot is often said to be "weightless."

This is an interesting problem in which centripetal force must be equated to weight.



Since the pilot cannot "turn off" gravity, we know that whatever other forces may or may not be acting he is always subject to the force of gravity, mg. This force acting alone results in an acceleration downward as though in free fall, $a = -9.8$ m/sec^2. If there is no apparent force from seat or belt opposing this motion, they too must be accelerating with $a = -9.8$ m/sec^2. This condition could occur if the pilot, seat, and belt were all falling freely. It also occurs when all three are moving in a circular path with a centripetal acceleration of 9.8 m/sec^2 downward. The speed that would provide this centripetal acceleration can be found from $a = v^2/R$:

$$v = \sqrt{aR} = \sqrt{9.8 \text{ m/sec}^2 \times 10^3 \text{ m}}$$
$$= \underline{99 \text{ m/sec}} \text{ (about 220 mi/hr).}$$

The pilot is said to be weightless because he does not experience any support for his weight such as normally would be supplied by the floor under a standing man.

From $a = v^2/R$ we can see than an increase in velocity (and/or a decrease in radius) will increase the acceleration. Hence if the velocity of a plane in a loop of radius 1 km is greater than 99 m/sec, the centripetal acceleration will be greater than 9.8 m/sec^2 and the pilot will be pushed down by the seat when he is at the top of the loop. A lesser velocity will result in less acceleration, and the pilot will hang from his seat belt.

12–21
Some people speak of an astronaut re-entering the atmosphere as weighing several times his own weight. We sometimes say that an astronaut, before re-entering the atmosphere, is weightless.
(a) Do these statements make sense in the light of our definition of weight as the force of gravity on an object?
(b) What is meant by "several times his own weight" and "weightless" above? Be prepared to discuss these questions in class.

This problem calls for a qualitative discussion which is related to the quantitative calculation of apparent weightlessness in Problem 20, and of apparent extra weight in Problem 22.
(a) No; the force of gravity on a given mass depends only on the gravitational field strength at its position.
(b) "Several times his own weight" and "weightless" refer to the force between the astronaut and the surface against which he is resting. Prior to entry, the space vehicle and everything in it are in free fall, and the conditions discussed in Section 12–12 apply; there is no force between the astronaut and any part of the space vehicle; the sensation is one of having no weight. When the space vehicle

enters the atmosphere, the resistance of the air exceeds the force of gravity on it, so that it decelerates. Everything within must decelerate at the same rate; hence the surface against which the astronaut rests must push against him with the force necessary to make his deceleration equal to that of the space vehicle. This force must be larger than the force of gravity, and during the entry will usually be several times the gravitational force. The sensation of this astronaut is the same as though his weight had increased several times.

12–22

A string 5.0 m long of diameter 2.0 mm just supports a hanging ball without breaking.

(a) If the ball is set to swinging, the string will break. Why?

(b) What diameter string of the same material should be used if the ball travels 7.0 m/sec at the bottom of its swing?

This problem concerns the net force required to produce centripetal acceleration.

(a) When the ball is swinging, the tension in the string must not only balance the weight of the ball but it must supply the required centripetal force. Apparently, the string breaks under a tension greater than the weight of the ball.

(b) The tension is greatest when the ball is at the bottom of its swing for at this point its weight is entirely along the string and its speed v is greatest (as it starts to rise again it will slow down under gravity).

The maximum value of the tension is

$$T = mg + mv^2/R = m(g + v^2/R).$$

From our values of v and R, we find

$$\frac{v^2}{R} = \frac{(7.0)^2}{5.0} = \frac{49}{5.0} = 9.8 \text{ m/sec}^2 = g.$$

Hence the tension is

$$T = 2mg.$$

Thus, the string tension for the swinging pendulum is twice the string tension for the pendulum at rest. The string for the swinging pendulum should be twice as strong. In order to provide double the strength, the string must have twice the cross-sectional area.

$$A_{new} = 2A_{old}, \text{ or } \frac{\pi d^2_{new}}{4} = 2 \times \frac{\pi d^2_{old}}{4},$$

$$\text{or } d^2_{new} = 2 \times d^2_{old}$$

$$d_{new} = \sqrt{2} \times d_{old} = \sqrt{2} \times 2.0 \text{ mm}$$

$$= \underline{2.8 \text{ mm.}}$$

12–23

The gravitational field at the surface of the moon is $\frac{1}{6}$ of that at the surface of the earth. The radius of the moon is about $\frac{1}{4}$ of the radius of the earth. Calculate the period of a lunar landing module circling the moon close to the surface.

From Section 12–6,

$$\frac{v^2}{R} = g \text{ and } vT = 2\pi R$$

Hence, $T = 2\pi \sqrt{\frac{R}{g}}$.

In Section 12–6, the period of an earth satellite was 5.6×10^3 sec. Since $R_M = \frac{1}{4} R_E$, and $g_M = \frac{1}{6} g_E$, the period of a satellite near the moon is equal to $5.6 \times 10^3 \times \sqrt{\frac{6}{4}} = 6.9 \times 10^3$ sec.

12–24

An electron (mass $= 0.91 \times 10^{-30}$ kg) under the action of a magnetic force moves in a circle of 2.0 cm radius at a speed of 3.0×10^6 m/sec. At what speed will a proton (mass $= 1.6 \times 10^{-27}$ kg) move in a circle of the same radius if it is acted upon by the same force?

For the electron, $F_e = \frac{m_e v_e^2}{R}$. For the proton, $F_p = \frac{m_p v_p^2}{R}$. We are given that $F_e = F_p$. Hence, $\frac{m_e v_e^2}{R} = \frac{m_p v_p^2}{R}$, or $m_e v_e^2 = m_p v_p^2$, and $v_p^2 = \frac{m_p v_p^2}{m_p}$.

Then

$$v_p = v_e \sqrt{\frac{m_e}{m_p}} = (3.0 \times 10^6) \sqrt{\frac{0.91 \times 10^{-30}}{1.6 \times 10^{-27}}}$$

$$= \underline{7.2 \times 10^4 \text{ m/sec.}}$$

12–25

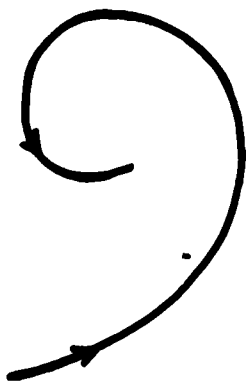
A body moving with a speed v is acted upon by a force that always acts perpendicular to the motion of the body but increases steadily in magnitude.

(a) Draw a sketch of the trajectory.

(b) Does the speed of the object increase, decrease, or remain unchanged?

This problem on circular motion should be handled qualitatively. A quantitative treatment will be beyond most high school students (or anyone else).

(a) We know that if the force were *constant* and perpendicular to the velocity, the body would travel in a circle with an acceleration of F/m perpendicular to its path. Since the force, and hence the acceleration, is increasing with time, the body is pulled around a sharper and sharper curve. In the figure we have sketched a path for such a force.



The exact solution of this motion shows that the path of the particle, for a force increasing linearly with time, is the "Cornu spiral," which is well known in optics. The exact solution is much too complicated to be undertaken with a high school class, so do not encourage students to do more than describe the qualitative characteristics of the motion.

(b) The force acting on the body is always perpendicular to the velocity. Thus the acceleration produced by this force is perpendicular to the velocity at all points. The magnitude of the velocity therefore does not change, but its direction does change.

12–26*

How must the force on an object vary with the displacement if simple harmonic motion is to take place? (Section 8.)

$\vec{\mathbf{F}} = -k\vec{\mathbf{x}}$ for some positive constant k.

12–27*

At what points on the circle in Fig. 12–16 will P have the same velocity as N? (Section 8.)

At the points where the circle intersects the y-axis.

12–28*

If the force constant of a spring is 8 newtons/meter, what suspended mass will give a period of 1 sec? (Section 8.)

$m = 0.2$ kg.

12–29*

What must be the length of a simple pendulum if it is to have a period of one second? (Section 8.)

$l = 0.25$ m.

12–30

A block of mass m rests on a horizontal platform. The platform is driven vertically in simple harmonic motion with an amplitude of 0.098 meter. When at the top of its path, the block just leaves the surface of the platform. (This means that at this point its acceleration is 9.8 m/sec^2 downward.)

(a) What is the period of the simple harmonic motion?

(b) When the block is at the bottom point of its path, what is its acceleration?

(c) What is the force exerted by the platform on the block at this bottom point?

(a) Since the block moves with the platform, its motion must also be simple harmonic, so the net force on it is

$$F_{\text{net}} = -ky,$$

where y is the displacement of the block from the midpoint of its path and the upward direction is taken positive. Then at the top of the path, where $y = R$ (the amplitude of the oscillation),

$$ma = -mg = -kR, \text{ and } k = mg/R.$$

The period T of the motion is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mR}{mg}} = 2\pi\sqrt{\frac{R}{g}}$$
$$= 2(3.14)\frac{0.098}{9.8} = \underline{0.63 \text{ sec.}}$$

Alternate method

The period of the simple harmonic motion is the same as the period of the matching circular motion about a circle of radius $R =$ 0.098 meter. For the circular motion,

$a = 4\pi^2 R/T^2$ (see Section 5)

so

$$T = 2\pi\sqrt{R/a} = 2(3.14)\left(\frac{0.098}{9.8}\right)^{1/2} = \underline{0.63 \text{ sec.}}$$

(b) At the top of the path the acceleration of the block is $-g$. At the bottom the acceleration must have the same magnitude but the opposite direction, so it is g (upward).

(c) At the bottom point, the *net* force on the block is $+mg$. This is the resultant of the weight $-mg$ and the force exerted by the platform:

$$F_{net} = mg = F_{plat} - mg$$
$$F_{plat} = \underline{2\ mg} \text{ (upward).}$$

12–31

(a) What is the period of a pendulum consisting of a mass of 2.0 kg suspended on a light cord 2.4 meters long if $g = 9.8$ newtons/kg?

(b) Notice that we can use a pendulum to measure g. If the period of this pendulum is 2.0 sec, what is g?

This problem illustrates how a pendulum can be used to measure g quite accurately.

(a) The period T for a simple pendulum is given by the equation

$$T = 2\pi\sqrt{\frac{l}{g}} = 2 \times 3.14\left(\frac{2.4}{9.8}\right)^{1/2}$$
$$= 6.28 \times 0.495 = \underline{3.1 \text{ sec.}}$$

(b) If l remains 2.4 m, but T becomes 3.0 sec instead of 3.1 sec, what would g have to be? The above relation gives:

$$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g},$$

and

$$g = l\left(\frac{2\pi}{T}\right)^2 = 2.4\left(\frac{6.28}{3.00}\right)^2 = \underline{10.5 \text{ nt/kg.}}$$

Be sure that students realize how easy it would be to find this difference. If the swings of the pendulum are counted for six minutes, for example, there would be 116 swings for part (a) and 120 swings for part (b). It is easy to set up a simple pendulum as a class demonstration. Accurate values of the period can be obtained by timing a large number of swings. You can verify that the period varies as $\sqrt{l}$, and also obtain a surprisingly accurate value for g.

The formula we have used was derived under the assumption that the angle of swing is "small." If the angle of swing is kept less than about 30°, the error will not be appreciable.

12–32

A 2-kg ball is suspended from a spring. When disturbed in a vertical direction, the ball moves up and down in simple harmonic motion at a frequency of 4 cycles per second.

(a) What is the period of the motion?

(b) What is the upward force exerted on the ball by the spring when the ball is at the midpoint of its up-and-down path?

(c) How much did the spring stretch when the ball was first attached to its end (before the oscillatory motion was started)?

(a) $T = \frac{1}{f} = \underline{1/4 \text{ sec.}}$

(b) At the midpoint of the path, the acceleration of the ball is zero. Therefore, there is no net force on the ball, and the upward force exerted by the spring at this point must be exactly equal to the downward force of gravity:

$$2 \text{ kg} \times 9.8 \text{ m/sec}^2 = \underline{20 \text{ newtons.}}$$

(c) Since $T = 2\pi\sqrt{m/k}$, $k = 4\pi^2 m/T^2$.
When the ball is attached to the spring in equilibrium, the spring must stretch an amount $x_0 = F/k = mg/k$. Thus,

$$x_0 = \frac{mg}{4\pi^2 m/T^2} = \frac{gT^2}{4\pi^2}$$
$$= \frac{9.8 \text{ m/sec}^2 \times (0.25 \text{ sec})^2}{4\pi^2} = \underline{0.016 \text{ m.}}$$

12–33

A ball is thrown vertically upward with a velocity of 24 meters/sec from a railroad flat car moving horizontally with a velocity of 4 meters/sec. Describe the path of the ball as seen by an observer (a) on the flat car, and (b) by the observer on the ground nearby.

This problem asks the student to describe the motion of an object when seen from two different reference frames. The effect of the air on the ball is neglected in the solution given below.

(a) The ball appears to go straight up and down when observed from a position on the flat car.

(b) An observer on the ground will see a constant horizontal velocity superimposed on the up and down velocity of the ball. This is just like the projectile motion we studied in Section 12–4, and the path is a parabola (with both branches).

12–34

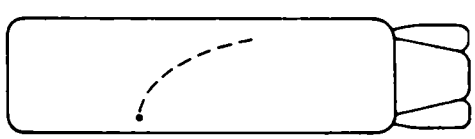
While a bus is in motion along a level, straight road, we roll a marble from one side to the other across the floor of the bus. Its path is a straight line relative to the bus. Later we roll it again, and this time the path is a parabola which bends toward the front of the bus (concave toward the front of the bus). Describe the motion of the bus in each case. Be prepared to discuss in class.

This is a problem involving relative motion and the effect of a moving or accelerating frame of reference.

In solving this problem, one must realize (idealize) that the marble is not subject to any force from the bus. Thus, while it is rolling, the marble must move in a straight line *relative to an inertial system such as the fixed stars.* The marble can, of course, have any initial velocity but that velocity then cannot change.

When the marble moves in a straight line relative to the bus, a man in the bus sees no marble acceleration, so he concludes that no force is acting on the marble. We have already noted that in an inertial frame, no force acts on the marble, so in this case the bus must be an inertial frame; i.e., it is moving at constant velocity (or is at rest).

When the marble moves in a parabola toward the front of the bus, the observer in the bus sees a constant apparent acceleration, $\vec{\mathbf{a}}$, (relative to the bus) toward the front of the bus. He concludes that a force $F = ma$ is pulling the marble toward the front of the bus.



However, we know that in an inertial frame the marble has no force acting on it. Therefore we can conclude that rather than the marble accelerating forward, the bus must be accelerating backward in the inertial frame.

This is a deceleration, and it must have the value $\vec{\mathbf{a}}$ derived (left) by the rider. The force deduced by the rider is then what we call a fictitious force. In the noninertial frame it must be assumed to be acting in order to understand the observed motion, but in an inertial frame it does not exist.

12–35

A long-range gun is located in the Northern Hemisphere. You wish to hit a point due north of it. In what direction should you aim the gun? (Section 11.)

Slightly west of due north.

Every point on the earth's surface (except at the poles) has a linear speed due to the rotation of the earth; this speed depends on the latitude of the point, being maximum at the equator and minimum at the poles. When the gun is fired, the velocity is the vector sum of the northward velocity imparted by the gun and the west-to-east velocity imparted by the rotating earth. As the projectile moves toward the north, it retains unchanged this west-to-east velocity. As the points on the ground over which it passes are at increasingly greater latitudes, these points have increasingly smaller west-to-east velocities due to the earth's rotation. Thus the projectile continually moves to the east with respect to the longitude from which it was fired. Hence to hit a point on the same longitude as the gun, the gun must initially be pointed slightly to the west of north.

(Actually, other factors also influence the exact trajectory of the projectile, and an accurate prediction of its path is complex.)

12–36

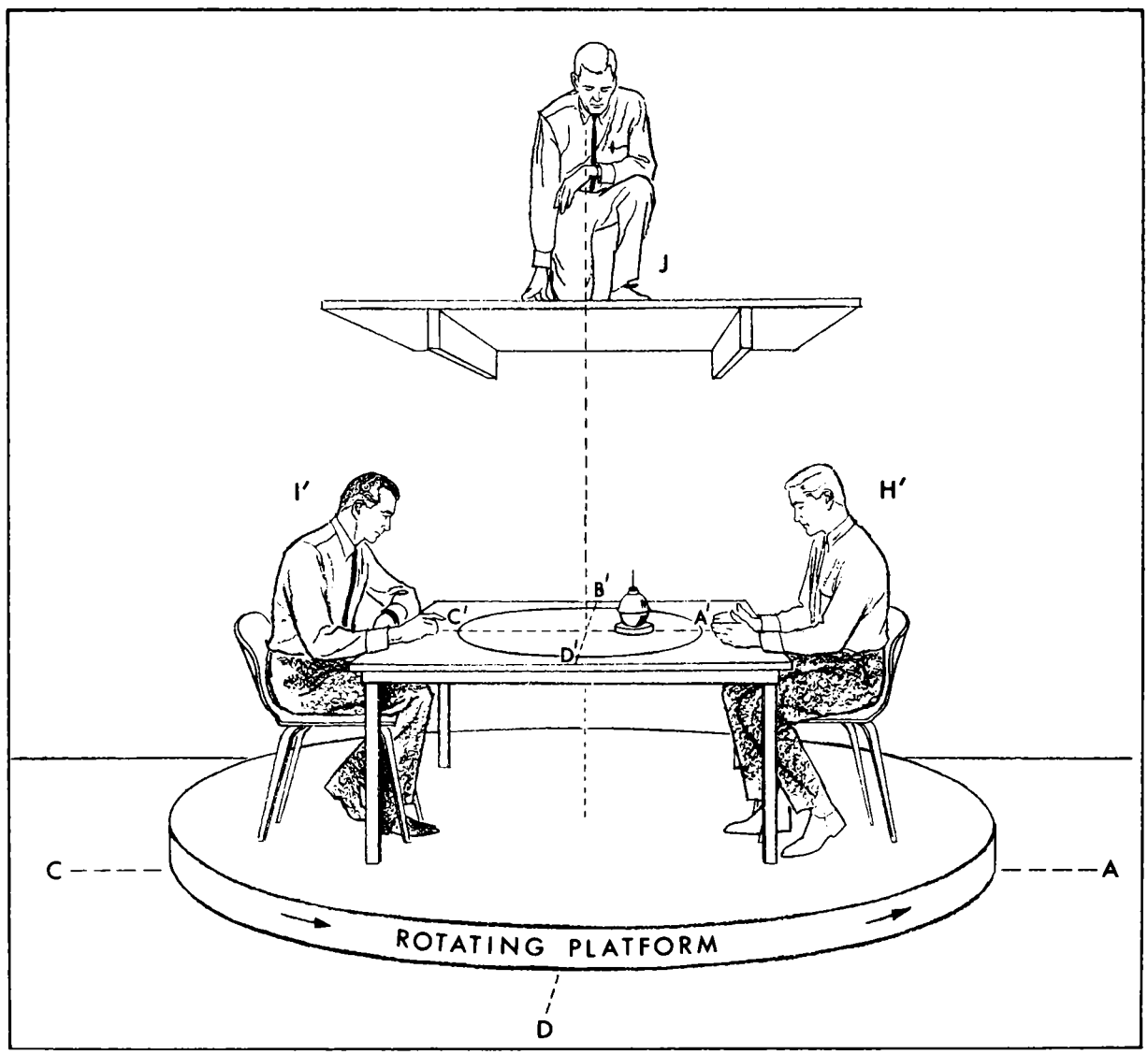
When a spacecraft is moving through space on its way from the earth to the moon, the gravitational force of the earth will at first exceed the gravitational force of the moon. Later on, when the spacecraft is nearer to the moon, the gravitational force of the moon will exceed the gravitational force of the earth. In the science-fiction story "From the Earth to the Moon," written about 100 years ago, Jules Verne has his astronauts falling from the rear end to the front end of the spacecraft when it passed the point in space where the gravitational force from the earth was equal to the gravitational force from the moon. Do you think Verne had a correct understanding of the physics of the case? Explain how you think the men in the spacecraft would feel when they passed that point in space.

As discussed in Section 12–12, the frame of reference of a coasting spacecraft behaves just like an inertial frame of reference with no gravitational forces. It does not matter that the gravitational force acting on the spacecraft is the net force due to the gravitational fields of the sun, earth, and moon (and indeed of all other bodies in the universe). Verne's understanding of this case was deficient, as no such effects as those he described are observed.

The astronauts will be "weightless" at all times when the spacecraft is coasting between the earth and the moon.

12–37

For some experiments in relative motion, a smooth, level table, shown in Fig. 12–22, is centered on a platform which rotates uniformly at a rate of one revolution in 12 seconds. Two perpendicular lines are drawn through the center of the table, intersecting a circle of 1.20-meter radius at the points A', C', B', and D'. Two men, H' and I', sit on the platform at opposite ends of the line $A'C'$. A third man, J, is above the table so that he can observe the motion of a Dry Ice puck in a stationary frame of reference. He has four marks on the floor, forming two perpendicular reference lines AC and BD through the center of the table. (In the diagram, B is behind the table.)



(a) H' holds the puck in his hand at A'. What is its velocity in J's frame of reference at the instant A' crosses the line AC?

(b) As H' passes A he gives the puck a sudden push so that it travels along line AC with a speed of 0.40 m/sec. Construct a vector diagram to show the velocity that H' gave the puck in J's frame of reference.

(c) Construct a diagram which shows the positions of the puck and of A' at 1, 2, 3, 4, 5, and 6 seconds in J's frame of reference.

(d) Make use of the diagram constructed for (c) to construct a diagram which will show the motion of the puck on the table, as seen by H' and I'. (*Hint:* Construct this moving-frame diagram on tracing paper which can be laid over the diagram of the fixed-frame motion. Plot the position of the puck for each 15° rotation of the moving frame. These points can be connected to give a picture of the motion in the moving frame.)

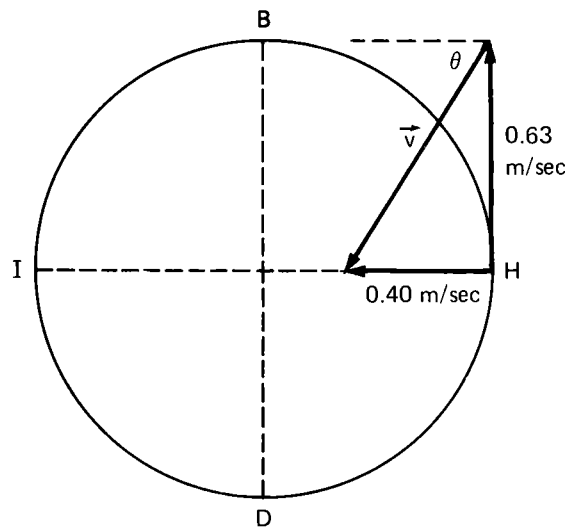
It would be best to give this problem after your students have seen the film "Frames of Reference."

(a) In J's frame, the puck is moving with a speed of

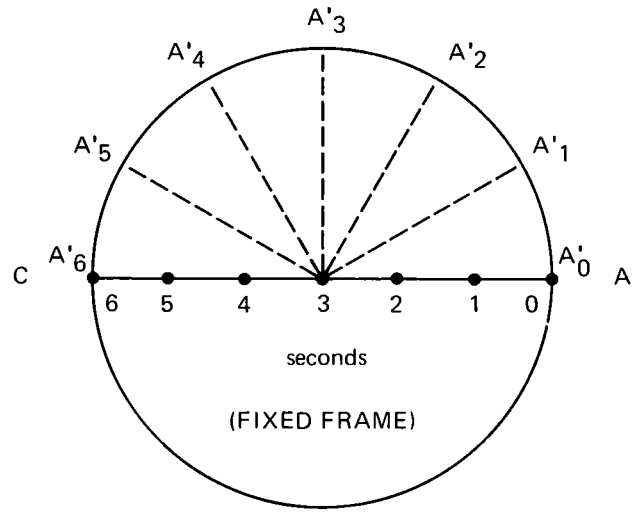
$$\frac{2\pi \times 1.20}{12.0} = \underline{0.63 \text{ m/sec.}}$$

It is moving parallel to the line DB in the direction DB.

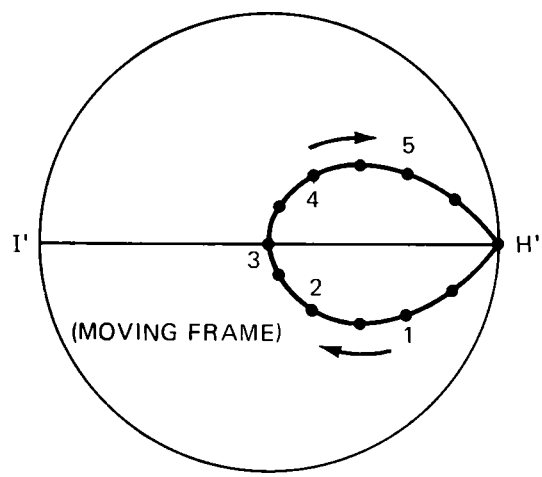
(b) H' must give the puck a velocity v of 0.74 m/sec at an angle, θ, of 57° from the line AC pointing toward the side of the table which lies at D.



(c) In J's frame of reference, the puck moves along line AC with a constant speed of 0.40 m/sec and will be in the position shown at each second of time. H' will be at the corresponding positions A'_1, A'_2, etc.



(d) By rotating a piece of tracing paper (the moving frame) over the above diagram (fixed frame) the points 1, 2, 3, . . . can be plotted as H' is at A'_1, A'_2, A'_3, ... Points between these can be plotted by placing H' at estimated midpoints of arc $A'_0A'_1$, $A'_1A'_2$, etc. Connecting these points gives the path H' and I' seen in the moving frame.



12–38

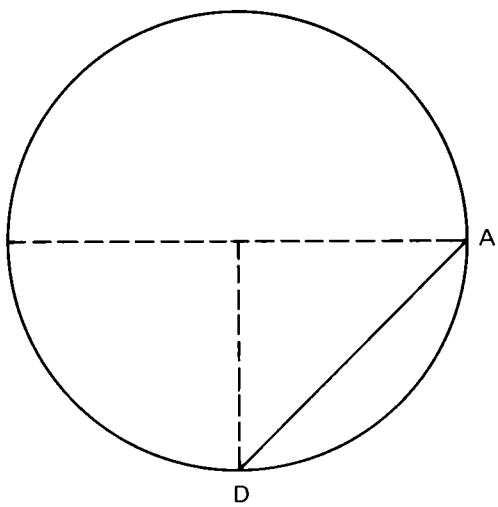
With the same situation as in Problem 37:

(a) Suppose that H' launches the puck as he passes A so that I' will catch the puck as he passes D. Construct a diagram which shows the motion of the puck in J's frame of reference. What is the speed of the puck in this frame of reference?

(b) Make use of the diagram constructed for (a) to construct a diagram which will show the motion of the puck on the table as seen by H' and I'.

(c) Construct a vector diagram to show the velocity that H' gave the puck so that it could travel as in (a).

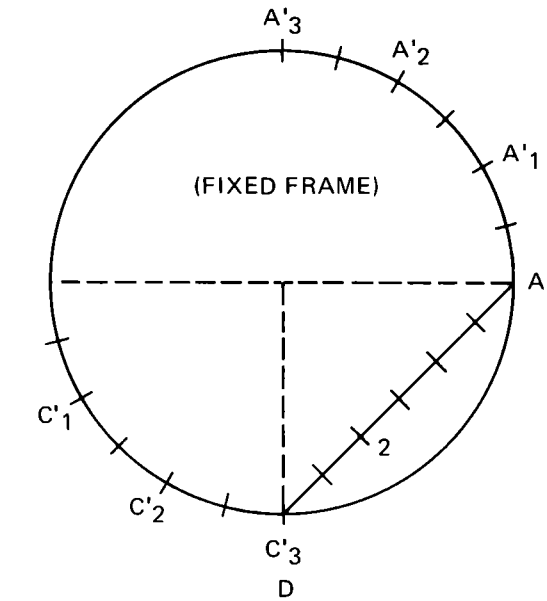
(a) The path of the puck in J's frame of reference will be the chord of one-quarter circle of radius 1.20 meters, which is 1.70 meters. The speed is 1.70 m/3 sec = 0.57 m/sec.

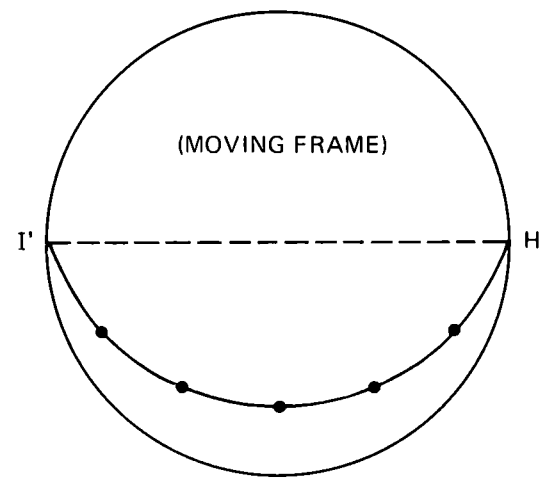


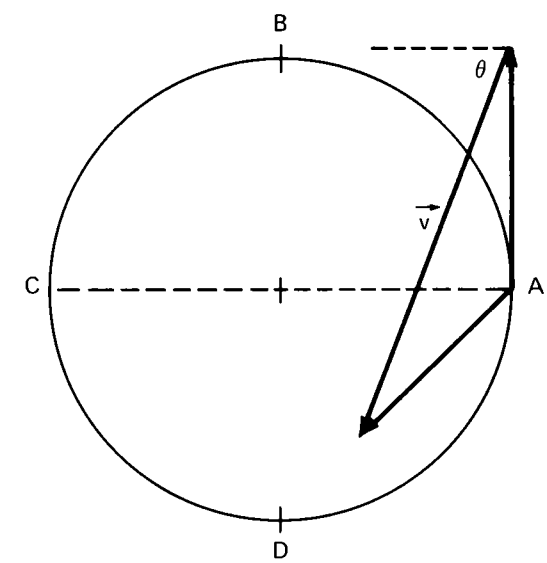
(b) On the fixed frame diagram repeated in (1) above, mark the positions of the puck at half-second intervals and the positions of H' and I'.

By rotating the tracing paper with the circle representing the moving frame, diagram (2), over the fixed frame (1), the positions of the puck in the moving frame can be plotted.

(c) H' had to give the puck velocity $\vec{v}$ as shown in the diagram. The value is 1.10 m/sec at an angle of 69° from line CA in the direction D.







12–39

With the same situation as in Problem 37:

(a) With what speed and in what direction can H' launch the puck as he passes A so that, as J sees it, the puck remains at A?

(b) What is the motion of the puck as seen by H' and I'?

(c) The puck has a mass of 0.50 kg. After H' has let go of it, what force acts on it according to J? What force acts on it according to H' and I' to make it move as it does in their frame of reference?

(a) As in Problem 37(a), the puck has a velocity in J's frame of 0.63 m/sec parallel to the line DB and in the direction DB while H' is holding it. To make it stand still in J's frame; H' must give it a velocity of 0.63 m/sec perpendicular to the line $A'C'$ and toward his own left.

(b) In the frame of H' and I' the puck will move along the circumference of the table with a steady speed of 0.63 m/sec.

(c) Since J sees the puck at rest, he says that no force is acting on it. Since H' and I' see it moving around a circle with a steady speed, they say that there is a force on it of

$$\frac{0.50 \times (0.63)^2}{1.20} = 0.17 \text{ newton},$$

directed toward the center of the table.

This 0.17 newton is the fictitious force which H' and I' must invent to make Newton's law of motion apply in their frame of reference. For the particular motion of this problem the fictitious force is constant in magnitude and is directed toward the center of the table. The fictitious force in Problems 37 and 38 is more complex. In Problem 38, to a good approximation, we can treat the motion of the puck in the rotating frame as though it moved at a constant speed along a circle whose center is displaced along the line $B'D'$ in the direction of B'. To the same approximation, the fictitious force is constant in magnitude and directed to this fixed center, which is not, however, at the center of the table. In Problem 37, the path of the puck is a spiral and the force cannot be expressed in simple terms.

CHAPTER 13

13–1

Three fireflies X, Y, and Z are on a moving bicycle at night. X is at the very center of one of the axles, which turns with the wheel. Y is on the rim of the wheel. Z is on the frame of the bike outside the circumference of the rim. Draw sketches and use a few words to describe the motions

(a) of X and Y as viewed by Z.

(b) of Y and Z as viewed by X.

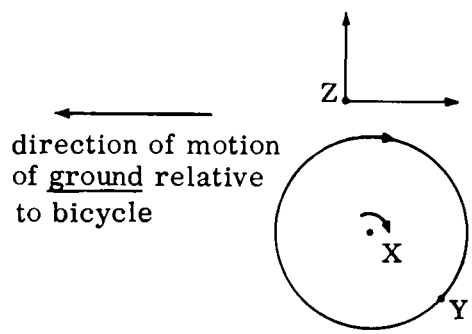
(c) of X and Z as viewed by Y.

(d) of all three as seen by an observer standing near the bicycle.

This moderately difficult problem is best used for class discussion. It can be used to illustrate different reference frames—an example of the problems involved in relating heliocentric and geocentric descriptions of the universe.

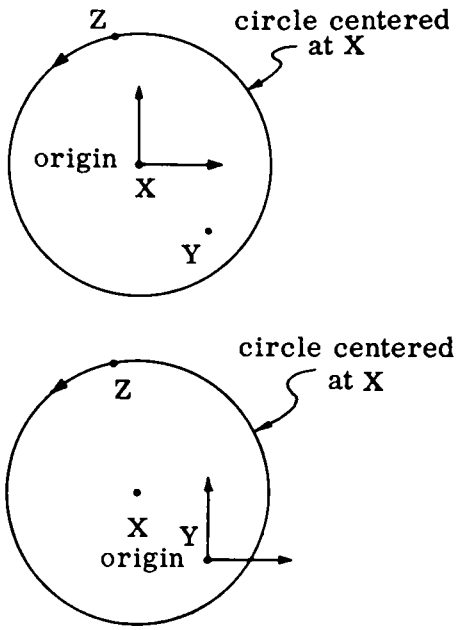
In this problem we are given four reference systems. We can most easily think of them in terms of the origin and orientation of their coordinate axes. If we assume that Z, X, and the circle described by Y are all in the same plane, two coordinates are enough.

(a) From Z's position on the frame of the bicycle, outside the wheel, X does not move, but spins at a fixed position in space. Y describes a circle around X, but since Z is in the plane of the circle, it will appear to Z that Y is in simple harmonic motion.



(b) From X's point of view, spinning with the axle, it appears that Y, on the wheel rim, is at rest. Z on the frame is at a fixed distance from X. Hence, as X rotates with the axle, Z seems to revolve around X in the opposite sense.

(c) From Y's point of view on the rim, X will



appear to be at rest, but Z moves in a circle centered at X.

(d) For an observer standing near the bicycle, Z, on the bicycle frame, moves with uniform speed in a straight horizontal line. X, on the axle, also moves with uniform speed in a straight horizontal line, but is spinning. Y, on the wheel rim, moves in a cycloid. As the wheel rolls, Y's horizontal velocity is zero at the instant Y's part of the wheel is in contact with the ground. Thus we get the figure at the bottom of the next page.

The simplest way of describing the motion of Y is to say that Y moves in a circle relative to the axes which are fixed to the bicycle frame [part (a)], and that these axes move horizontally at a constant speed without turning relative to the ground.

13–2*

(a) In Fig. 13–5, what is the ratio of the greatest distance to the shortest distance between the earth and Mars?

(b) What about Fig. 13–8? (Section 4.)

(a) 1.05;

(b) About 6.

13–3

The fixed stars go around in about one day as seen by an observer on the earth. About how long do the following revolutions appear to take?

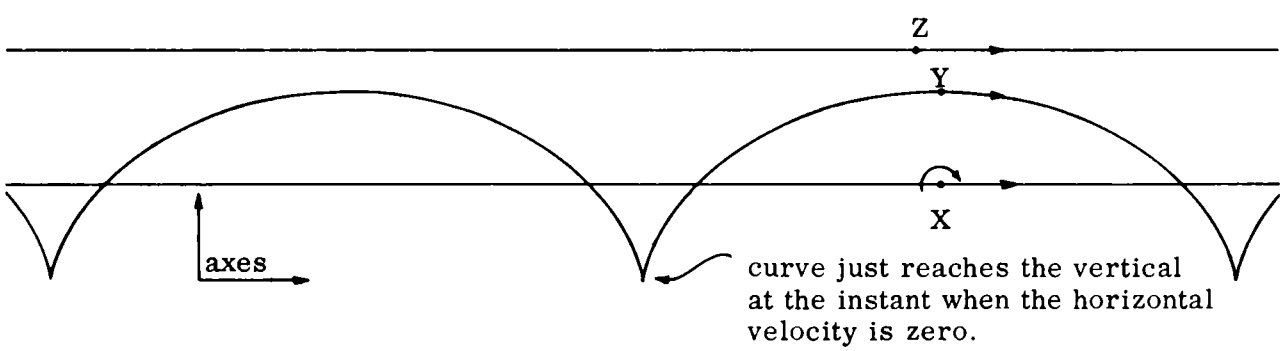
(a) The sphere of the fixed stars as seen by someone on the moon. (From the earth, we always see the same side of the moon.)

(b) The earth about the moon as seen from the moon.

(c) The sun as seen from the moon. Does the sun appear to rotate as if it were one of the fixed stars? Does it appear to rotate faster or slower? Remember that the earth and moon go around the sun once in every year.

This question can be answered at several different levels. At this stage a brief qualitative answer should be expected.

(a) We know that, because the earth rotates on its axis, the stars appear to go around the earth once a day. But what about the stars



as seen from the moon? Does the moon rotate on its axis once a day as does the earth?

The key is in properly interpreting the fact that, from the earth, we always see the same "face" of the moon. As the moon travels through an angle θ in its revolution around the earth, the moon must also rotate on its axis through the same angle in order to keep the same "face" toward the earth. Hence the period of the moon's rotation on its axis is the same as the period of the moon's revolution around the earth. In earlier science work some students may have learned this period as 28 days (more accurately, 27.3 days).

(b) Since the moon keeps the same "face" toward the earth, an observer on the moon would always see the earth in the same position in the sky. From the moon, the earth would not appear to be revolving around the moon.

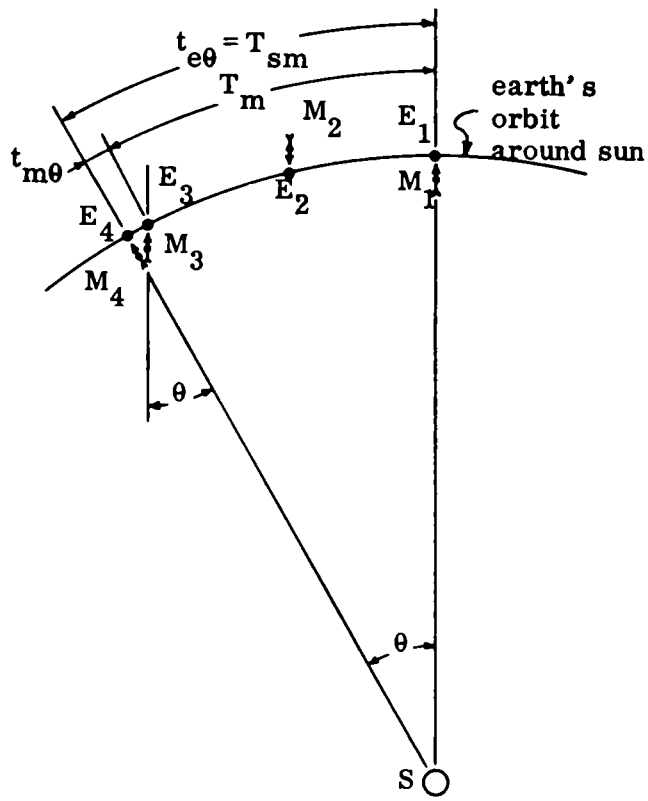
(c) Since from part (a) we found that the moon rotates on its axis about once in four weeks, this is the approximate time required for the sun to move from one "high noon" position to the next as seen from a particular point on the moon. If the moon were only rotating on its own axis, and not moving around the sun, then for a moon man the sun would appear to move with the fixed stars. However, because the moon is going around the sun as well as rotating on its own axis, the moon must make a little more than one revolution on its axis in order for the sun to reappear at a "high noon" position. Hence, the sun will appear to rotate around the moon a little more slowly than the fixed stars. (The actual rate at which the sun will appear to rotate around the moon is once in 29.5 days.)

13–4

Does Tycho Brahe's geocentric system account for the fact that there is a maximum angle between Venus and the sun as seen from the earth?

Yes; from the diagram in Figure 13–8 one can see that the maximum angle between the direction from the earth to the sun and the direction from the earth to Venus is about 38°.

In Tycho's system, the sun, together with all the planets, revolves around the earth once in 24 hours. Hence there is a time interval of $38°/360°$ (24 hr) = 2.5 hr between the time we see the sun and the time we see Venus at the same position in our sky, when they are at this maximum angular separation. Since we cannot see any of the stars or planets during the daytime, this means that the latest we can see Venus is 2.5 hr after sunset, and the earliest is 2.5 hr before sunrise. It would never be visible more than 38° above the horizon.



The Copernican system makes essentially the same predictions about Venus (and Mercury) as does Tycho's system (see Figure 13–7).

13–5

The earth's orbit around the sun is nearly circular, and the moon moves in a nearly circular orbit around the earth. The radius of the earth's orbit is 1.5×10^{11} meters, and that of the moon's orbit is 4×10^8 meters.

(a) How often is the moon between the earth and the sun?

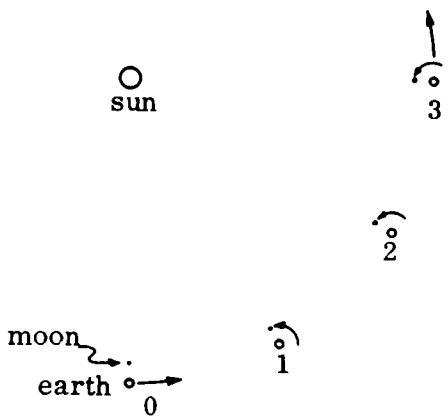
(b) How far has the moon moved around the sun in the interval between two successive times when the moon is between the earth and the sun?

(c) Sketch the orbit of the earth going around the sun and on the same figure sketch the orbit of the moon going around the sun.

(d) Does the moon show retrograde motion as seen by an observer at the sun? (Retrograde motion is motion like that shown in Fig. 13–2.)

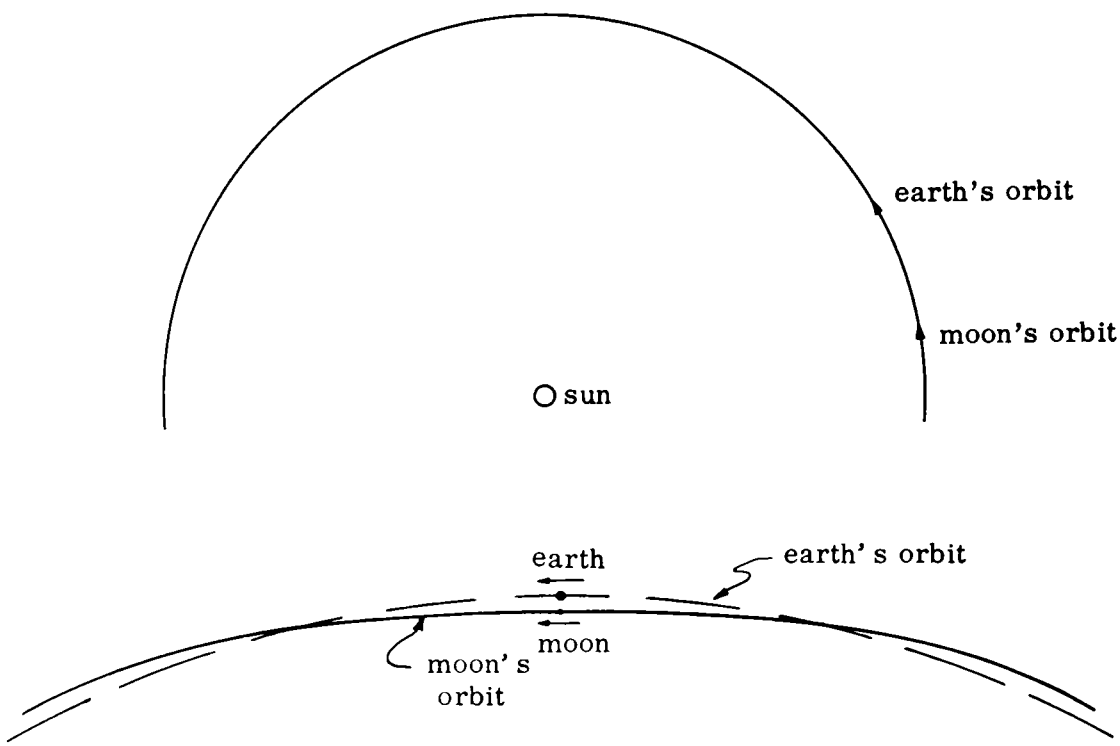
(a) Since the moon takes about four weeks to go around the earth, and the earth takes 52 weeks to go around the sun, the moon revolves around the earth about 13 times in one year. (This can also be found from the periods for the earth and moon given in the solar system table on page 284 in the text:

$$\frac{T_e}{T_m} = \frac{3.16 \times 10^7}{2.36 \times 10^6} = 13.4.)$$



In these annual motions of the earth and moon, the moon will be between the earth and sun 12 times. This can be seen from the diagram which shows the positions of earth, sun, and moon at one-month intervals for 3 months or $\frac{1}{4}$ of a year. It can be seen that during the time the earth has moved from 0 to 1, the moon must make a little more than one revolution of the earth to be between earth and sun. During the $\frac{1}{4}$ year required for the earth to move from 0 to 3, the moon must make an extra quarter of a revolution around the earth in order again to be between the earth and sun. Therefore, a total of 3.25 revolutions per quarter year are required for the moon to appear between the earth and the sun 3 times. Hence in one year (13 revolutions of the moon around the earth) the moon will be between the earth and sun 12 times or once a month. (A more precise answer is once every 29.5 days.) A similar argument shows that the earth turns on its axis 367 times during leap year and 366 times during other years.

(b) During one month, the moon (and earth) have moved $\frac{1}{12} \times$ circumference of the earth's orbit, or $\frac{1}{12} \times 2\pi \times 1.5 \times 10^{11}\text{m} \approx$

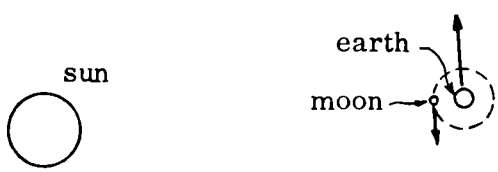


8×10^{10}m. (More precisely, $29.5/365 \times 2\pi \times 1.5 \times 10^{11}$ m $= 7.6 \times 10^{10}$ m.)

(c) In the sketch (left), the earth's orbit around the sun has a radius of 2 inches. The ratio of the radius of the moon's orbit around the earth to that of the earth around the sun is $3.8 \times 10^8/1.5 \times 10^{11} = 1/400$; therefore the moon's orbit around the earth, on the same sketch, would have a radius of 2/400 inches. This would be less than the width of the line used to indicate the earth's orbit.

Frequently diagrams are drawn showing the moon weaving back and forth across the earth's orbit. These are seldom correct, since the moon's orbit around the sun is always concave toward the sun. This can be seen from the fact that the gravitational force between the sun and moon is about twice as great as the force between the earth and the moon. Hence the moon is always accelerated toward the sun. The moon's acceleration toward the earth is 2.7×10^{-3} m/sec^2; its acceleration toward the sun is 5.9×10^{-3} m/sec^2. Thus even when the moon is between the earth and sun, as shown in the diagram, the net acceleration of the moon is toward the sun.

(d) To an observer on the sun, the moon would not show retrograde (reverse) motion. If the moon were to show retrograde motion, its speed "backward" at a point such as shown in the diagram would have to be greater than the "forward" speed of the earth-moon system.



The speed of the earth-moon system in its orbit around the sun is:

$$\frac{2\pi \times 1.5 \times 10^{11}\text{ m}}{365 \times 8.6 \times 10^4\text{ sec}} \approx 30\text{ km/sec.}$$

The speed of the moon relative to the earth is:

$$\frac{2\pi \times 3.8 \times 10^8\text{ m}}{27 \times 8.6 \times 10^4\text{ sec}} \approx 1\text{ km/sec.}$$

Since the moon and earth travel around the sun at 30 km/sec, the moon's speed of 1 km/sec around the earth is too small to cause retrograde motion.

13–6*

What area is swept out by Jupiter in making one fourth of a revolution around the sun? (See Table 2.) (Section 5.)

4.75×10^{23} m^2.

13–7

How fast, in m^2/sec, is area swept out by the radius from sun to earth?

This is a straightforward problem related to Kepler's second law. Using the data given in Problem 5:

Area swept out by radius from earth to sun per second $= \pi R^2/T$, where T is the time in seconds for one complete revolution. Since the earth revolves once in 365 days (3.2×10^7 sec), the area swept out by earth-to-sun radius per second =

$$\frac{\pi \times (1.5 \times 10^{11}\text{ m})^2}{3.2 \times 10^7\text{ sec}} = 2.2 \times 10^{15}\text{ m}^2\text{/sec.}$$

13–8*

Figure 13–12 shows the orbits of the planets about the sun. When does Pluto move

(a) fastest?

(b) slowest? (Section 5.)

(a) When it is closest to the sun.

(b) When it is farthest from the sun.

13–9*

Table 1 does not include Uranus, Neptune, and Pluto. What value do you predict for the ratio $\frac{R^3}{T^2}$ for each of these planets? (Section 5.)

3.35×10^{18} m^3/sec^2.

13–10*

If a small planet were discovered whose distance from the sun was eight times that of the earth, how many times longer would it take to circle the sun? (Section 5.)

$(8^3)^{1/2} = 23$ times longer.

13–11

Between September 21 and March 21 there are three days fewer than between March 21 and September 21. These are the dates when day and night are of equal length, and between them the earth moves 180° around its orbit with respect to the sun. From this and Kepler's law of equal areas, explain how you can determine the part of the year during which the earth is closest to the sun.

Kepler's law states that the line from the sun to the earth sweeps out equal areas in equal times. When the earth is moving more rapidly along its orbit, this line must be shorter. Since the period from September 21 to March 21 is the shorter, this is the time of the more rapid motion, and so the earth passes closest to the sun during the winter of the Northern Hemisphere.

Because of the inclination of the earth's axis, the sun's radiation falls more slantingly on that hemisphere and for a small number of hours each day. These effects are far stronger than the relatively slight decrease in the distance from the sun, about 3 percent. Therefore, the Northern Hemisphere receives less heat in December than in June.

13–12

Astronomers have observed that Halley's comet has a period of 75 years and that its smallest distance from the sun is 8.9×10^{10} meters, but its greatest distance from the sun cannot be measured because it cannot be seen. Use this information together with the second footnote in Section 21–5 to compute its greatest distance from the sun. (It was Newton who told Halley how to compute the orbit of a comet. Halley found and calculated the orbit and period of the comet that bears his name in the course of a general analysis he made of comets' orbits.)

This problem involves the orbit of a comet, and Kepler's third law. Since we know $\overline{R}^3/T^2$ is constant, and we know T, we can compute $\overline{R}$, which is defined as the average of the shortest and longest radii. Then, knowing the shortest radius, we can compute the longest.

From the table for Kepler's third law, we take $\overline{R}^3/T^2 = 3.4 \times 10^{18}$ m^3/sec^2. Since the period, T, of Halley's comet is 75 years $= 2.4 \times 10^9$ sec, $T^2 = 5.8 \times 10^{18}$ sec^2, and

$$R = \left(3.4 \times 10^{18} \frac{\text{m}^3}{\text{sec}^2} \times 5.8 \times 10^{18} \text{ sec}^2\right)^{1/3}$$

$$= 2.7 \times 10^{12} \text{ m}.$$

Since $\overline{R}$ is defined as $\frac{1}{2}$ the sum of the shortest and longest radii,

$$\overline{R} = \frac{R_{max} + R_{min}}{2},$$

and

$$R_{max} = 2\overline{R} - R_{min} = 2 \times 2.7 \times 10^{12} \text{ m} - 8.9 \times 10^{10} \text{ m} = \underline{5.3 \times 10^{12} \text{ m}}.$$

(The distance of closest approach, 8.9×10^{10} m is negligible compared to the long radius.) If this orbit is drawn to scale together with that of the earth, it is evidently highly elongated, as shown below.

13–13*

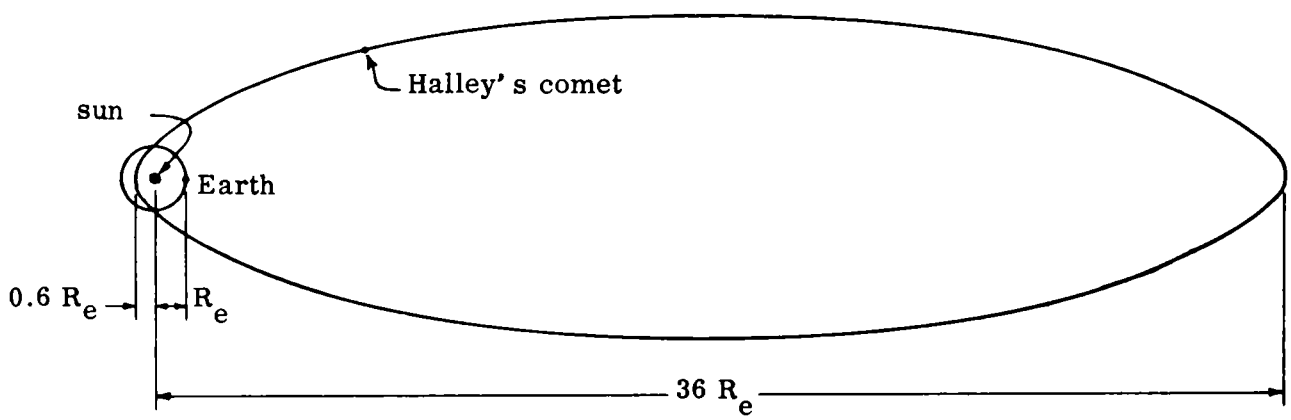
The radius of the moon's orbit is 60 times greater than the radius of the earth. How many times greater is the acceleration of a falling body on the earth than the acceleration of the moon toward the earth? (Section 7.)

3.6×10^3 times greater.

13–14*

(a) Calculate K_c from Table 2.
(b) Using the value of K_s from Table 1 and your answer to (a), determine the ratio of the mass of the earth to the mass of the sun. (Section 8.)

(a) $K_c = 9.9 \times 10^{12}$ m^3/sec^2.
(b) $m_e/m_s = 3.0 \times 10^{-6}$.



13–15*

At what height above the earth's surface will a rocket have half the force of gravitation on it that it would have at sea level? Express your answer in earth radii. (Section 8.)

At a height equal to $(\sqrt{2} - 1) = 0.41$ times the earth's radius.

13–16

(a) At what height will a satellite moving in the plane of the equator stay over one place on the equator of the earth? One way to get the answer is to compare this satellite with the moon, which is at 59.5 earth radii from the center of the earth and takes 27 days to go around the earth.

(b) How fast is the satellite accelerating toward the earth?

(c) Using the inverse-square law and g at the surface of the earth, find the gravitational field strength at the height of the satellite. Compare with your answer to (b).

(a) The satellite will be stationary relative to a point on the earth if its period, T, is the same as that of the earth, very nearly one day (actually 365/366 days).

The radius of the satellite's orbit can be found from Kepler's third law and the knowledge that the moon is 59.5 earth radii from the center of the earth and revolves once every 27 (27.3) days. Let R_s and R_m be the radii of the satellite and moon orbits, and T_s and T_m be their periods:

$$\frac{R_s^3}{T_s^2} = \frac{R_m^3}{T_m^2}.$$

$$R_s = \left(\frac{R_m^3 T_s^2}{T_m^2}\right)^{1/3}$$

$$= \left[\frac{(59.5 \text{ earth radii})^3 \times (1 \text{ day})^2}{(27 \text{ days})^2}\right]^{1/3}$$

$$= 6.6 \text{ earth radii.}$$

Since the radius of the earth is 6.4×10^6 m,

$R_s = 6.6 \times 6.4 \times 10^6 \text{ m} = 4.2 \times 10^7$ m.

The height of the satellite above the earth's surface will be 4.2×10^7 m $- 6.4 \times 10^6$ m $= \underline{3.6 \times 10^7 \text{ m, or about 22,000 miles.}}$

(b) The satellite is accelerating toward the earth with an acceleration:

$$a = \left(\frac{2\pi}{T}\right)^2 R = \frac{4 \times \pi^2 \times 4.2 \times 10^7 \text{ m}}{(1 \times 24 \times 60 \times 60)^2 \text{ sec}^2}$$

$$= \underline{0.22 \text{ m/sec}^2.}$$

(c) The force due to gravity, and thus the acceleration due to gravity, is proportional to $1/R^2$. Thus the acceleration, g, at the height of the satellite and g at the earth's surface are related as follows:

$$g_s = \frac{k}{R_s^2}, \quad g_e = \frac{k}{R_e^2}, \quad \text{where } k \text{ is a proportionality constant,}$$

$$g_s = \frac{g_e R_e^2}{R_s^2} = 9.8 \text{ m/sec}^2 \times \left(\frac{6.4 \times 10^6 \text{ m}}{4.2 \times 10^7 \text{ m}}\right)^2 = \underline{0.23 \text{ m/sec}^2.}$$

Thus, we see in part (b) that the acceleration can be found from R and T without using an inverse square law of force, and in part (c) that it can be found from g at the earth's surface using the inverse square law. These are the two methods Newton used in calculating the acceleration of the moon.

13–17

Find the weight of a 100-kg man on Jupiter.

Since the weight is simply the force of gravity on the man, it equals

$$F = \frac{G \text{ mass}_{Jup} \text{ mass}_{man}}{(\text{rad}_{Jup})^2}$$

$$= \frac{(0.667 \times 10^{-10})(1.90 \times 10^{27})(100)}{(7.18 \times 10^7)^2}$$

$$= \underline{2.46 \times 10^3 \text{ newtons.}}$$

13–18

A 70-kg boy stands 1 meter away from a 60-kg girl. Calculate the force of attraction (gravitational) between them.

$$F = \frac{Gm_1m_2}{R^2}$$

$$= \frac{0.667 \times 10^{-10} \frac{\text{nt} \cdot \text{m}^2}{\text{kg}^2} \times 60 \text{ kg} \times 70 \text{ kg}}{1 \text{ m}^2}$$

$$= \underline{2.8 \times 10^{-7} \text{ newton.}}$$

This force of attraction is only a small fraction of the girl's weight ($F/W = 4.8 \times 10^{-10}$). Obviously, it must be something other than gravity which attracts boys and girls!

13–19

A satellite circles the earth once every 98 minutes at a mean altitude of 500 km. Calculate the mass of the earth. The masses of planets are actually calculated from satellite motions, and one reason for establish-

ing artificial earth satellites is to get a better value for the mass of the earth.

Using the law of universal gravitation, we may equate the centripetal force with the gravitational force on the satellite (if the orbit is stable). Thus, if we designate the mass of the satellite by m_s, and that of the earth by m_e,

$$\frac{m_s 4\pi^2 R_s}{T_s^2} = G\frac{m_s m_e}{R_s^2}$$

$$\therefore m_e = \frac{4\pi^2 R_s^3}{GT_s^2}.$$

Substituting, $R = 6.36 \times 10^6\ \text{m} + 5.00 \times 10^5\ \text{m} = 6.87 \times 10^6\ \text{m}$, and $T = 98\ \text{min} \times 60\ \text{sec/min} = 5.88 \times 10^3\ \text{sec}$.

$$m_e = \frac{4\pi^2 \times (6.87 \times 10^6\ \text{m})^3}{0.667 \times 10^{-10}\ \frac{\text{m}^3}{\text{kg-sec}^2} \times (5.88 \times 10^3\ \text{sec})^2}$$

$$= \underline{5.55 \times 10^{24}\ \text{kg}.}$$

13–20

(a) If T is the period of a satellite circling around just above the surface of a planet whose average density is ρ, show that ρT^2 is a universal constant.
(b) What is the value of the constant?

(a) As in Problem 19, we may write:

$$\frac{m_s 4\pi^2 R}{T^2} = G\frac{m_p m_s}{R^2},$$

and

$$m_p = \frac{4\pi^2 R^3}{GT^2}.$$

If $R \approx R_p$, the radius of the planet, we may write:

$$m_p = \frac{4\pi^2 R_p^3}{GT^2}.$$

But $m_p = \frac{4}{3}\pi R_p^3 \rho$, where ρ = the density. Thus,

$$\frac{4\pi R_p^3 \rho}{3} = \frac{4\pi^2 R_p^3}{GT^2}.$$

Solving for ρT^2, we get:

$$\underline{\rho T^2 = \frac{3\pi}{G},}$$

which is a universal constant since G is a constant.

(b) Evaluating,

$$\rho T^2 = \frac{3\pi}{0.667 \times 10^{-10}\ \text{m}^3/\text{kg-sec}^2}$$

$$= \underline{1.41 \times 10^{11}\ \text{kg-sec}^2/\text{m}^3.}$$

13–21

The earth is acted upon by the gravitational attraction of the sun. Why doesn't the earth fall into the sun? Be prepared to discuss your answer.

This problem is intended primarily for class discussion to help clarify students' thoughts about planetary motion.

The class should understand that the earth is falling *freely* toward the sun. If it started from rest, it would fall on a straight line into the sun. The actual situation may be appreciated by thinking of the problem of bombing a target from a plane. The bomb is not released directly over the target. If it were, it would overshoot the mark. The faster the plane, the greater the overshot. An orbit results when the speed is within a range such that:

(1) The speed is great enough that the overshot causes the bomb to miss the earth;
(2) The speed is not great enough for the bomb to escape completely from the earth's gravitational field.

In the same manner, the earth does not fall into the sun simply because it *is* in a stable orbit; it has a tangential velocity such that it "misses" the sun as it falls toward the sun.

13–22

(a) What is the speed of the moon around the sun compared to that of the earth around the sun?
(b) If the earth could be removed suddenly without disturbing the motion of the moon, what would be the subsequent path of the moon?
(c) Calculate the ratio of the force of attraction exerted by the sun on the moon to the force exerted by the earth on the moon.
(d) Why does the sun not capture the moon, taking it away from the earth?

This is a difficult problem for students to work through with any degree of rigor. The major point to bring out is that both earth *and* moon are attracted to the sun. (The earth and moon revolve about their common mass center which in turn goes about the sun. The center of mass of the earth-moon system is about $\frac{5}{6}$ of the radius of the earth from the center of the earth.)

(a) In a year, both the earth and the moon complete one trip around the sun. Their speeds

around the sun must be the same.

(b) From Kepler's third law we know that, for all objects moving in a stable orbit around the sun, the value of R^3/T^2 will be a constant, *independent of the objects' masses.* Thus, since R and T are the same for both moon and earth (and the system of the two together), the present orbit would be an equilibrium orbit for either moon or earth alone as well as the two together.

Some students may point out that the moon's orbit around the sun would be dependent upon the exact point in the moon's orbit around the earth at which the earth "disappeared." This is because of small variations in the speed and direction of the moon's motion around the sun. At some points in the moon's orbit around the earth, the moon is moving faster than the earth in the moon-earth motion around the sun, at other points slower. The curvature of the moon's path around the sun also varies slightly with different positions of the moon relative to the earth and sun. However, compared to the moon-earth motion around the sun, the moon's motion around the earth is inconsequential. The speed of the earth-moon system around the sun is

$$\frac{2\pi \times 1.5 \times 10^{11}\ \text{m}}{365 \times 8.6 \times 10^{4}\ \text{sec}} = 3 \times 10^{4}\ \text{m/sec.}$$

The speed of the moon around the earth is

$$\frac{2\pi \times 3.8 \times 10^{8}\ \text{m}}{27 \times 8.6 \times 10^{4}\ \text{sec}} = 10^{3}\ \text{m/sec.}$$

Hence the moon's speed around the sun is 30 times greater than its speed around the earth. Annihilating the earth would not substantially change the length of a year for a moon resident!

(c) There are two alternatives:

(1) We can write

$$\frac{F_{\text{sun on moon}}}{F_{\text{earth on moon}}} = \frac{m_{\text{sun}}}{m_{\text{earth}}} \times \frac{R_{\text{moon earth}}^{\,2}}{R_{\text{moon-sun}}^{\,2}}.$$

Or, (2) we can write the force between the moon and sun, F_{ms}, as:

$$F_{ms} = \frac{4\pi^2 M_m R_{ms}}{T_{ms}^{\,2}},$$

and

$$F_{me} = \frac{4\pi^2 M_m R_{me}}{T_{me}^{\,2}}.$$

Therefore,

$$\frac{F_{ms}}{F_{me}} = \frac{R_{ms}}{T_{ms}^{\,2}} \times \frac{T_{me}^{\,2}}{R_{me}}$$

$$= \frac{1.5 \times 10^{11}\ \text{m}}{3.8 \times 10^{8}\ \text{m}} \left(\frac{27.3\ \text{days}}{365\ \text{days}}\right)^2 = \underline{2.2.}$$

(d) The sun exerts forces on *both* the moon and the earth. If we could maintain the sun's attraction for earth and moon, but "turn off" the attraction of the earth and moon for each other, they would both continue to orbit around the sun at the same radius and at the same speed. When we add the attraction between the earth and the moon, unless they go about each other they would fall together. Thus we see there is no capture even without an attraction between moon and earth. This question is sometimes put in a slightly different form, "When the moon is toward the sun from the earth, why doesn't the sun steal the moon?" A good answer is, "It already has it."

13–23

Assume the earth is perfectly round and has a radius of 6400 km.

(a) How much less does a man with a mass of 100 kg apparently weigh at the equator than at the poles because of the rotation of the earth?

(b) How fast would the earth have to spin in order that he would exert no force on a scale at the equator?

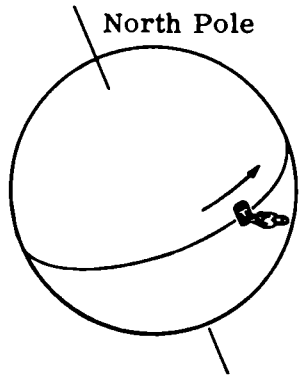
(c) How many times larger is the speed of rotation in (b) than the actual speed?

This is a moderately hard problem aimed at understanding the consequences of the earth's spin. We are asked about the *apparent* weight of a man—the reading of a scale on which he weighs himself.

(a) Suppose a man stands on a scale, first at the pole, then at the equator.

At the pole, the man will turn with the earth, once each 24 hours, but the rotation of the man does not result in an acceleration toward the center of the earth. He is in static equilibrium. The force of the scale upward, F_s, balances the force of gravity, F_g, downward, giving $F_g = F_s$. The gravitational force, F_g, equals mg = 100 kg × 9.8 newtons/kg = 980 nt. The scale will read 980 newtons.

Now let us look from the North Pole at the man on the scale at the equator. The forces acting on the man are F_g, the force of gravity



toward the earth's center, and F_s, the force exerted by the scale away from the earth's center. But in this case, $F_g > F_s$. This is because the man, moving in uniform circular motion, is accelerating toward the earth's center. The force required to produce this acceleration is supplied by the gravitational force F_g. If this force, F_g, were not opposed by the force of the scale, it would result in an acceleration of 9.8 m/sec^2. But it is opposed by the scale, limiting the acceleration to $4\pi^2R/T^2$. Thus the force of the scale upward, F_s, which is opposed to F_g downward, must be such as to result in the net force downward being of a magnitude that will result in an acceleration of $4\pi^2R/T^2$. The force that will produce this acceleration is $m4\pi^2R/T^2$. Hence:

$$F_g - F_s = \frac{4\pi^2 Rm}{T^2}$$

$$= \frac{4\pi^2 \times (6.4 \times 10^6 \text{ m}) \times 100 \text{ kg}}{24^2 \times 60^4 \text{ sec}^2}$$

$$= \underline{3.4 \text{ newtons.}}$$

The man's apparent weight at the equator, as read by the scale, would be 3.4 newtons less than the 980 newtons read at the pole. This would be a "loss" of 0.35%.

(b) If a man were to exert no force on a scale at the equator, it would be necessary that $4\pi^2R/T^2 = v^2/R = g$. Then, $v^2 = Rg = 6.4 \times 10^6$ m $\times$ 9.8 m/sec^2, and $v = 7.9 \times 10^3$ m/sec.

The period, T', required for this speed would be:

$$T' = \frac{2\pi R}{v} = \frac{2\pi \times 6.4 \times 10^6 \text{ m}}{7.9 \times 10^3 \text{ m/sec}}$$

$$= 5.1 \times 10^3 \text{ sec} = \underline{1.4 \text{ hours.}}$$

(c) The speed found in part (b) and the actual speed will be inversely proportional to the respective periods:

$$\frac{v'}{v_{\text{actual}}} = \frac{T}{T'} = \frac{24}{1.4} = \underline{17.}$$

13–24

A 10,000-kilogram space ship is drifting on a long mission toward the outer edge of the solar system. It has put out a small experimental satellite which revolves around it at a distance of 120 meters under their mutual gravitational attraction.

(a) What is the period of revolution of the satellite?

(b) What is the speed of the satellite?

The solution of the problem is a straightforward application of Kepler's third law, $T^2 = R^3/K$, and Newton's law of gravitation, $4\pi^2K = GM$, where M is the mass of the space ship:

(a) $$K = \frac{0.667 \times 10^{-10} \times 10^4}{4\pi^2} = 1.69 \times 10^{-8},$$

and

$$T^2 = \frac{(120)^3}{1.69 \times 10^{-8}} = 1.02 \times 10^{14},$$

so that

$$T = \underline{1.01 \times 10^7 \text{ sec or } 120 \text{ days.}}$$

(b) Then the speed of the satellite in its orbit is

$$v = \frac{2\pi \times 120}{1.01 \times 10^2} = 7.5 \times 10^{-5} \text{ m/sec}$$

$$= \underline{0.075 \text{ mm/sec.}}$$

As this very tiny speed indicates, this is hardly a practical problem. To place such a satellite in orbit would be an extremely delicate operation. As we have set up the solution, we have assumed that the gravitational force between the space ship and the satellite would hold it in a circular orbit. This will happen in reality only if the general gravitational field through which the space ship moves is so nearly uniform that its change across the 240-meter diameter of the satellite orbit is small compared to the gravitational field of the space ship itself. It can be calculated that this would be true for the sun's gravitational field at a distance of about ten times the radius of the earth's orbit.

When you come to the section on kinetic energy, it might be rewarding to come back to this problem again and calculate the kinetic energy of the satellite supposing that it has a mass of 100 kg. This will illustrate again how difficult it would be to put it in the orbit specified.

As the problem shows, gravitational forces become important only for bodies of enormous mass.

13–25

An astronomer observes a planet with a small satellite revolving around it in a circular orbit of radius r with a period of time T.

(a) What is the mass of the planet?
(b) What is the acceleration of the satellite toward the planet?
(c) What is the gravitational force on the satellite?
(d) The astronomer measures the radius of the planet and finds it to be $\frac{1}{10}$ of the radius of the orbit of the satellite. What is the gravitational field strength of the planet at its surface?

(a) Let M = mass of planet. From the definition of G,

$$GM = 4\pi^2 K, \text{ where } K = r^3/T^2.$$

Hence,

$$M = \frac{4\pi^2 r^3}{GT^2}.$$

(b) The (centripetal) acceleration of the satellite toward the planet is

$$a = \frac{4\pi^2 r}{T^2}.$$

(c) We cannot calculate the gravitational force on the satellite if we do not know the value of the mass of the satellite; if the mass m is known, then the force is $\underline{ma = 4\pi^2 mr/T^2}$.

(d) By the inverse square law, the gravitational field strength on the surface of the planet is 10^2 times that at the satellite. Since the gravitational field strength at the satellite equals the acceleration of the satellite toward the planet, which was found in part (b), the value at the planet's surface is $\underline{400\pi^2 r/T^2}$.

13–26†

(See the Appendix in the text.)

13–27

Astronomical observations indicate that the sun is describing a circular orbit around the center of our galaxy. The radius of the orbit is about 30,000 light-years ($= 2.7 \times 10^{20}$ m) and the period of one complete revolution is about 200 million years. In this motion the sun is acted on by the gravitational pull of the great quantity of stars lying inside its orbit.

(a) Calculate the total mass of these stars from the data given.
(b) How many stars of mass equal to the sun (2×10^{30} kg) does this represent?

This problem supplies a practical answer to the question, "How can we estimate the number of stars in our galaxy?" The data on which it is based are deduced from observations of the relative motion of many stars with respect to the solar system. In spite of the simplifying assumptions and crude approximations which we make to solve it, we can still get some notion of the order of magnitude of this number.

(a) Let us assume that the galaxy is a spherical mass of stars, and that the stars are symmetrically distributed throughout the sphere. Then we calculate the gravitational force on the sun as though the whole mass of the stars lying inside its orbit were concentrated at the center of the galaxy, as we do when we calculate the gravitational attraction of the earth (Section 13–8). In a spherical mass, the stars lying outside the sun's orbit would exert no net gravitational force on it:

$$F = \frac{GMm}{R^2} = \frac{m4\pi^2 R}{T^2},$$

and

$$M = \frac{4\pi^2 R^3}{GT^2}$$

$$= \frac{4\pi^2 \times (2.7 \times 10^{20})^3}{0.67 \times 10^{-11} \times (6 \times 10^{15})^2}$$

$$= \underline{3 \times 10^{41} \text{ kg.}}$$

(b) Suppose there are N stars, each of mass m ($= 2 \times 10^{30}$ kg); then total mass is $M = Nm$. It is certainly not true that all the stars in the galaxy are the same size as the sun, but the sun is a typical star and its mass will approximate the average mass. Then

$$N = \frac{3 \times 10^{41}}{2 \times 10^{30}} \approx \underline{10^{11}} = 100 \text{ billion.}$$

What we have now is only the number of stars lying within 30,000 light-years of the center of the galaxy. There is reason to believe that this is about half the distance to the edge. For a spherical galaxy, the whole number of stars would then be 2^3 times the number we have just computed, or about one order of magnitude larger. Since our other approximations are certainly no better than this, we can be content with 10^{11} as giving the number of stars in our galaxy within perhaps a factor of 10^2 or 10^3.

CHAPTER 14

14–1*

You push a body with a force of 3 newtons for ½ sec. What impulse do you give the body? (Section 1.)

1.5 newton-sec.

14–2*

How great is the impulse that gives an 8.00-kg mass a change in velocity of 4.00 m/sec? (Section 1.)

32.0 kg-m/sec = 32.0 newton-sec.

14–3*

Suppose you throw a ball against a wall and catch it on the rebound.

(a) How many impulses were applied to the ball?

(b) Which impulse was the greatest? (Section 1.)

(a) Four: the throw, the bounce, the catch, and the continuous impulse due to gravity.

(b) The bounce. Here the ball experienced about twice the change in velocity that it did during the throw or catch. The gravitational impulse was the smallest of all.

14–4

A 3-kg body has been accelerated by a constant force of 12 newtons from 10 m/sec to 18 m/sec.

(a) What impulse was given to the body?

(b) How long was the force acting?

(a) For straight-line motion,

$$\begin{aligned}\text{impulse} &= m(v' - v)\\ &= (3\text{ kg})(18\text{ m/sec} - 10\text{ m/sec})\\ &= \underline{24\text{ newton-sec.}}\end{aligned}$$

(b) Since $F\,\Delta t$ = impulse = 24 newton-sec, then

$$\Delta t = \frac{24\text{ nt-sec}}{12\text{ nt}} = \underline{2\text{ sec.}}$$

Note that the vector equation

$$\vec{\mathbf{F}}\,\Delta t = m(\vec{\mathbf{v}}' - \vec{\mathbf{v}}) = m\,\Delta\vec{\mathbf{v}}$$

leads to $F\,\Delta t = m(v' - v)$

only if the initial and final velocities are parallel. In general, the magnitude of $(\vec{\mathbf{v}}' - \vec{\mathbf{v}})$ does not equal $(v' - v)$.

14–5

A constant force applied to a 2.0-kg object at rest moves it 4.0 m in 2.0 sec. What impulse was applied to the object?

There are two approaches to this problem. The first involves finding the force and multiplying it by the time to get the impulse; the second involves finding the change in velocity directly and equating $m\Delta v$ to the impulse.

Since the force is constant, the acceleration is constant and can be found from $\Delta x = \frac{1}{2}a(\Delta t)^2$:

$$a = \frac{2\,\Delta x}{(\Delta t)^2}.$$

Then

$$\begin{aligned}\text{impulse} &= F\,\Delta t = (ma)\,\Delta t = 2m\,\Delta x/\Delta t\\ &= 2(2.0\text{ kg})(4.0\text{ m})/2.0\text{ sec}\\ &= \underline{8.0\text{ newton-sec.}}\end{aligned}$$

In the second approach we note that the object's *average* velocity during this period is $\Delta x/\Delta t$. Its final velocity must be twice this, since it starts from rest and the force is constant. Therefore the change in velocity is $2\,\Delta x/\Delta t$ and we have

$$\begin{aligned}\text{impulse} &= m\,\Delta v = 2m\,\Delta x/\Delta t\\ &= \underline{8.0\text{ newton-sec,}}\text{ as before.}\end{aligned}$$

14–6

A railroad freight car with a mass of 5.0×10^4 kg is rolling along a level track at 0.30 m/sec (0.7 mi/hr). A rope trails behind it.

(a) A reasonable estimate of the largest force you could apply to stop the car by pulling on the rope is 250 newtons. How long would it take you to bring the car to rest?

(b) Ten meters from the point where you start pulling, another car is standing. Will there be a crash?

(a) $$\begin{aligned}F\,\Delta t &= m\,\Delta v,\ \Delta t = m\,\Delta v/F\\ &= \frac{(5.0 \times 10^4\text{ kg})(0.30\text{ m/sec})}{250\text{ nt}}\\ &= \underline{60\text{ seconds.}}\end{aligned}$$

(b) The car will continue to roll for

$$\Delta x = v_{\text{av}}\,\Delta t = \frac{(0.30\text{ m/sec} + 0\text{ m/sec})}{2} \times 60\text{ sec} = 9\text{ m.}$$

Therefore, there will not be a crash.

14–7*

What average force is necessary to stop a hammer with 25 newton-sec momentum in 0.05 sec? (Section 2.)

$\vec{\mathbf{F}}$ = 500 newtons, in the opposite direction to the momentum.

14–8

A man is at rest in the middle of a pond on perfectly frictionless ice. How can he get himself to shore?

This standard problem can serve nicely as the basis for reviewing isolated systems as well as conservation of momentum.

If we suppose for the moment that there is no air, then there is no way for the whole man to get to shore. He can move in one direction only if he throws something in the other. Fortunately it can be very small, since he will move at a constant speed once he gets going. As long as we are supposing, if there is air *friction* he can use his hands as "propellers." With a still wilder supposition of no air friction but still air to breathe, he can make a jet with his breath—and turn his head to breathe in.

14–9

An object with a mass of 10 kg moves along a straight line at a constant velocity of 10 m/sec. A constant force then acts on the object for 4.0 sec, giving it a velocity of −2 m/sec.

(a) Calculate the impulse acting on the object.
(b) What is the magnitude and direction of the force?
(c) What is the momentum of the object before and after the force acts?

(a) Impulse $= m(v' - v)$ for motion along a straight line

$$= (10\text{ kg})(-2\text{ m/sec} - 10\text{ m/sec})$$
$$= \underline{-120\text{ kg-m/sec}}$$
$$= -120\text{ newton-sec.}$$

We see from the minus sign that the impulse is in the direction opposite from the initial velocity.

(b) $F\,\Delta t$ = impulse, so

$$F = \frac{-120\text{ nt-sec}}{4.0\text{ sec}} = -30\text{ newtons.}$$

That is, a force of 30 newtons acts in a direction opposite to the original velocity.

(c) Initial momentum:

$$mv = 10\text{ kg} \times 10\text{ m/sec} = 100\text{ kg-m/sec.}$$

Final momentum:

$$mv' = 10\text{ kg} \times (-2)\text{ m/sec} = \underline{-20\text{ kg-m/sec.}}$$

The change in momentum is, of course, equal to the impulse.

14–10*

An impulse is applied to a moving object at an angle of 120° with respect to its velocity vector. What is the angle between the impulse vector $\vec{F}\Delta t$ and the change in momentum vector $\Delta\vec{p}$? (Section 2.)

Zero. The net change in momentum is always in the direction of the total impulse.

14–11

In Fig. 14–12, how is diagram (b) constructed from diagram (a)? (Section 4.)

Each velocity vector in (a) is multiplied by the mass of the body whose motion it represents, and a new scale is chosen to represent these new vectors.

14–12*

Two ice pucks initially at rest were driven apart by an explosion with velocities 5 cm/sec and −2 cm/sec. What is the ratio of their masses? (Section 4.)

(Mass of faster)/(Mass of slower) $= \frac{2}{5}$

14–13

A 20-kg cart is moving with a velocity of 2.0 m/sec. A boy whose mass is 60 kg jumps off the cart. When he hits the ground, he is

(a) moving at the same velocity as the cart
(b) not moving relative to the ground
(c) moving with twice the initial velocity of the cart.

In each case, what is the change in velocity of the cart?

Initially cart (20 kg) and boy (60 kg) are moving with a velocity of 2.0 meters/sec. Hence the total momentum of the system is $(20 + 60\text{ kg}) \times 2.0\text{ m/sec} = 160\text{ kg-m/sec}$.

(a) In this case the boy has a final momentum of $60\text{ kg} \times 2.0\text{ m/sec} = 120\text{ kg-m/sec}$, which leaves $160 - 120 = 40$ kg-m/sec for the cart. Since it has a mass of 20 kg, it has a velocity of $40 \div 20 = 2.0$ m/sec. There is no change in the velocity of the cart. This is almost trivially obvious since, to "land" at the same velocity as the cart, the boy would have to drop off the side of the cart, imparting no impulse to the cart.

(b) If the boy ends up with no velocity and hence no momentum, the cart must have all the original 160 kg-m/sec. It then is going with a velocity $= 160/20 = 8.0$ meters/sec. Therefore the change is 6.0 m/sec. Physically this

comes about because the boy must jump backward off the cart so as to arrive on the ground with no momentum. In doing this he pushes the cart forward.

(c) If the boy moves with twice the initial velocity of the cart, he is moving with a velocity of 4.0 meters/sec. His momentum is then $60 \times 4.0 = 240$ kg-m/sec. If the final momentum of the cart is p_f, then

$$p_f + 240 = 160; \; p_f = -80 \text{ kg-m/sec.}$$

The cart has a velocity of $-80/20 = -4.0$ or 4.0 meters/sec in the direction opposite to that in which it was originally moving. The change is therefore $\underline{-6.0 \text{ m/sec.}}$

14–14

Two heavy frictionless carts are at rest. They are held together by a loop of string. A light spring is compressed between them (Fig. 14–18). When the string is burned, the spring expands from 2.0 cm to 3.0 cm, and the carts move apart. Both hit the bumpers fixed to the table at the same instant, but cart *A* moved 0.45 meter while cart *B* moved 0.87 meter. What is the ratio of:

(a) the speed of *A* to that of *B* after the interaction?
(b) their masses?
(c) the impulses applied to the carts?
(d) the accelerations of the carts while the spring pushes them apart?

This problem deals with momentum conservation, but also can be used to introduce the ideas of Newton's third law, which appears in Section 8 of the text.

(a) Cart *A* moved a distance $d_a = 0.45$ meter during a time t, and Cart *B* moved a distance $d_b = 0.87$ meter in the same time. Their speeds then were

$$v_a = \frac{d_a}{t}, \text{ and } v_b = \frac{d_b}{t}.$$

$$\therefore \frac{v_a}{v_b} = \frac{d_a}{d_b} = \frac{0.45 \text{ m}}{0.87 \text{ m}} = \underline{0.52.}$$

(b) Since the two momenta must be equal in magnitude:

$$m_a v_a = m_b v_b, \text{ or } \frac{m_a}{m_b} = \frac{v_b}{v_a} = \frac{1}{0.52} = \underline{1.9.}$$

(c) Since the momenta were equal and opposite, the impulses were also $\underline{\text{equal but opposite.}}$

(d) Since average $a = \frac{\Delta v}{\Delta t}$, $\frac{a_a}{a_b} = \frac{\Delta v_a/t}{\Delta v_b/t} = \frac{\Delta v_a}{\Delta v_b} = \underline{-0.52}$ (since the changes in velocity are in $\underline{\text{opposite}}$ directions).

14–15

A proton (mass 1.67×10^{-27} kg) with a speed of 1×10^7 m/sec collides with a motionless helium nucleus, and the proton bounces back with a speed of 6×10^6 m/sec. The helium nucleus moves forward with a speed of 4×10^6 m/sec after the bombardment.

(a) Can you compute the mass of the helium nucleus? If so, what is it?
(b) Can you compute the force that acted during the collision? If so, what is it?
(c) If you answered "no" to either (a) or (b), be prepared to discuss in class why you gave this answer.

This is a conservation of momentum problem with nuclear physics flavor.

(a) Let m_p be the mass of the proton and v_p its velocity before the collision. Since the helium nucleus is at rest, the total momentum before the collision is $m_p v_p$. After the collision it is $m_{He} v_{He} - m_p v'_p$ where v'_p is the final velocity of the proton, and the minus sign occurs because velocity v'_p is opposite in direction to all the other velocities:

$$m_p v_p = m_{He} v_{He} - m_p v'_p,$$

and

$$m_{He} = \frac{m_p v_p + m_p v'_p}{v_{He}} = \frac{m_p(v_p + v'_p)}{v_{He}}$$

$$m_{He} = \frac{1.67 \times 10^{-27} \text{kg} \, (1 \times 10^4 + 6 \times 10^4 \text{ m/sec})}{4 \times 10^6 \text{ m/sec}}$$

$$= 4 \times 1.67 \times 10^{-27} \text{ kg} = \underline{7 \times 10^{-27} \text{ kg.}}$$

(b) No. Force cannot be computed.

(c) The force during the collision $\underline{\text{cannot be computed}}$ unless the length of time of the collision interaction is known.

14–16

A stationary refrigerator car with mass 2.0×10^4 kg is rammed by a loaded gondola car with mass 3.0×10^4 kg. Before impact, the gondola car was going 1.0 m/sec. If they lock together, what is the new velocity?

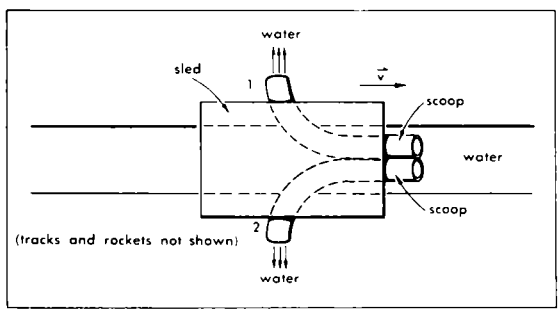
Before the collision, the total momentum of the two-car system is just the momentum of the gondola car because only the gondola car is moving. After the collision the total momentum of the two-car system is the same as before. Since the collision couples the two cars, both move with the same velocity after the collision:

$$p_f = p_i$$
$$(m_g + m_r)v_f = m_g v_g, \text{ and}$$
$$v_f = \frac{m_g v_g}{m_g + m_r}$$
$$= \frac{3.0 = 10^4 \text{ kg} = 1.0 \text{ m/sec}}{3.0 = 10^4 \text{ kg} + 2.0 = 10^4 \text{ kg}}$$
$$= \underline{0.60 \text{ m/sec.}}$$

Most students have seen the jerky banging together of two freight cars being coupled. It may be well to call attention to the fact that although complicated forces are at work, the overall result is simple to comprehend. If the question is raised in discussion, you might tell your students that it is kinetic energy that is lost in the banging around. However, try to steer clear of energy at this point. It will be the subject of the next three chapters.

14–17

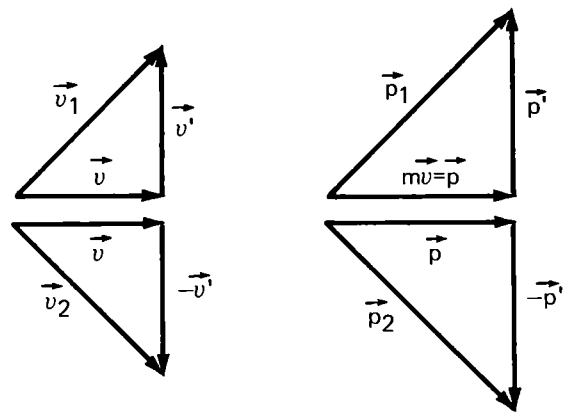
An experimental rocket sled is slowed down by a scoop which dips into a trough of water. As shown in Fig. 14–19, the scoop is designed so that it ejects the water at right angles to the moving sled, and so that the water leaves the tubes with a speed equal to the speed of the sled at any instant.



(a) What will be the change in momentum of a small mass of water m ejected toward either side at the instant the sled is traveling with a speed v?
(b) What is the change in momentum of the sled for each mass of water m scooped up?
(c) Is any advantage gained by ejecting half of the water from each side? Why not eject all of it from one side?

(a) The water leaves at right angles to the direction of motion of the sled with a speed v equal to that of the sled. The velocities of the two jets of water relative to the ground will be $\vec{v}_1$ and $\vec{v}_2$ as shown in the diagram. The change in momentum of the water ejected from each tube is then $\sqrt{2}\,mv$ in one of the directions shown by $\vec{p}_1$ and $\vec{p}_2$ in the diagram.

(b) The loss in momentum of the sled is equal to the gain in momentum of the water in the direction of motion of the sled. This gain in momentum of the water as shown in the diagram is $\vec{p} = m\vec{v}$. The change in momentum of the sled is $-m\vec{v}$.



(c) When the water is ejected equally on both sides, the total momentum of the water ejected relative to the sled is zero. If all the water were ejected from one side, there would be a change in momentum occurring relative to the sled which would push sideways on the sled and probably derail it.

You can extend the problem by asking the student to again answer part (b) if the discharge tubes are extended through another 90° so they discharge the water directly forward with a speed relative to the sled, equal to that of the sled at any instant.

In this case the water relative to the ground will have changed velocity by $2\vec{v}$. Therefore, the change in momentum of a small mass of water, m, will be $2\vec{v}m$ and the change in momentum of the sled will be $-2\vec{v}m$.

14–18

An astronaut with a total mass M (including his equipment) has been accidentally separated from his spacecraft and is at rest with respect to the ship at a distance d. His tank contains a mass m_0 of oxygen ($m_0 \ll M$) and is equipped with a nozzle through which the gas can be very rapidly ejected at an average speed of v. He must release oxygen for propulsion to get back to the spacecraft but also needs oxygen for breathing. He breathes oxygen at a rate R.

(a) If he releases an amount m of oxygen for propulsion, what speed V will he acquire? How long will it take him to reach the spacecraft?
(b) How long can he breathe on the remaining oxygen?
(c) If he is to return successfully to his craft, his breathing time, t_B, must be equal to (or greater than) his travel time t_T. What condition does this place on m?
(d) Calculate how much oxygen the astronaut may safely release for propulsion if $M = 100$ kg, $m_o = 0.5$ kg, $d = 45$ m, $v = 50$ m/sec, and $R = 2.5 \times 10^{-4}$ kg/sec.

(a) From the conservation of momentum,

$$MV = mv, \text{ or } V = mv/M.$$

Hence, the time to reach the spaceship will be $t_T = d/V = Md/mv$.

(b) The astronaut releases a mass m of the oxygen for propulsion. He now has a mass $m_o - m$ left for breathing. At a breathing rate R this will last for a time

$$t_B = \frac{m_o - m}{R}.$$

(c) If we set $t_T = t_B$ (the case in which he just makes it back), then

$$\frac{Md}{mv} = \frac{m_o - m}{R}, \text{ or } RMd = mm_o v - m^2 v,$$

$$\text{or } vm^2 - m_o vm + RMd = 0,$$

which is a quadratic equation for m.

The amount of oxygen released for propulsion can have any value between the limits set by the roots of this equation, which are:

$$m = \frac{m_o v \pm \sqrt{(m_o v)^2 - 4vRMd}}{2v}.$$

(d) Putting in the values for our particular situation,

$$m = \frac{25 \pm \sqrt{625 - 225}}{100} = 0.45 \text{ or } 0.05 \text{ kg}.$$

A numerical check will easily convince the students that 0.05 kg $< m <$ 0.45 kg will get the astronaut back, whereas with $m <$ 0.05 kg or $m >$ 0.45 kg, he will run out of oxygen before reaching the ship. For the above value the times are 200 seconds and 1800 seconds.

14–19*

What is the direction of motion of the center of mass of the balls in Fig. 14–9 after the collision? (Section 6.)

The direction of motion of the center of mass after collision is the same as it was before: it is upward, about a degree counterclockwise from the direction of the ball entering from the bottom of the picture.

14–20†

(See the Appendix in the text.)

14–21

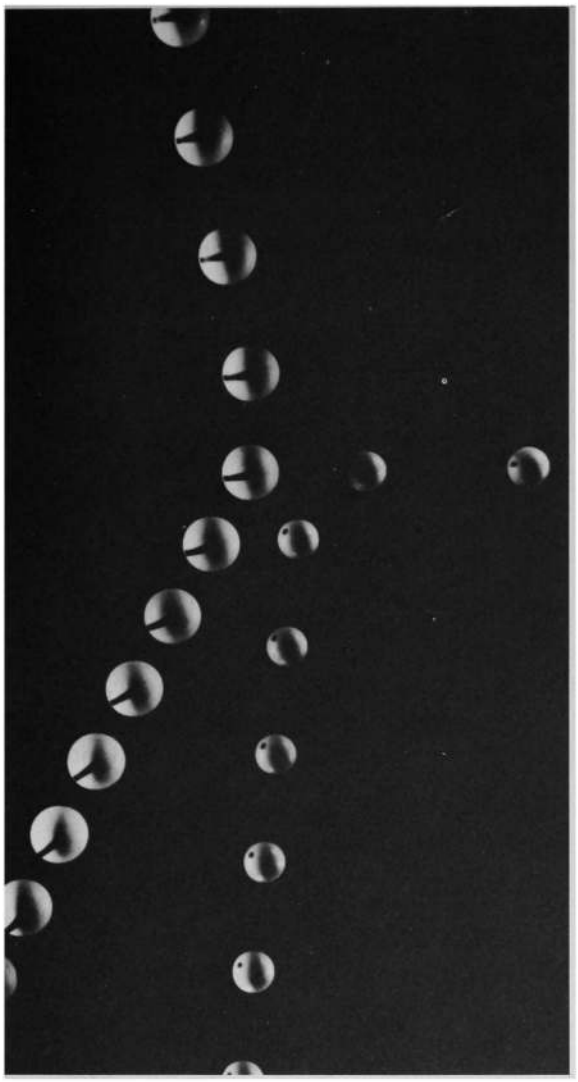
A rocket is out in free space shooting out a stream of exhaust gases and picking up speed in the opposite direction. What happens to the center of mass of all the matter—that which is ejected and that which is left in the rocket?

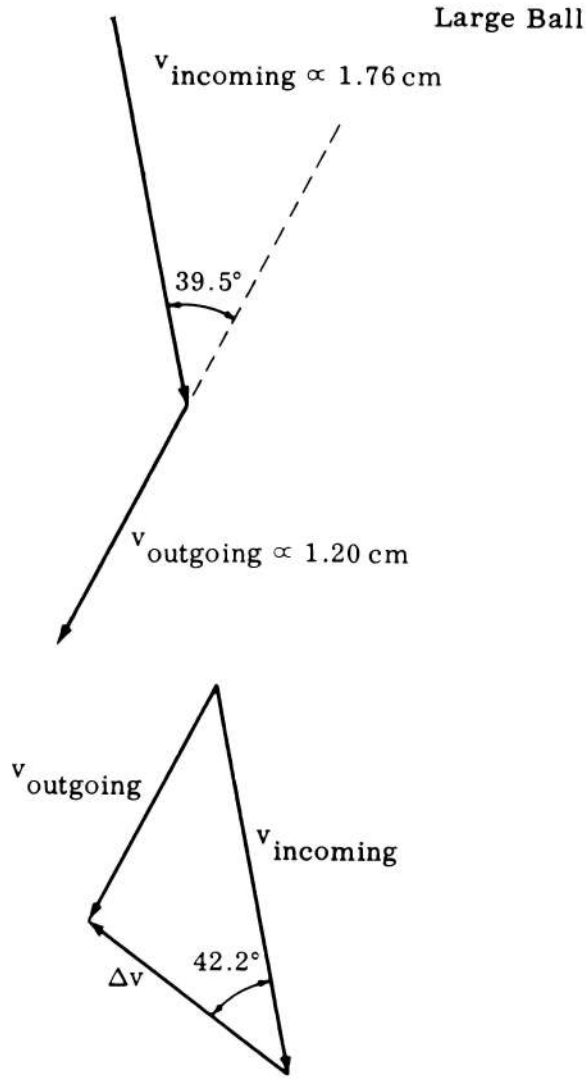
A rocket moving in "free space" represents an isolated system. The system consists of the ejected matter and that which is left in the rocket. Since no external forces are acting on the rocket system, the center of mass of the matter in the system cannot be accelerating. Therefore the velocity of the center of mass is constant.

14–22

In Fig. 14–20, the large ball came in at the top of the picture and the little ball at the bottom. As you see, a collision took place in the middle.
(a) Draw the vectors which represent the change in the velocity of the large ball and the change in velocity of the small ball. Plot these vectors to the same scale and make sure that each one is in the right direction.
(b) Are these changes of velocity opposite in direction?
(c) Are they equal in magnitude?
(d) If their magnitudes differ, what should be their ratio?
(e) The mass of the large ball is 201 gm. What is the mass of the small ball?

Students should be encouraged to work to 0.1 millimeter and 0.1° accuracy although the distortions in the picture are probably greater than this. Depending on how much practice students have had with this sort of graphical problem it may be well to point out in assigning it that they can plot velocity vectors *proportionally* even though they do not know either the real distance scale or the time between flashes. The velocity of an object may be plotted as proportional to the distance between successive flashes. Directions of travel can be determined by drawing along one side of the line of images of the ball, using tracing paper if it





is not permissible to mark the text. Distances can be determined by using the line of centers, measuring between the more brightly lighted edges.

(a) The vector diagrams are given below. The vector equation is $\vec{\mathbf{v}}_{in} + \Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_{out}$. Therefore $\Delta\vec{\mathbf{v}}$ is the vector from the head of $\vec{\mathbf{v}}_{in}$ to the head of $\vec{\mathbf{v}}_{out}$. (We have drawn each vector three times as long as the distance between centers on the photograph.)

(b) The changes of velocity are within 0.5° of opposite directions. Any measurement between 0 and 1.5° might be accepted.

(c) No. From the vector diagrams, the ratio of the magnitudes is

$$\frac{\Delta v_{big}}{\Delta v_{small}} = \frac{1.13}{2.68} = 0.422.$$

(d) From momentum conservation, the ratio of the changes in velocity should be inversely proportional to the ratio of the masses:

$$m_{big}\,\Delta v_{big} = m_{small}\,\Delta v_{small}$$

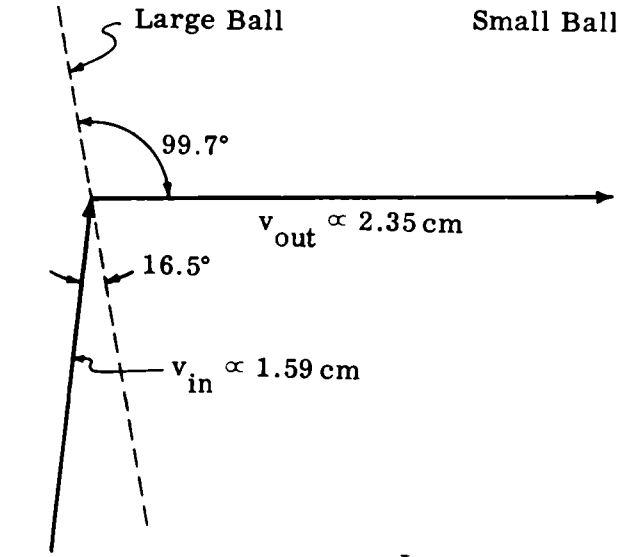
$$\frac{\Delta v_{big}}{\Delta v_{small}} = \frac{m_{small}}{m_{big}}.$$

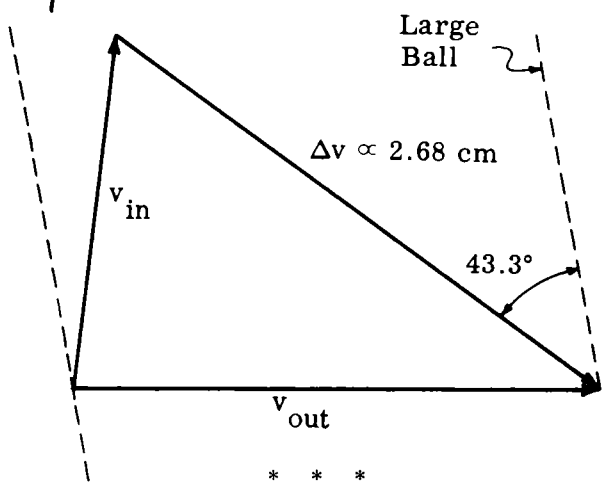
(e) Since

$$\frac{m_{small}}{m_{big}} = \frac{\Delta v_{big}}{\Delta v_{small}} = 0.422,$$

$m_{small} = 0.422 \times 201 =$ 84.8 gm (a value between 80 and 90 is acceptable).

Clearly these balls are the 201 and 85 gm pair that has been used in other figures.





* * *

Note: You may be interested in whether energy was conserved. You may wish to ask students to save their solutions to this problem for analysis of energy conservation when you deal with this subject later. The initial energy was proportional to

$$\tfrac{1}{2} \times 201 \text{ gm} \times (1.76 \text{ cm/t})^2 + \tfrac{1}{2} \times 84.8 \text{ gm} \times (1.59 \text{ cm/t})^2 = 418 \text{ gm-cm/t}^2$$

and the final energy to

$$\tfrac{1}{2} \times 201 \text{ gm} \times (1.20 \text{ cm/t})^2 + \tfrac{1}{2} \times 84.8 \text{ gm} \times (2.35 \text{ cm/t})^2 = 378 \text{ gm-cm/t}^2.$$

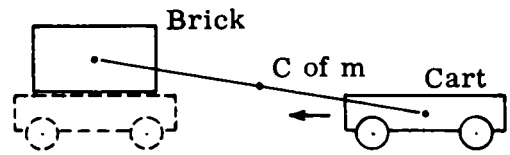
Thus about 9.5% of the energy was lost; the collision was *not* elastic, but close.

14–23

A 2.0-kg brick with no horizontal motion is dropped on a 2.0-kg cart moving across a frictionless table at 0.40 meter/sec.

(a) What is the change in velocity of the cart?
(b) What is the velocity of the center of mass of the system composed of cart and brick before the brick is dropped?

(a) The initial velocity of the cart is 0.40 m/sec. Since momentum must be conserved when its mass is doubled, its velocity must be halved. Therefore the change in its velocity is 0.20 m/sec.



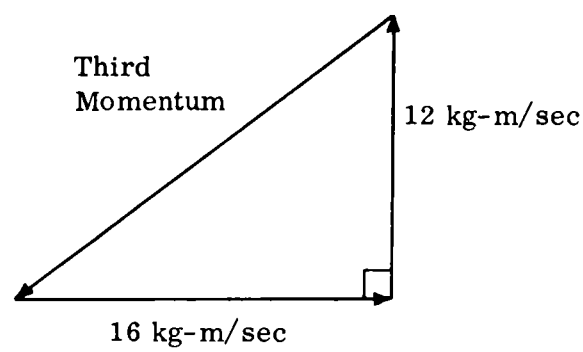
(b) The center of mass of the 2-kg brick and 2-kg cart lies on the line joining them and halfway between them. Consequently, the center of mass of the cart and brick moves half as fast as the cart does before the "collision." Therefore the center of mass moves at 0.20 m/sec before the collision (and after the collision).

14–24

An explosion blows a rock into three parts. Two pieces go off at right angles to each other, a 1.0-kg piece at 12 m/sec and a 2.0-kg piece at 8.0 m/sec. The third piece flies off at 40 m/sec.

(a) Draw a diagram to show the direction in which the third piece goes.
(b) What is its mass?

(a) We must presume the rock to be initially at rest. It then had zero momentum before the explosion and hence must have zero momentum after the explosion. The momentum of the 1.0-kg piece is 12kg-m/sec; the momen-

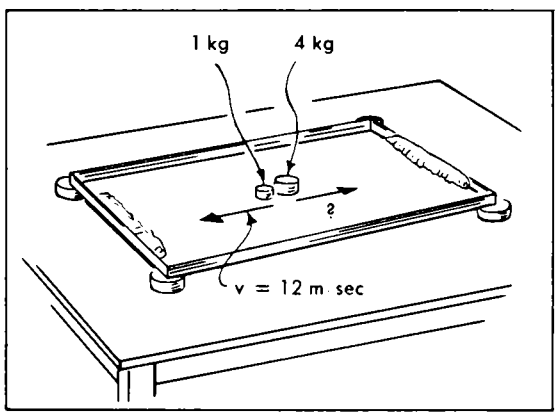


tum of the 2.0-kg piece is 16 kg-m/sec at right angles to the first. The third piece must have a momentum $\vec{\mathbf{p}}$ given by the vector diagram at the right which shows that the three momenta add to zero.

(b) The diagram shows the direction in which the third piece flew off, and from the 3, 4, 5 right triangle, it is seen to have a momentum of 20 kg-m/sec. Since its velocity was 40 m/sec, its mass was $\frac{1}{2}$ kg.

14–25

The system shown in Fig. 14–21 consists of a 5.0-kg frame with a 1.0-kg and a 4.0-kg mass in the middle. At either end of the frame are stops made of putty. The two masses and the frame are mounted on Dry Ice pucks. (The mass of the Dry Ice is included in the values given above.) The center of mass of the whole system is at the center of the frame. An explosion pushes the two masses apart. The 1.0-kg mass goes off with a speed of 12 m/sec. Eventually each mass is trapped by the putty.



(a) What is the speed of the 4.0-kg mass immediately after the explosion?
(b) Where is the center of mass of the whole system after 1 sec? After 2 sec?
(c) What is the velocity of the frame and the attached masses after 100 sec?
(e) How far does the frame move?

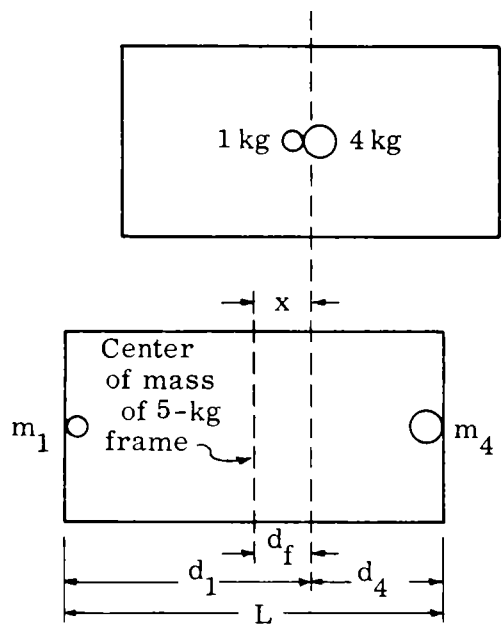
(a) Momentum must be conserved in the explosion between the 1.0- and 4.0-kilogram masses. The 1.0-kg mass has a momentum of 12 kg-m/sec after the explosion. The 4.0-kg mass must have an equal and opposite momentum and therefore a velocity of 3.0 meters/sec.

(b) Since there are no external forces acting on the system, the center of mass of the whole system remains fixed at all times.

(c) Since there are no external forces acting, and since before the explosion the momentum of the whole system was zero, the momentum remains zero. When the frame and two masses are joined, they all have the same velocity—zero. (With reasonable dimensions the masses have become attached to the frame long before 100 seconds have passed.)

(d) With the explosion, the 1.0- and 4.0-kilogram masses fly apart with speeds of 12 and 3 meters/sec, respectively. The 1.0-kg mass hits first and sticks, moving the frame in its direction. The momentum of frame and 1.0-kg mass is 12 kg-m/sec, the momentum of the mass before the collision. Next the 4.0-kg mass hits and transfers 12 kg-m/sec in the opposite direction, thus stopping the frame and masses, which then remain at rest.

(e) The distance the frame moved can be calculated by doing part (d) quantitatively. However, the simplest method is to remember that the center of mass does not move.



Suppose the frame moved a distance x. Then it acts like a 5.0-kg mass a distance x from the old (and the new) center of mass.

Where L is the length of the frame, the 1.0-kg mass is $(L/2 + x)$ from the center of mass, and the 4.0-kg mass is $(L/2 - x)$ from the center of mass:

$$d_1 m_1 + d_f m_f = d_4 m_4$$

$$\left(\frac{L}{2} + x\right) m_1 + x m_f = \left(\frac{L}{2} - x\right) m_4$$

$$\frac{L m_1}{2} + x m_1 + x m_f = \frac{L m_4}{2} - x m_4$$

$$x(m_1 + m_4 + m_f) = \frac{L(m_4 - m_1)}{2}$$

$$x(1.0 + 4.0 + 5.0) = \frac{L(4.0 - 1.0)}{2}$$

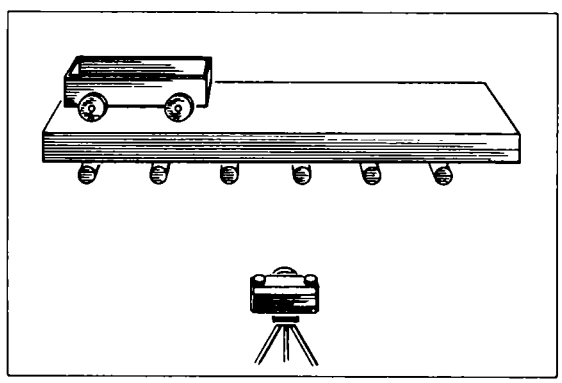
$$x = 0.15L.$$

Many students who are unable to solve this part of the problem using L will be able to do so if they are given a specific value for L.

14–26

A 20-kg car stands at rest on an 80-kg platform as shown in Fig. 14–22. The car can be driven along the platform by a motor and the platform, mounted on rollers, is free to move along the laboratory floor substantially without friction. A stroboscopic camera records any motions that occur.

When the motor is turned on, a study of the photograph shows that over a period of 3.0 sec the platform acquired a velocity of 0.30 m/sec.



(a) What does the same photograph show for the velocity of the car at the end of the 3.0-sec period?

(b) With what force did the wheels of the car push against the platform?

(c) What was the relative velocity of the car along the platform at the end of the 3.0 sec?

(d) If you know only the relative motion of the car along the platform, what force would you calculate was needed to give the car this velocity in 3.0 sec? Can you explain why the force you calculate here is different from the one you found in Part (b)?

The positive direction is taken to be the direction of the platform's motion.

(a) Since the car and platform are originally at rest, and since no outside force acts on either of them, the change in their total momentum is

$$\Delta p = 0 = m_c \Delta v_c + m_p \Delta v_p,$$

where v_c is the velocity of the car and v_p is the velocity of the platform;

$$\Delta v_c = v_c - 0, \text{ so}$$

$$v_c = -\frac{80 \text{ kg} \times 0.30 \text{ m/sec}}{20 \text{ kg}}$$

$$= \underline{-1.20 \text{ m/sec.}}$$

The car moves in the direction opposite to the platform's motion.

(b) If we set the impulse of the force equal to the change in momentum of the platform,

$$F\Delta t = m_p \Delta v_p$$

$$F = m_p \Delta v_p / \Delta t$$

$$= (80 \text{ kg})(0.30 \text{ m/sec})/(3.0 \text{ sec})$$

$$= \underline{8.0 \text{ newtons.}}$$

(c) The velocity of the car relative to the moving platform was

$$v_{rel} = v_c - v_p = (-1.20 \text{ m/sec}) - (0.30 \text{ m/sec}) = \underline{-1.50 \text{ m/sec.}}$$

(d) If we use the impulse-momentum relation to calculate the force on the car, we get

$$F = m_c \Delta v_{c\,rel} / \Delta t$$

$$= (20 \text{ kg})(-1.50 \text{ m/sec})/(3.0 \text{ sec})$$

$$= \underline{-10 \text{ newtons.}}$$

This force is not equal in magnitude to the one we calculated was acting on the platform, because here we took the moving platform as our frame of reference. Since the platform is being accelerated, it does not provide an inertial frame of reference within which Newton's law of motion can be used. The application of Newton's law in an accelerated frame of reference will give a fictitious force, in this case amounting to 2 newtons, which depends upon the frame of reference rather than any action of the bodies involved.

14–27*

What happens to the momentum of a car when it comes to a stop? (Section 8.)

Most of the momentum is transferred to the earth (through the road) and a small amount to the air molecules surrounding the car.

14–28

A double star consists of two large masses that attract each other gravitationally. By observing the motion of both masses, we can see that they rotate around each other.

(a) What do you think happens to the momentum of each of the masses in a double star as time goes on? Explain your answer.

(b) When observed carefully, the bright star Sirius seems to wobble about, instead of having a uniform motion of the center of mass. From this and other evidence astronomers believe that Sirius has a dark companion. It is really a double star. How does this explain the peculiar observed motion?

This problem asks the student to discuss conservation of momentum in an astronomical problem.

(a) The momentum of one member of a double star does not have to remain constant because there is an external force—the gravitational attraction of the other member—acting upon it. (Momentum changes due to emitted light are negligible.) Therefore the momentum of each member changes, but the *sum* of the two momenta remains constant since we presume the double star system is far enough away from all other stars that it may be considered isolated from them. This means that when one momentum changes, the other must change by an equal amount in the opposite direction. The changes are periodic, i.e., repeat themselves as the two stars revolve about each other.

(b) Assume that Sirius and its dark companion have equal masses and are revolving around each other as together they move through space. The point half-way between the two stars would move uniformly, as shown above. How this motion would appear to an observer depends upon the magnitude and frequency of revolution of the binary system, and upon the angle from which the motion is viewed. If the observer is in the plane of revolution, one star periodically eclipses the other star. This is called an eclipsing binary. From other angles, the visible star will appear to wobble in its path. The magnitude and frequency of wobble will depend on the stars' relative masses and the shapes of their orbits.

14–29*

Consider a collision between two particles in which light is emitted in a given direction. Since light exerts a pressure, it must have momentum. If the overall momentum (including the momentum of the light) is conserved, are the forces of interaction between the two particles equal and opposite?

No; the vector sum of the impulses due only to the forces of interaction cannot be zero because the total impulse, which includes the momentum of the light, must equal zero.

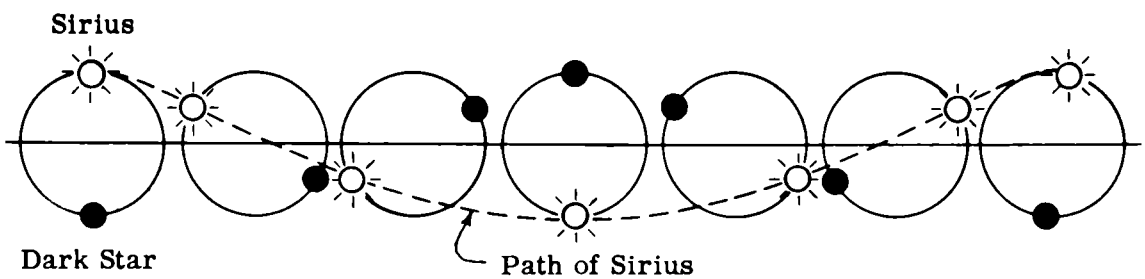
14–30

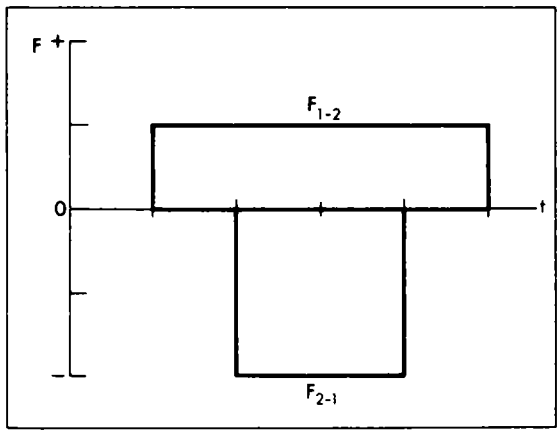
Figure 14–23 shows the hypothetical force-time graphs between two bodies; $F_{1,2}$ and $F_{2,1}$ are the forces with which the two bodies act on each other.

(a) Is the total final momentum different from the total initial momentum?

(b) Is momentum conserved throughout the collision?

(a) The total impulse which the first body gives to the second is exactly equal and opposite to the total impulse that the second body





gives to the first, since the areas between the force curves and the t-axis are equal and on opposite sides of the axis. Hence the net change in momentum is zero, and the total momentum is the same as it was before.

(b) Momentum is not conserved throughout the collision; for example, during the time interval $F_{1,2}$ is greater than zero but $F_{2,1}$ is zero, the momentum of the second body is increasing while the momentum of the first body stays the same.

CHAPTER 15

15–1*

What is the kinetic energy of a 1-kg hammer moving at 20 m/sec? (Section 1.)

200 joules.

15–2*

How does the kinetic energy of a car change when its speed is doubled? (Section 1.)

The kinetic energy increases by a factor of 4.

15–3

A force of 10.0 newtons acts on a 2.00-kg roller skate initially at rest on a frictionless table. The skate travels 3.00 meters while the force acts.

(a) How much work is done?
(b) How much energy is transferred to the skate?
(c) What is the final speed of the skate?

(a) The work done is 10.0 nt × 3.00 m = 30.0 joules.

(b) The energy transferred to the skate is measured by the work done, 30.0 joules.

(c) The final speed of the skate is obtained by knowing that the kinetic energy of the moving skate, $\frac{1}{2}mv^2$, equals the energy that has been transferred to the skate:

$$\tfrac{1}{2}mv^2 = 30.0 \text{ joules, or}$$

$$v = \sqrt{\frac{2 \times 30.0}{2.00}} = 5.48 \text{ m/sec.}$$

15–4

Compare the kinetic energies of two objects, A and B, identical in every respect except one. Assume that the single difference is:

(a) Object A has twice the velocity of B.
(b) Object A moves north, B south.
(c) Object A moves in a circle, B in a straight line.
(d) Object A is a projectile falling vertically downward; B is a projectile moving vertically upward at the same speed.

(a) Since the kinetic energy is $\frac{1}{2}mv^2$, A has 2^2 or 4 times the kinetic energy of B.

(b) Kinetic energy is a scalar, independent of direction; hence the kinetic energy of A is equal to that of B.

(c) Same as (b), the kinetic energies are equal, since the magnitudes of the velocities are equal.

(d) Again, same as (b), the kinetic energies are equal, since the magnitudes of the velocities are equal. (Of course, in this case, the kinetic energies will not be equal a moment later, since that of A is increasing, while that of B is decreasing.)

15–5*

Two bodies of unequal mass each have the same kinetic energy and are moving in the same direction. If the same retarding force is applied to each, how will the stopping distances of the bodies compare? (Section 2.)

They will be equal, since they depend only on the force and the kinetic energy.

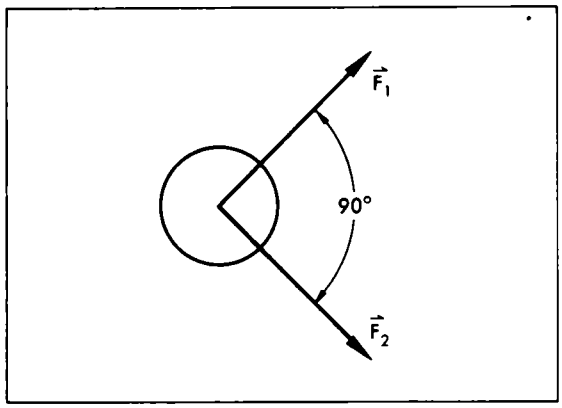
15–6*

Is it possible to exert a force and yet not cause a change in kinetic energy? (Section 2.)

Yes; no energy is transferred when a force acts on a moving body at right angles to its motion.

15–7

A Dry Ice puck is acted upon by two forces $\vec{F}_1$ and $\vec{F}_2$ as shown in Fig. 15–14. Assume that in each case the body starts from rest.



How much work is done on the body as it moves 2 meters if:

(a) $\vec{F}_1 = 10$ newtons and $\vec{F}_2 = 0$?

(b) $\vec{F}_1 = 0$ and $\vec{F}_2 = 10$ newtons?

(c) $\vec{F}_1 = 10$ newtons and $\vec{F}_2 = 10$ newtons?

This problem illustrates that the work done is the product of the distance moved times the component of the *net* force taken along the direction of motion. In this case, since the puck starts from rest and the forces are constant, the net force is in the same direction as the motion.

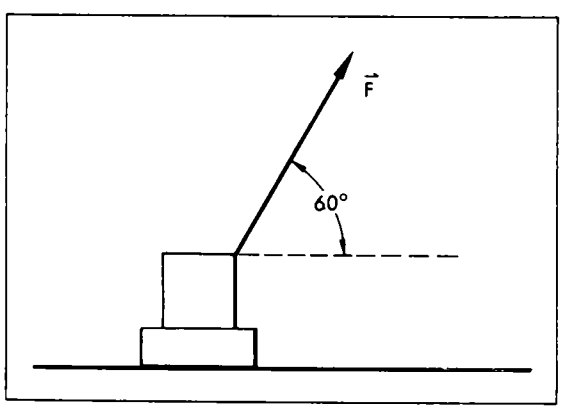
(a) $W = F_1 s = (10 \text{ nt})(2 \text{ m}) = \underline{20 \text{ joules.}}$

(b) $W = F_2 s = (10 \text{ nt})(2 \text{ m}) = \underline{20 \text{ joules.}}$

(c) $W = F_{\text{net}} s = (10\sqrt{2} \text{ nt})(2 \text{ m}) = \underline{28 \text{ joules.}}$

15–8

A Dry Ice puck has a mass of 5 kg. It is subjected to a constant force $\vec{F}$ of 50 newtons at an angle of 60° above the horizontal for 2 seconds (Fig. 15–15).



(a) If the puck starts from rest, what is its change in momentum in the first two seconds the force is applied?

(b) How much work was done in these two seconds?

$$\left(\textit{Note: } E_K = \tfrac{1}{2}mv^2 = \frac{(mv^2)}{2m} = \frac{p^2}{2m}.\right)$$

(c) What would happen if the force were doubled?

(a) Only the horizontal component of $\vec{F}$ does work on the puck:

$$\Delta p = F_h \, \Delta t = (50 \cos 60° \text{ nt})(2 \text{ sec}) = \underline{50 \text{ kg-m/sec.}}$$

Since the vertical component of $\vec{F}$ is less than the weight:

$$F_v = 50 \sin 60° \text{ nt} = 43 \text{ nt} < mg = (5 \text{ kg})(9.8 \text{ nt/kg}) = 49 \text{ nt},$$

there is no net force in the vertical direction, and the motion is entirely horizontal.

(b) $$W = E'_K - E_K = \frac{(50 \text{ kg-m/sec})^2}{2(5 \text{ kg})} - 0 = \underline{250 \text{ joules.}}$$

(c) The puck would leave the table and accelerate in the direction of the resultant of $\vec{F}$ and $m\vec{g}$.

15–9

A force of 30 newtons accelerates a 2.0-kg object from rest for a distance of 3.0 meters along a level, frictionless surface; the force then changes to 15 newtons and acts for an additional 2.0 meters.

(a) What is the final kinetic energy of the object?

(b) How fast is it moving?

(a) The kinetic energy of the object equals the energy transferred to it by the net force that acts on it. The energy transferred to the object equals the work done on it by the force:

$$30 \text{ nt} \times 3.0 \text{ m} + 15 \text{ nt} \times 2.0 \text{ m} = \underline{120 \text{ joules.}}$$

(b) The object's speed can be determined by making use of the fact that its kinetic energy is $\frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2 \times E_K}{m}} = \sqrt{\frac{2 \times 120}{2.0}} = \underline{11 \text{ m/sec.}}$$

15–10

Estimate your kinetic energy (in joules) when you ride your bicycle on the road.

The purpose of this problem is to let the student relate the joule to something with which he is familiar. Let him make his own estimates. A reasonable answer in round numbers is: Mass of student and bicycle is 10^2 kg. Speed $\approx$ 10–15 miles/hour or around 5 m/sec. Hence kinetic energy $\approx$ 10^3 joules.

15–11

A 2.0-kg stone whirls around on the end of a 0.50-m string with a frequency of 2.0 revolutions per second.
(a) What is its kinetic energy?
(b) What is the centripetal force on it?
(c) How much work is done by the centripetal force in one revolution?

(a) To find the kinetic energy of the stone we must know its speed as well as its mass. The speed of an object moving in a circle of radius R with period T is

$$v = 2\pi R/T = 2\pi Rf,$$

where f is the frequency. Hence the kinetic energy is

$$E = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(2\pi Rf)^2$$
$$= 2(2.0\text{ kg})(3.14)(0.50\text{ m})(2/\text{sec})^2$$
$$= 39\text{ joules}.$$

(b) The centripetal force is

$$F = mv^2/R = 2E_K/R$$
$$= 2(39\text{ joules})/0.50\text{ m} = 160\text{ newtons}.$$

15–12*

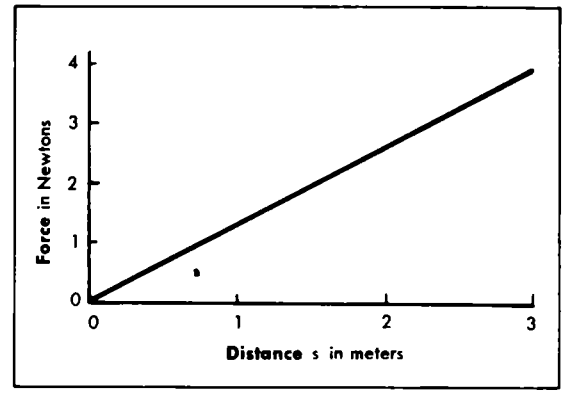
In Fig. 15–5, do the two bodies repel or attract each other? (Section 3.)

They repel each other during the interaction.

15–13

A 10.0-kg mass moves 2.00 meters against a retarding force that increases linearly by 4.00 newtons for every 3.00 meters the mass moves (see Fig. 15–16). If the force is zero at the beginning, how much kinetic energy is lost?

The energy lost by the 10.0-kg mass is measured by the work it does. The work done by the mass in pushing against a retarding force that changes with distance can be found from the area under the graph of force vs. distance. Between 0 and 2.00 m, $E_K = W = \frac{1}{2} \times 2.67\text{ nt} \times 2.00\text{ m} = 2.67$ newton-meters or joules.



15–14*

When do we call a collision *elastic?* (Section 6.)

When the force between the colliding bodies depends only on the separation of the bodies.

15–15*

When a putty ball is thrown at a wall and sticks to it, does the force between the wall and the putty ball depend on the separation alone? (Section 7.)

On the way in, the putty ball experiences a repulsive force at small separation, which slows it down; after the putty gets to the wall, the force suddenly becomes attractive, so it is not the same as on the way in. Hence, we know that the force does not depend on distance alone.

15–16*

A 3-kg ball with a speed of 6 m/sec strikes another 3-kg ball moving 6 m/sec in the opposite direction. After collision each ball has a velocity equal in magnitude but opposite in direction to its initial velocity. Is this an elastic collision? (Section 6.)

Yes. Since kinetic energy is conserved, we may assume that in this collision the interaction force is a function only of separation.

15–17*

A rapidly moving mass m_1 strikes a mass m_2 at rest in a head-on elastic collision. How does m_1 compare with m_2 if m_1 comes back along its path at a very low speed? (Section 7.)

Mass of m_1 < mass of m_2.

15–18

A projectile (which does not contain an explosive charge) from an antitank gun has a mass of about 10 kg and a speed of 10^3 m/sec. A freight car being

moved around in a switchyard has a mass of about 10^4 kg and a speed of 1 m/sec. What are their momenta? Their kinetic energies? Why does the projectile do much more damage than the freight car when it hits something?

Both shell and freight car have a momentum of 10^4 kg-m/sec. The shell has a kinetic energy of $\frac{1}{2}(10 \times 10^6) = 5 \times 10^6$ joules and the car $\frac{1}{2}(1^2 \times 10^4) = 5 \times 10^3$ joules. When something is damaged, work is done against the forces which hold it together. This explains why the shell, having 10^3 times more kinetic energy, does the greater damage.

15–19

Two 3.0-kg bodies interact. At a given moment the first body is moving to the right at 0.50 m/sec and the second body is moving to the right at 0.30 m/sec, and so at that instant the speed of approach of one body as measured from the other is 0.20 m/sec. If the force of interaction is repulsive and equal to 0.10 newton, at what rate is the total kinetic energy decreasing at that time?

In a short time interval Δt, the first body is displaced by $\Delta x_1 = v_1 \Delta t$ while the second body is displaced by $\Delta x_2 = v_2 \Delta t$. The force of repulsion on the first body acts opposite to the direction of travel of the first body, but the repulsive force on the second body is in the direction of travel of the second. Hence, during the given time interval, the kinetic energy of the first body changes by $-F\Delta x_1$, that of the second body changes by $F\Delta x_2$ (where F is the magnitude of the repulsive force). Therefore the net change in total kinetic energy is $\Delta E_K = F\Delta x_2 - F\Delta x_1$, and the rate at which kinetic energy is changing is $\Delta E_K/\Delta t$ or:

$$\frac{\Delta E}{\Delta t} = \frac{F\Delta x_2 - F\Delta x_1}{\Delta t} = F\left(\frac{\Delta x_2}{\Delta t} - \frac{\Delta x_1}{\Delta t}\right)$$
$$= F(v_2 - v_1)$$
$$= 0.10 \text{ newton} \times (0.30 - 0.50 \text{ m/sec})$$
$$= -0.02 \text{ joule/sec.}$$

The minus sign tells us that the total kinetic energy is decreasing at the rate of 0.02 joule/sec. (Students may wonder what happens to the "lost" kinetic energy. If the collision is elastic, it has been stored as potential energy of the two-body system. This subject will be discussed in the next chapter.)

15–20

A 1.5-kg body is at rest. It is "hit" head-on by a body of mass 0.50 kg moving with a speed of 0.20 m/sec. The interaction force depends only on the separation of the two bodies.

(a) What is the final velocity of each body?

(b) In what direction does each move after the interaction?

This problem can be worked out from scratch using the approach described in Section 15–7. For students who do not go through that development, it serves as an exercise in the application of the equations that are derived in that section.

(a) Let the positive direction be the initial direction of motion of the 0.50-kg body. Taking m_1 as the moving 0.50-kg body, and m_2 as the 1.5-kg body that is "struck":

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1$$
$$= 2 \times \frac{0.50 \text{ kg}}{2.0 \text{ kg}} \times 0.20 \text{ m/sec}$$
$$= 0.10 \text{ m/sec.}$$

Similarly,

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$
$$= \frac{0.50 - 1.5 \text{ kg}}{2.0 \text{ kg}} \times 0.20 \text{ m/sec}$$
$$= -0.10 \text{ m/sec.}$$

(b) The significance of the positive sign in the answer for v'_2 is that m_2, the second body (the struck one), moves in the same direction the incident ball did before the collision. The significance of the negative sign in the answer for v'_1 is that m_1, the incident ball (which is lighter), bounces back in the collision so that its final velocity is in the opposite direction from its velocity before the collision.

15–21

An object of mass m_1 and kinetic energy E_K collides head on with an object of mass m_2 at rest. Assuming the force of interaction depends only on the separation, calculate the kinetic energy transferred if

(a) $m_2 = 0.01\, m_1$

(b) $m_2 = m_1$

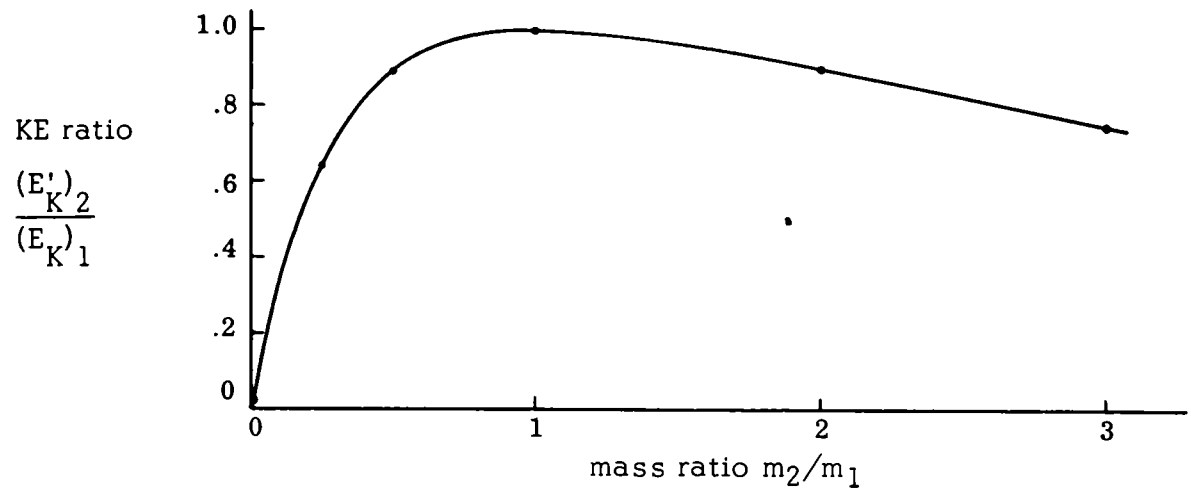
(c) $m_2 = 100\, m_1$

What is the ratio of m_1 and m_2 when the transfer of kinetic energy is

(d) a maximum?

(e) very small?

(*Note:* To see this relationship more clearly, you may wish to find the energy transferred for additional values of m_2, and make a graph.)



We use the equations for head-on collisions to express the final kinetic energy of m_2 in terms of the initial kinetic energy of m_1.

The kinetic energy of m_2 after the collision is:

$$\tfrac{1}{2}m_2v_2'^2 = \tfrac{1}{2}m_1\left(\frac{2m_1}{m_1+m_2}v_1\right)^2$$

$$= \tfrac{1}{2}m_1v_1^2\frac{4m_1m_2}{(m_1+m_2)^2}$$

$$= \tfrac{1}{2}m_1v_1^2\frac{4\frac{m_2}{m_1}}{\left(1+\frac{m_2}{m_1}\right)^2}$$

$$= \tfrac{1}{2}m_1v_1^2\frac{4r}{(1+r)^2},$$

where $\frac{1}{2}m_1v_1^2$ is simply the initial kinetic energy of m_1, 1 joule in this problem, and $r = m_2/m_1$, the mass ratio.

(a) $r = \dfrac{0.01}{2} = 0.005,$

and

$$\frac{4r}{(1+r)^2} = \frac{0.020}{1.01} = 0.0198$$

$(E'_K)_2 = 0.02 \times 1\text{ joule} = \underline{0.02\text{ joule.}}$

(b) $r = \dfrac{2.0}{2} = 1,$ and $\dfrac{4r}{(1+r)^2} = \dfrac{4}{4} = 1$

$(E'_K)_2 = \underline{1.0\text{ joule.}}$

(c) $r = \dfrac{400}{2} = 200,$

and

$$\frac{4r}{(1+r)^2} = \frac{800}{40201} = 0.02$$

$(E'_K)_2 = 0.02 \times 1\text{ joule} = \underline{0.02\text{ joule.}}$

(d) The graph of the function

$$\frac{4r}{(1+r)^2} = (E'_K)_2/(E_K)_1$$

is given above, showing a maximum at $\underline{m_2/m_1 = r = 1.}$

(e) The function has very small values both for very small and for very large r. (The zero value for $r = 0$ is easy to see analytically. The approach to zero for very large r can be understood as follows: When r is very large $1 + r \approx r$ and the function becomes:

$$\frac{4r}{(1+r)^2} = \frac{4r}{r^2} = \frac{4}{r},$$

and $4/r$ goes to zero as r goes to infinity.)

15–22

A 5.0-kg body is at rest. A 10-kg body approaches it with a velocity of 0.20 m/sec. The interaction force is zero when the separation is greater than 0.10 meter and is 4.0 newtons when the separation is less than this distance. (Note that some of the answers in this problem are easier to get if you use the conservation of momentum.)

(a) What is the kinetic energy of the masses before the interaction?

(b) What will be the kinetic energy of each mass after the interaction is complete?

(c) What will be the kinetic energy of each mass when the separation is at a minimum? Recall

that the velocities at minimum separation are equal.

(d) What is the minimum separation? [Your answer to part (c) tells you the net loss of kinetic energy of the two masses at minimum separation].

(a) $(E_K)_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2} \times 10(0.20)^2$
$= \underline{0.20 \text{ joule.}}$

$(E_K)_2 = \frac{1}{2}m_2v_2^2 = \underline{0 \text{ joules.}}$

(b) $(E'_K)_2 = (E_K)_1 \times \frac{4r}{(1+r)^2}$ (see solution to Problem 21)

$r = \frac{5}{10} = \frac{1}{2}$, and $\frac{4r}{(1+r)^2} = \frac{8}{9}$

$(E'_K)_2 = \frac{8}{9} \times 0.20 \text{ joule} = \underline{0.18 \text{ joule.}}$

Since kinetic energy must be the same in the initial and final states (when the interaction force is zero),

$(E'_K)_1 = (E_K)_1 - (E'_K)_2$
$= 0.20 - 0.18 = \underline{0.02 \text{ joule.}}$

(c) During the interaction period kinetic energy is *not* conserved. (Energy is temporarily stored as potential energy.) However, momentum *is* conserved since no external forces act. When the separation of the masses is a minimum, their velocities are equal. Let this velocity be denoted as v''. Then the momentum of the two masses is: $(m_1 + m_2)v''$, and is equal to m_1v_1 by conservation of momentum. Hence,

$$v'' = \frac{m_1}{m_1 + m_2} v_1.$$

The kinetic energy of the 10-kg mass at this time is then

$$\frac{1}{2}m_1v''^2 = \frac{1}{2}m_1\left(\frac{m_1}{m_1 + m_2}\right)^2 v_1^2$$

$$= (\tfrac{1}{2}m_1v_1^2)\left(\frac{m_1}{m_1 + m_2}\right)^2$$

$$= 0.20 \times \tfrac{100}{225} = \underline{0.089 \text{ joule.}}$$

The kinetic energy of the 5.0-kg mass is similarly

$$(E_K)_2 = \tfrac{1}{2}m_2v''^2 = (\tfrac{1}{2}m_1v_1^2)\frac{m_1m_2}{(m_1 + m_2)^2}$$

$$= 0.20 \times \tfrac{50}{225} = \underline{0.044 \text{ joule.}}$$

The total kinetic energy is $0.089 + 0.044 = 0.133$ joule, which is less than the original kinetic energy for the reason noted above. Since the original kinetic energy was 0.20 joule, the net loss of kinetic energy is 0.07 joule.

(d) The net loss in kinetic energy is the work done against the interaction force:

$$\Delta E_K(\text{tot}) = F\,\Delta s,$$

so

$$\Delta s = \Delta E_K(\text{tot})/F = \frac{-0.07 \text{ joule}}{4.0 \text{ nt}} = -0.02 \text{ m.}$$

The quantity $\Delta E_K(\text{tot})$ is negative because kinetic energy was lost; the negative sign in the value of Δs indicates that the separation decreased by 0.02 meter. Hence the minimum separation is $0.10 - 0.02 = \underline{0.08 \text{ meter.}}$

15–23

An antiaircraft shell has a kinetic energy E_K and a momentum $\vec{\mathbf{p}}$. Just then it explodes. What can you say about:

(a) the momentum of the pieces?

(b) the kinetic energy of the pieces?

(a) Since no external force acts on the shell, the total momentum of the shell must remain unchanged by the explosion. The vector sum of the momenta of all of the pieces is therefore equal to the momentum of the shell just before it exploded.

(b) The total kinetic energy of the pieces must equal the kinetic energy of the shell plus the work done (the energy transferred into kinetic energy of the pieces) in the explosion, and so it is greater than the initial kinetic energy of the shell above.

15–24

In one of the experiments that led to the determination of the mass of the neutron, Chadwick measured the velocity of protons that had been hit head on by neutrons. The velocity of the protons was 3.3×10^7 m/sec.

(a) What was the velocity of the neutrons before and after collision with the protons?

(b) Chadwick also measured the velocity of nitrogen atoms hit head on by the neutrons. What was it?

(c) What was the velocity of the neutrons after each kind of collision?

(a) This is the case of a head-on collision of one object with another of equal mass, initially

at rest. The kinetic energy of the projectile is entirely transmitted to the target. The projectile is brought to rest. Analytically:

$$v_1 = \frac{m_1 + m_2}{2m_1} v_2' = v_2' = \underline{3.3 \times 10^7 \text{ m/sec;}}$$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 = 0.$$

(b) $m_2 = 14\, m_1$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 = \frac{2m_1}{m_1 + 14\, m_1} v_1$$

$$= \frac{2 \times 3.3 \times 10^7}{15} = \underline{4.4 \times 10^6 \text{m/sec.}}$$

(c) From part (a), after colliding head-on with a proton, the neutron's velocity will be 0. After colliding head-on with nitrogen, the neutron will have a velocity

$$v_1' = \frac{m_1 - 14\, m_1}{m_1 + 14\, m_1} \times 3.3 \times 10^7$$

$$= \underline{-2.9 \times 10^7 \text{ m/sec.}}$$

(After colliding with nitrogen, the neutrons bounce backward.)

15–25

Suppose you wish to determine the force your bicycle brakes are exerting on the bicycle when you brake hard without skidding. Assume the brakes grip the rim of the wheel. At your disposal is a tape to measure length and a road of known constant slope. Also, you know the total weight of the bicycle and yourself. You do not have a watch. How would you go about it?

You might proceed as follows: Starting from rest, coast downhill a measured distance d_1. Apply the brakes and then measure the additional distance d_2 it takes you to stop. If the combined weight of you and the bicycle is W, the slope α, and the braking force B, then as you coast downhill you and the bike gain an amount of energy equal to

$$(\Delta E_K)_1 = F_x x = (W \sin \alpha)\, d_1.$$

The braking force acts upward along the slope of the hill, while the weight acts vertically down. Hence, when the brakes are applied the net change in kinetic energy is

$$(\Delta E_K)_2 = -B\, d_2 + W \sin \alpha\, d_2.$$

Since you come to a stop, $(\Delta E_K)_1$ must equal $-(\Delta E_K)_2$, or

$$W \sin \alpha\, d_1 = B\, d_2 - W \sin \alpha\, d_2$$

$$B = (W \sin \alpha) \frac{d_1 + d_2}{d_2}.$$

If a student will actually measure d_1 and d_2 experimentally, he is likely to find $d_2 \ll d_1$. Then

$$B \approx (W \sin \alpha) \frac{d_1}{d_2}$$

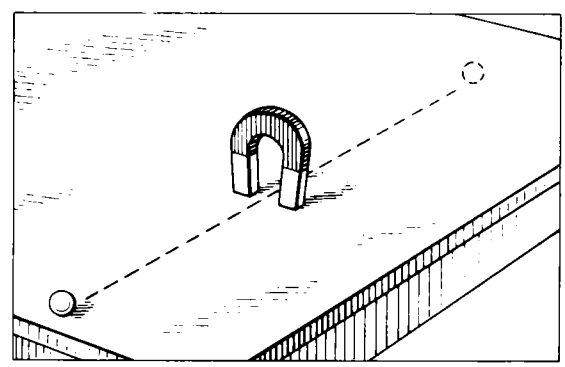
gives a good enough approximation. If the brakes grip a drum, as is the case with coaster brakes, the force acts along a distance $d_2(r_1/r_2)$, where r_1 and r_2 are the radii of the drum and wheel, respectively.

15–26

A horseshoe magnet of mass m stands on end on a frictionless table. A steel ball bearing of mass m is rolled toward the magnet from far away with velocity v and goes through the magnet and far beyond. Assume the force of attraction F changes with distance and is the same in front and behind the magnet. (See Fig. 15–17.)

(a) What is the final velocity of the ball?

(b) What is the final velocity of the magnet?



This problem involves an elastic collision, although the students may not recognize it as a collision. Furthermore, this collision is novel in that the forces are attractive. It is only because the ball can scoot between the legs of the magnet that the ball can go away from the collision instead of sticking to the magnet. Why is it elastic? Because the force between the two objects depends only on the separation distance.

Both the magnet and the ball are acted on by the magnetic force, so both are accelerated toward each other; the magnet accelerates from rest, and the ball accelerates from its initial velocity.

As the ball and magnet move closer together, the force (and the accelerations) increase steadily until the ball passes through the magnet. The force then rapidly goes through zero and reverses sign. As the ball recedes from the magnet, the force on both ball and magnet is a decelerating force. At every value of separation distance, the decelerating force is equal in magnitude but opposite in direction to the accelerating force which acted when the ball and magnet approached each other. At a very large separation distance the magnet will have been brought to rest. To conserve momentum and energy, the ball must then have been slowed to its initial velocity.

CHAPTER 16

16–1*

In Fig. 16–1, where is the acceleration of the block the greatest? (Section 1.)

In the middle picture where the force is greatest.

16–2*

How could you determine the k for the spring in Fig. 16–3? (Section 1.)

The slope of the graph is k.

16–3

A mass of 4.0 kg sliding with a velocity of 3.0 m/sec on a frictionless horizontal table collides with a queer kind of spring bumper. The bumper exerts a constant force of 120 newtons on the mass as it moves in (compressing the spring) and the same force on the way out until the spring is back where it was.

(a) Is this an elastic collision? How do you know?
(b) What is the kinetic energy at the beginning of the interaction?
(c) How much is the spring compressed?
(d) What is the ratio of kinetic energy to potential energy when the spring has been compressed 10 cm?

(a) The collision is elastic because the force depends only on the distance.

(b) $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 4.0\text{ kg} \times (3.0\text{ m/sec})^2$
$= 18$ joules.

(c) The compression of the spring will continue until the kinetic energy is reduced to zero, that is, when the speed of the mass is zero.. The initial kinetic energy of the mass then has been transferred to the spring as potential energy. The work done in compressing the spring, $F\Delta x$, is stored as potential energy and is equal to the initial kinetic energy the instant the mass stops moving.

$$F\,\Delta x = E_{K_i}$$
$$\Delta x = \frac{E_{K_i}}{F} = \frac{18\text{ joules}}{120\text{ nt}} = 0.15\text{ m}.$$

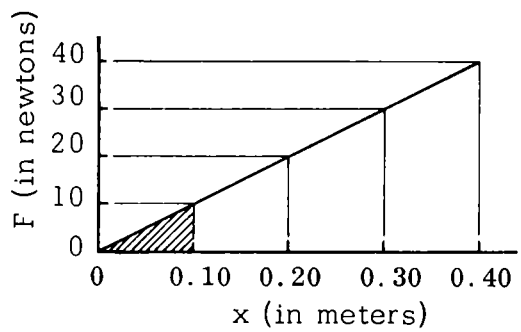
(d) The potential energy at this point is 120 nt × .10 m = 12 joules. The potential energy has increased at the expense of the initial kinetic energy . . . interaction.

16–4

A 3.0-kg mass moving with a velocity of 2.0 m/sec collides with a spring bumper which exerts a force $F = 100x$, where F is the force in newtons and x is the compression in meters.

(a) Draw a graph of F versus x from $x = 0$ to $x = 0.40$ m.
(b) What is the potential energy stored in the spring when $x = 0.10$ m? What is the kinetic energy of the mass at this point?
(c) What will happen if the spring is compressed 0.10 m by hand and then the 3-kg mass is placed in contact with the spring and the hand is removed?

(a)



(b) The work done by an external agent in compressing the spring a distance of 0.10 m is the shaded area under the force-compression curve shown in part (a). The work done by the mass in moving from $x = 0$ to $x = 0.10$ m is therefore:

$$W = \tfrac{1}{2} \times 10\text{ newtons} \times 0.10\text{ m}$$
$$= 0.5\text{ joules}.$$

The potential energy stored in the spring is the same as the amount of work done on the spring, 0.5 joule.

The initial kinetic energy of the moving mass was:

$$E_K = \tfrac{1}{2} \times 3.0 \text{ kg} \times (2.0 \text{ m/sec})^2$$
$$= 6.0 \text{ joules.}$$

The increase in potential energy occurs at the expense of the kinetic energy of the mass. Therefore the kinetic energy must decrease by the 0.5 joule that the potential energy increased. Hence, at $x = 0.10$ m, the kinetic energy of the mass is $6.0 - 0.5 = 5.5$ joules.

(c) The work done in compressing the spring 0.10 m is the same whether it is done by a moving mass or by hand:

$$W = \tfrac{1}{2} \times 10 \text{ newtons} \times 0.10 \text{ m}$$
$$= 0.5 \text{ joule.}$$

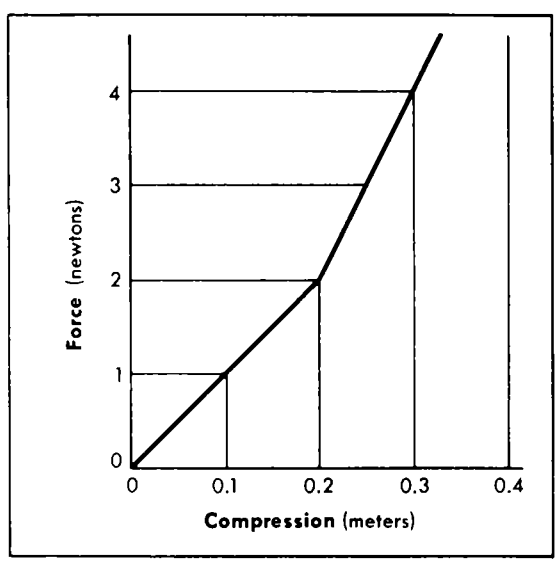
At this point the potential energy of the spring has increased from 0 to 0.5 joule. This energy is available to accelerate the mass when the hand is removed. When the spring has returned to its normal position, the interaction ends and the transfer of potential energy to kinetic energy is complete. Hence, at this point the kinetic energy $= \frac{1}{2}mv^2 =$ 0.5 joule.

16–5

The force-compression curve of a spring is shown in Fig. 16–13.

(a) How much work is done in compressing the spring 0.3 m?

(b) What is the potential energy of the spring when compressed this amount?



(c) Place a 2-kg mass at rest against the spring when it is compressed 0.3 m. Let go. What is the kinetic energy of the mass as it passes the point where the spring is compressed 0.2 m?

This problem is conceptually the same as Problem 4, but involves a nonlinear force curve.

(a) The work done in compressing the spring is the area under the force-distance curve. In this case, between 0 and 0.3 m, the total area under the curve is the sum of the area of a triangle and the area of a trapezoid, these figures being the two parts of the area if it is cut vertically at 0.2 m.

Area of triangle $= \frac{1}{2}(2 \text{ newtons})(0.2 \text{ m})$
$= 0.2$ newton-meter or joule.

Area of trapezoid $= \frac{1}{2}(2 \text{ nt} + 4 \text{ nt})(0.1 \text{ m})$
$= 0.3$ nt-m or joule.

Sum = 0.5 joule = work of compression.

(b) The potential energy of the spring = work done by the applied force = 0.5 joule (from above).

(c) The gain in kinetic energy is equal to the loss of potential energy and this is the area under the force-distance curve from $x = 0.3$ to $x = 0.2$ m. This area is just the area of the trapezoid which was found in part (a) to be 0.3 joule. Therefore,

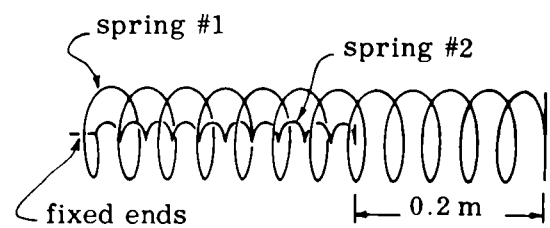
$\frac{1}{2}mv^2 = 0.3$ joule = gain in kinetic energy.

16–6

Suppose you are given two springs of different sizes. How would you set them up so that the force-compression curve of the system looks like Fig. 16–13?

This problem is a challenge to the student with a flair for inventing mechanical devices.

In order to get a change of slope at 0.2 meter in the force-compression curve, there must be an accompanying change in the force constant of the spring at the 0.2-meter compression point. There are a number of ways to accomplish this. One way is to set up two springs so that the movable end of



one spring is 0.2 meter behind the movable end of the other. If the springs have different diameters, one can be placed inside the other, as shown in the sketch.

As an object pushes in from the right, it first comes in contact with spring #1. It is subject to the force $F_1 = k_1 x$ until it has compressed spring #1 a distance 0.2 meter. From the graph, we see that when $F_1 = 2$ newtons, $x = 0.2$ meter; therefore, $k_1 = 10$ nt/m. At the point $x = 0.2$ m, the object comes in contact with spring #2. From there on it is subject to the force

$$\begin{aligned} F &= F_1 + F_2 = k_1 x + k_2(x - 0.2) \\ &= (k_1 + k_2)(x - 0.2) + 0.2k_1 \\ &= 0.2k_1 + (k_1 + k_2)\,\Delta x, \end{aligned}$$

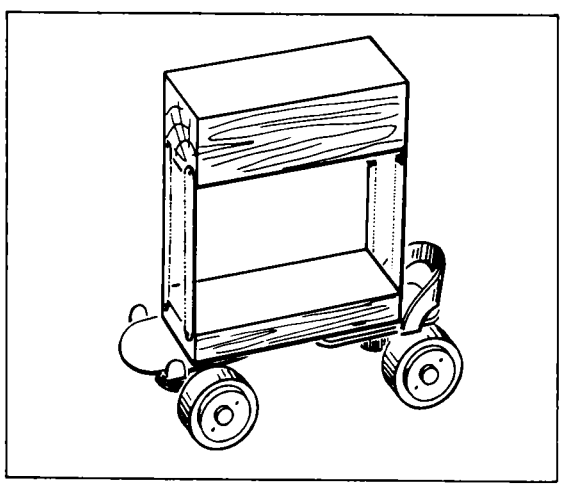
where Δx represents the distance of compression beyond the point where $x = 0.2$ meter. From the graph, the slope of the straight line for $x > 0.2$ is

$$\begin{aligned} \Delta F/\Delta x &= (4 - 2)/(0.3 - 0.2) \\ &= 20 \text{ newtons/meter} = k_1 + k_2. \end{aligned}$$

Therefore, k_2 also has the value 10 newtons/-meter.

16–7

Figure 16–14 shows a roller skate with a mass M mounted on it by four hacksaw blades attached to a base. The mass M is equal to the mass of the roller skate plus the mass of the base. If you hold the roller



skate and pull the mass M out to the left, and then let the whole system go, the mass moves to the right and the skate to the left. Be prepared to describe the further motion, indicating the form of the energy at various stages.

For the solution to this problem we shall assume that all of the motions are frictionless and that the entire system is at rest at the instant the displaced mass is released.

Since no external forces act on the skate-mass system, conservation of momentum tells us that the center of mass must always remain at rest. The motion of the system will consist of back-and-forth oscillations of its top and bottom halves, one always moving in the opposite direction from the other. The initial potential energy stored in the distorted blades is transformed into kinetic energy of the motion of the parts. When the blades are vertical, the potential energy is zero and all of the energy resides in the equal speeds of the two masses, $E_K = 2 \times \frac{1}{2}mv^2$. When the blades are at the extremes of their distortion, the motion of the masses is zero and all of the energy of the system is now in the form of potential energy.

If we describe the motion more accurately we can take into account the vertical motion of the upper mass as the blades sweep it through an arc. This up-down motion "stretches" and "relaxes" the "spring" of the gravitational field. However, in the undistorted position of the hack-saw blades where the kinetic energy is a maximum, the gravitational potential energy is also a maximum. Thus the useful energy of the system is never entirely kinetic. There is always some available potential energy stored either in the spring or in the gravitational field.

If we take friction into account, energy will gradually be transformed into heat, and the motion will stop. Insofar as the friction is rolling friction on a surface, an external force will act on the skate-mass system and its center of mass, also, will describe gradually diminishing back-and-forth oscillations instead of remaining stationary as described.

16–8

A linear elastic spring is compressed 0.2 m by a force of 20 newtons.

(a) What is the force constant k (or force-compression ratio) of the spring?

(b) What is the equation for the potential energy stored by the spring as a function of its compression?

(a) The spring is linear and elastic. Therefore, $F = kx$ relates the magnitude of the force to

the amount of compression. Rewriting, we get

$k = F/x$; in particular,
$k = 20\text{ nt}/0.2\text{ m} = \underline{100\text{ nt/m.}}$

(b) Since the spring is linear and elastic,

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(100\text{ nt/m})x^2 = (50\text{ nt/m})x^2.$$

If x is measured in meters, U will be in joules.

16–9*

In Fig. 16–8, we see two masses which have a certain amount of stored energy at separation s and are moving to the right. If we placed the two masses at rest in the same positions and released them, which way would they go? (Section 3.)

Each would move in a straight line in the direction of the force acting on it.

16–10

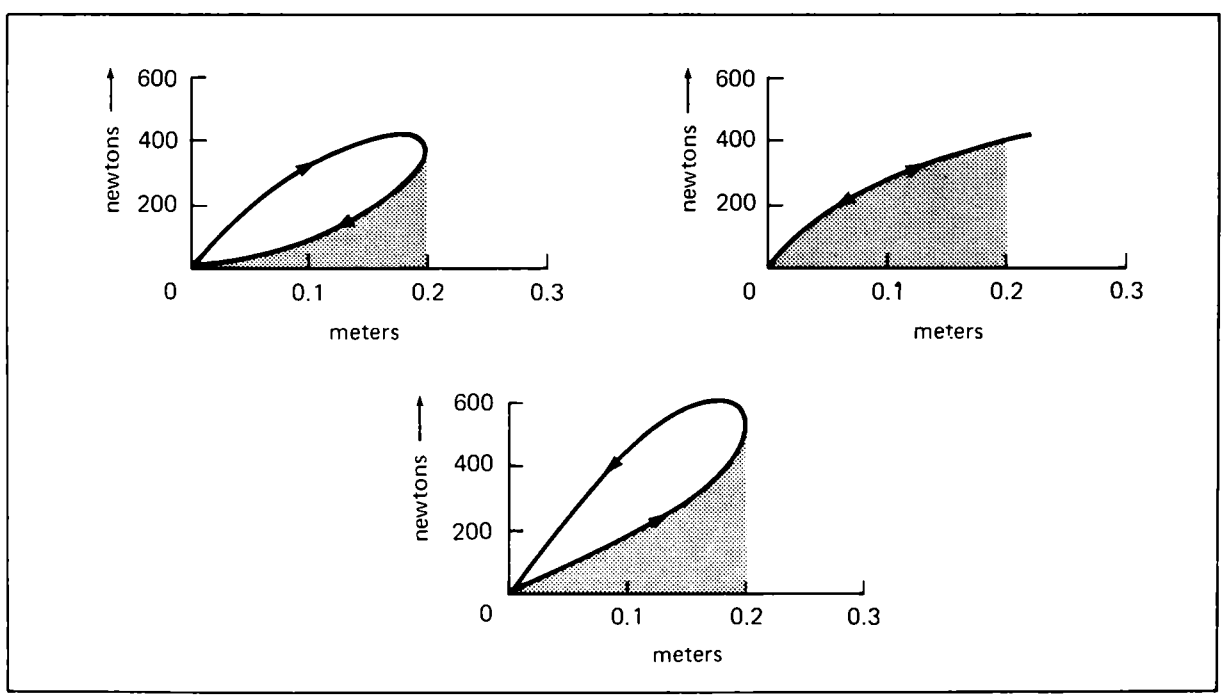
A 1.0-kg mass moving with a velocity of 10 m/sec strikes a spring, compressing it a distance of 0.20 m. The mass rebounds with a speed of 8.0 m/sec.

(a) What is the loss in kinetic energy of the mass?
(b) What happens to this lost energy?
(c) Which of the force-compression curves in Fig. 16–15 do you think is most likely correct for this spring?

(a) $(E_K)_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(1.0)(10)^2$
$= 50$ newton-meters.
$(E_K)_f = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}(1.0)(8.0)^2$
$= 32$ newton-meters.
$\Delta E = 50 - 32$
$= \underline{18\text{ newton-meters or joules.}}$

(b) As the mass is brought to rest, it compresses the spring, doing work equal to its loss in kinetic energy (50 joules). On the way out, the spring transfers back to the mass only 32 joules. While we cannot say definitely what happened to the "lost" energy, one possibility is that mechanical energy was transformed into thermal energy during the bending and unbending of the spring. Another possibility is that heat is generated through friction of the mass sliding along some surface.

(c) Qualitatively, graph (a) can account for the data; the area under the upper curve (compression varying from 0 to 0.2 m) is greater than the area under the lower curve (compression varying from 0.2 m to 0 m). The first area is equal to the work done on the spring in compressing it, the second area is equal to the work given back by the spring in expanding back to its normal length. The area under the upper curve should equal the



work (50 joules) required to stop the mass initially moving at 10 m/sec; and the area under the lower curve should equal the work (32 joules) required to accelerate the mass from rest to 8.0 m/sec. In graph (a), the first area is about 50 joules and the second area is about 30 joules. Curve (b) represents a case in which work put in equals energy transferred out. Curve (c) represents a case for which more energy is taken out than is put in. Neither of these situations corresponds with the facts of the problem.

16–11

What will happen if the 3.0-kg mass in Problem 4 hooks itself onto the spring and remains fastened to it?

If the mass remains attached to the spring, it will oscillate back and forth in simple harmonic motion.

We can obtain the amplitude of the oscillation by applying conservation of energy. Just as the mass reaches the spring (and hooks on), its energy is simply the kinetic energy

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(3.0\text{ kg})(2.0\text{ m/sec})^2 = 6.0\text{ joules}.$$

At the maximum compression (or extension) of the spring, the energy is wholly potential,

$$U = \tfrac{1}{2}kx^2 = 6.0\text{ joules}.$$

With $k = 100$ newton/meter, we get:

$$x^2 = \frac{2U}{k} = 2 \times \frac{6.0\text{ newton-meter}}{100\text{ newton/meter}} = 0.12\text{ m}^2$$

$$x = \pm\sqrt{0.12\text{ m}^2} = \pm 0.35\text{ meter}.$$

The mass oscillates back and forth from 0.35 meter to the right, to 0.35 meter to the left of the equilibrium position.

16–12

In Fig. 16–6, why does the large ball not gain kinetic energy while the force of the cage acts on it?

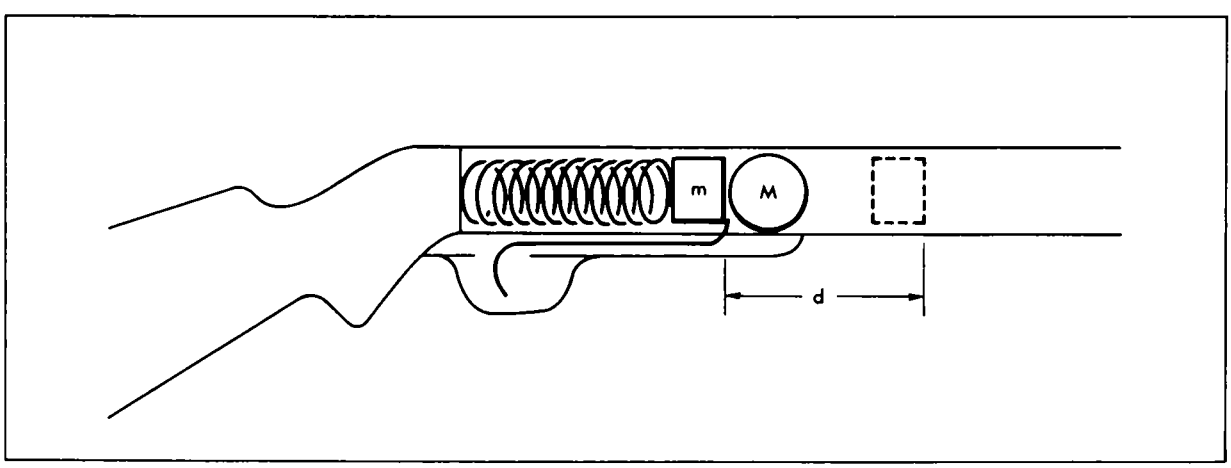
The force exerted on the large ball by the cage is just equal and opposite to the force exerted on the large ball by the small ball. Hence, the net force on the large ball is zero and it does not gain kinetic energy.

16–13

Suppose we have a spring-operated popgun something like the one in Fig. 16–16. It has a piston of mass m fastened to the end of a light spring whose force constant is k, and it shoots a ball of mass M. When the gun is loaded, the piston is pushed down the barrel until the spring is compressed through a distance d, and latched to the trigger.

Leaving out friction for the sake of simplicity, work out an argument to decide whether all the potential energy of the spring will be transferred to the ball.

While the shot is being fired, the total energy is the sum of the potential energy of the compression of the spring and the kinetic energy of the piston and the ball. The piston and ball will reach their maximum speed when the spring reaches its natural length. At this point the total energy is kinetic energy of the piston and the ball. Beyond this point, the spring stretches, increasing its potential energy and decreasing the kinetic energy and speed of the piston. The ball continues to move at a steady speed and draws away from the piston.



At the separation point,

$$\tfrac{1}{2}kd^2 = \tfrac{1}{2}(m + M)v^2,$$

where k is the force constant of the spring. Then the ball will carry off a kinetic energy

$$E_K = \frac{1}{2}Mv^2 = \frac{1}{2}\left(\frac{M}{m + M}\right)kd^2$$
$$= \frac{M}{m + M}U_{\text{spring}}.$$

Properly we should take account of the mass of the spring, which is distributed along its entire length. In this problem we have made a simplifying approximation by replacing the mass of the spring by the mass m at the end. This is a good approximation if m is one third the mass of the spring.

16–14

A linear spring with constant k is compressed between two Dry Ice pucks of mass m_1 and m_2. The spring is compressed a distance x and then tied with a thread. Both pucks are initially at rest.

(a) If the thread is burned, what is the total kinetic energy of both pucks?
(b) What are the momenta of the pucks, and how are they related?
(c) What is the kinetic energy of each puck in terms of its momentum and its mass?
(d) What fraction of the total energy does each puck acquire? What is their ratio?

There are no numbers in this problem. We suggest that you discuss it in general terms and then consider the special cases $m_1 = m_2$, $m_1 \ll m_2$, $m_1 \gg m_2$, rather than use numerical examples.

(a) The total kinetic energy after the thread is burned is equal to the energy $\underline{U = \tfrac{1}{2}kx^2}$ stored in the spring initially.

(b) If we call the velocities $\vec{v}_1$ and $\vec{v}_2$, the momenta are related by

$$\underline{m_1\vec{v}_1 = -m_2\vec{v}_2}, \text{ or } \vec{p}_1 = -\vec{p}_2.$$

(c) The kinetic energy of the first puck is

$$E_{K_1} = \tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}m_1\left(\frac{p_1}{m_1}\right)^2 = \underline{\frac{p_1^2}{2m_1}}.$$

Similarly,

$$E_{K_2} = \frac{p_2^2}{2m_2} = \frac{p_1^2}{2m_2}, \text{ from (b).}$$

(d) After the thread is burned, the total energy is entirely kinetic, so

$$E = E_{K_1} + E_{K_2} = \frac{p_1^2}{2m_1} + \frac{p_1^2}{2m_2}$$
$$= \frac{p_1^2(m_1 + m_2)}{2m_1m_2}$$

Therefore,

$$\frac{E_{K_1}}{E} = \frac{p_1^2/2m_1}{\dfrac{p_1^2(m_1 + m_2)}{2m_1m_2}} = \frac{m_2}{m_1 + m_2}.$$

Similarly,

$$\frac{E_{K_2}}{E} = \frac{m_1}{m_1 + m_2}.$$

The ratio of these two fractions is

$$\frac{E_{K_1}/E}{E_{K_2}/E} = \frac{E_{K_1}}{E_{K_2}} = \frac{m_2}{m_1}.$$

This ratio can also be obtained directly from (c). If the masses are equal, each puck takes half the energy. If one puck has a much smaller mass than the other, it will take almost all the energy. In all cases, the momenta are equal and opposite.

16–15

A spring bumper with restoring force $F = 200x$, where F is in newtons and x in meters, is compressed 0.100 meter. A 0.500-kg mass is placed next to the end of the spring and the whole thing let go.

(a) With what momentum will the mass leave the spring?
(b) We do the same thing with masses 0.125 kg, 2.00 kg, and 8.00 kg. What is the momentum of each as it leaves the spring?
(c) What is the energy of each as it leaves the spring?

(a) At the point of maximum compression, $x = x_o$, all of the energy is potential, $E = U = \tfrac{1}{2}kx_o^2$. When the spring is released the mass is accelerated until the spring reaches the equilibrium position. At this point all of the energy is kinetic, and the mass leaves the spring. Then $E = E_{K_o} = \tfrac{1}{2}mv_o^2$. Since the total energy is constant,

$$\tfrac{1}{2}mv_o^2 = \tfrac{1}{2}kx_o^2, \text{ and } v_o = x_o\sqrt{k/m}.$$

For this spring, $k = 200$ nt/m. The momentum of the 0.500-kg mass is then

$$mv_o = x_o\sqrt{km}$$
$$= 0.100 \text{ m} \times \sqrt{200 \text{ nt/m} \times 0.500 \text{ kg}}$$
$$= \underline{1.00 \text{ kg-m/sec}}.$$

(b) For a 0.125-kg mass,

$$mv_o = x_o\sqrt{km} = 0.100 \times \sqrt{200 \times 0.125} = \underline{0.500 \text{ kg-m/sec.}}$$

For a 2.00-kg mass,

$$mv_o = x_o\sqrt{km} = 0.100 \times \sqrt{200 \times 2.00} = \underline{2.00 \text{ kg-m/sec.}}$$

For an 8.00-kg mass,

$$mv_o = x_o\sqrt{km} = 0.100 \times \sqrt{200 \times 8.00} = \underline{4.00 \text{ kg-m/sec.}}$$

(c) Since the original potential energy is the same no matter what mass is put in motion by the spring, the kinetic energy is the same for each mass:

$$E_{Ko} = \tfrac{1}{2}kx_o^2 = \tfrac{1}{2} \times 200 \times 0.100^2 = \underline{1.00 \text{ joule.}}$$

Thus, when a fixed amount of energy is transferred to objects of different mass, the resulting momenta are not the same, but vary in proportion to the square root of the mass.

16–16*

A 70-kg boy climbs 5 meters up a rope. What is the increase of his potential energy? (Section 4.)

3.4×10^3 joules.

16–17

(a) A ball of mass 0.25 kg is thrown to the right at a speed of 2.0 meters per second. It starts along the frictionless surface at the left of Fig. 16–17. How high up the slope at the right will it go before coming momentarily to rest? What kind of motion will it perform?

(b) If the ball is released from rest at point *P*, what kind of motion will it perform? How high up the slope on the right will it rise? What is the binding energy (the extra energy needed to make the ball escape from the well in the center of the figure)?

(c) What is the binding energy in part (a)?

(a) The initial kinetic energy is

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.25 \text{ kg})(2.0 \text{ m/sec})^2 = 0.50 \text{ joule.}$$

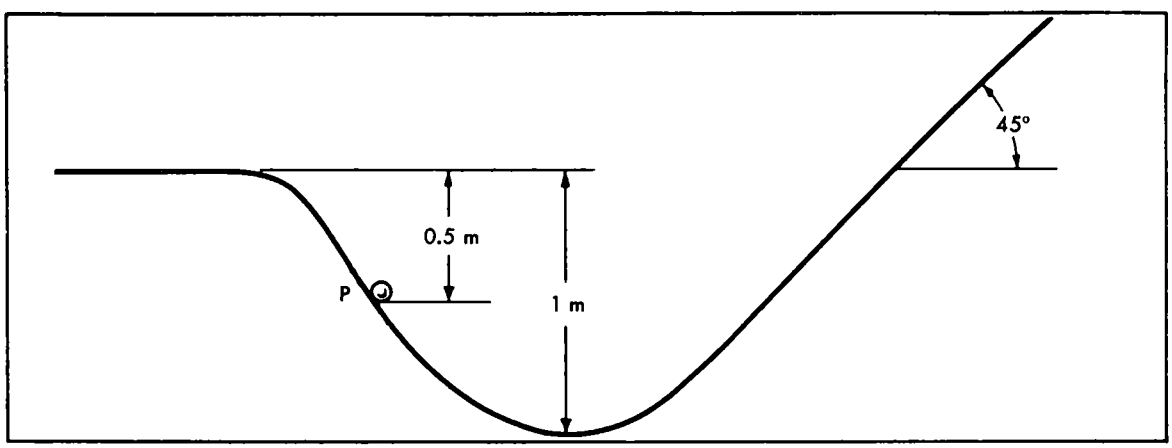
The ball gains kinetic energy (and loses potential) as it drops into the valley, but then loses kinetic energy as it rises up the other side until at the instant it reaches the original elevation it will again have 0.50 joule kinetic energy. It continues to rise to an additional vertical height h such that $mgh = 0.50$ joule. Thus,

$$h = E_K/mg = \frac{0.50 \text{ joule}}{(0.25 \text{ kg})(9.8 \text{ nt/kg})} = \underline{0.20 \text{ meter.}}$$

The additional distance it travels along the slope is $\dfrac{h}{\sin 45°} = \underline{0.29 \text{ m.}}$

The ball can be thought of as rebounding from an interaction; it comes in from the left, dips, rolls up the hill, and then reverses, retracing its path and going off to the left with the same speed it had initially.

(b) The motion will be oscillatory. The ball will rise as high on the right as it was initially on the left. To escape from the valley it would need additional energy in the amount mgh, where h is the additional height it must rise,



0.5 meter. The binding energy is then $0.25 \times 9.8 \times 0.5 = 1.2$ joules.

(c) The binding energy in part (a) is negative because the particle has kinetic energy to spare when it is free of the valley. One must take away 0.50 joule of energy to leave the ball just free of the well (with no kinetic energy on the horizontal surface). Thus the binding energy in (a) is -0.5 joule.

16–18

A linear spring whose force-extension ratio k is 40 newton/m hangs vertically, supporting a 0.80-kg mass at rest. The mass is then pulled down a distance of 0.15 m.

(a) How high will it rise?
(b) What will be its maximum velocity?
(c) How would the answers to (a) and (b) differ if the experiment were done on the moon?

This problem is straightforward if it is recognized that the constant force of gravity (mg downward) on the hanging mass has no effect beyond that of shifting the equilibrium position. This is easy to see if we consider first the forces acting on the mass when it is in the equilibrium position. The resultant force is zero; the downward pull of gravity is balanced by an upward pull by the spring of the same magnitude. What is the resultant force on the mass at other positions? The gravitational force does not change, but the spring force changes. If the mass is above the equilibrium position, the spring extension is smaller, the spring force is smaller and the resultant force is down, back toward the equilibrium position. Similarly, if the mass is below the equilibrium position, the extension of the spring is larger, the upward pull by the spring is larger and the resultant force is up, again back toward the equilibrium position. The resultant forces are just $-kd$ (where d is the displacement from equilibrium).

We can see this mathematically if we let x_0 be the amount the spring is stretched at equilibrium. Then if the spring is displaced from equilibrium by d, the total displacement of the spring relative to its unstretched position is $x = x_0 + d$. Hence, the spring exerts a force $-kx = -k(x_0 + d)$ on the mass. If we choose the positive direction as downward, the net force on the mass is $F = mg - k(x_0 + d)$.

Now, at equilibrium ($d = 0$), $F = 0$, so $mg = kx_0$. From this equation we see that x_0 is positive; according to the sign convention we chose, the spring is displaced downward from its unstretched position when the mass is hanging on it. Hence, $F = -kd$. If we take the potential energy to be zero at equilibrium, then $U = \frac{1}{2}kd^2$, just as if we had ignored gravity from the beginning.

This problem is similar to Experiment 32. The Lab Notes for that experiment contain further commentary. After these preliminaries, the questions can be answered readily.

(a) Since the force is linear we have simple harmonic motion, with the extreme points of the motion at equal distances below and above the equilibrium position. Therefore the mass will rise 0.15 meter above the equilibrium position (0.30 m above the release point).

(b) As the mass passes through the equilibrium position ($d = 0$), the potential energy will be a minimum ($U = 0$) and the kinetic energy will equal the total energy:

$$\tfrac{1}{2}mv^2 = E = \tfrac{1}{2}kd_0^2, \text{ where } d_0 = 0.15 \text{ meter}$$
$$v^2 = kd_0^2/m$$
$$v = d_0\sqrt{k/m}$$
$$= (0.15 \text{ m})\sqrt{(40 \text{ nt-m}/0.80 \text{ kg})}$$
$$= 1.1 \text{ m/sec.}$$

(c) Since both answers are independent of the gravitational field strength, the same answers are obtained on the earth, on the moon or anywhere else. The only effect of changing g is to change the extension of the spring at equilibrium, $x_0 = mg/k$.

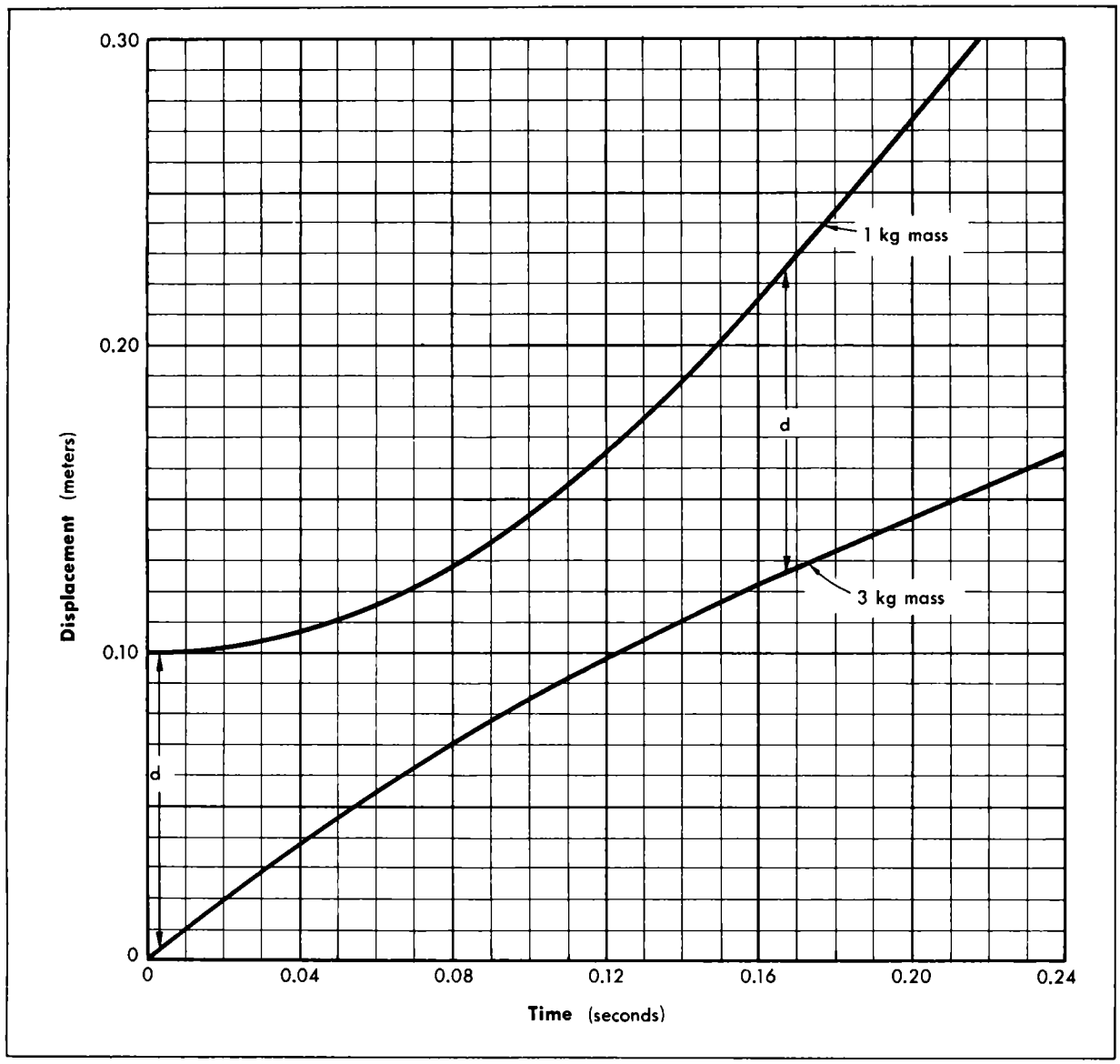
16–19

Figure 16–18 is the graph of the displacements as a function of time of two interacting masses $m_1 = 3$ kg and $m_2 = 1$ kg.

(a) What is the initial kinetic energy of m_1?
(b) What is the final kinetic energy of m_1?
(c) What is the final kinetic energy of m_2?
(d) What is the minimum total kinetic energy?
(e) What is the maximum potential energy?

To find the various kinetic energies we can measure velocities (slopes) of the appropriate displacement vs. time curve at the times of interest. To do this, draw tangent lines to the displacement vs. time curves and calculate the ratio $\Delta d/\Delta t$ for the tangent line.

(a) The initial velocity of m_1 is



$$v_1 = \frac{\Delta x}{\Delta t} = \frac{(0.20 - 0.00)\text{ m}}{(0.20 - 0.00)\text{ sec}}$$
$$= 1.0\text{ m/sec at } t = 0\text{ seconds.}$$

The initial kinetic energy of m_1 is then

$$E_{K_1} = \tfrac{1}{2}m_1{v_1}^2 = \tfrac{1}{2}(3\text{ kg})(1.0\text{ m/sec})^2$$
$$= \underline{1.5\text{ joules.}}$$

(b) The final velocity of m_1 is

$$v_1' = \frac{\Delta x}{\Delta t} = \frac{(0.16 - 0.04)\text{ m}}{(0.23 - 0.00)\text{ sec}}$$
$$= 0.52\text{ m/sec at } t > 0.17\text{ sec.}$$

The final kinetic energy of m_1 is

$$E_{K_1}' = \tfrac{1}{2}m_1(v_1')^2 = \tfrac{1}{2}(3\text{ kg})(0.52\text{ m/sec})^2$$
$$= \underline{0.4\text{ joule.}}$$

(c) The final velocity of m_2 is

$$v_2' = \frac{\Delta x}{\Delta t} = \frac{(0.23 - 0.04)\text{ m}}{(0.17 - 0.04)\text{ sec}}$$
$$= 1.5\text{ m/sec at } t > 0.17\text{ sec.}$$

The final kinetic energy of m_2 is

$$E_{K_2}' = \tfrac{1}{2}m_2(v_2')^2 = \tfrac{1}{2}(1\text{ kg})(1/.5\text{ m/sec})^2$$
$$= \underline{1.1\text{ joules.}}$$

We note that energy is conserved in the interaction. Before the interaction started,

$$E = (E_K)_1 + (E_K)_2 = 1.5 + 0.0 = 1.5 \text{ joules.}$$

After the interaction is over,

$$E = (E'_K)_1 + (E'_K)_2 = 0.4 + 1.1 = 1.5 \text{ joules.}$$

(d) The kinetic energy is a minimum and the potential energy a maximum at the distance of closest approach. Moving a piece of paper along the two curves shows that the distance $x_2 - x_1$ is at least at $t = 0.085$ second. At this time the relative velocity is zero, that is, both masses have the same velocity. As a check, the two slopes can be measured:

$$v_1 = \frac{\Delta x}{\Delta t} = \frac{(0.16 - 0.01)\text{ m}}{(0.20 - 0.00)\text{ sec}} = 0.75 \text{ m/sec at } t = 0.085 \text{ sec.}$$

$$v_2 = \frac{\Delta x}{\Delta t} = \frac{(0.22 - 0.07)\text{ m}}{(0.20 - 0.00)\text{ sec}} = 0.75 \text{ m/sec at } t = 0.085 \text{ sec.}$$

The kinetic energy at this time is:

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}(m_1 + m_2)v^2 = \tfrac{1}{2}(3 + 1)(0.75)^2 = \underline{1.1 \text{ joules.}}$$

(e) The total energy equals the initial (or final) kinetic energy, if we take the potential energy to be zero at separation greater than the range d of the interaction. The total energy is therefore 1.5 joules. At $t = 0.085$ second, the kinetic energy was 1.1 joules. Thus the potential energy at this time must have been:

$$U = E - E_K = 1.5 \text{ joules} - 1.1 \text{ joules} = \underline{0.4 \text{ joule.}}$$

16–20

A 0.200-kg stone is thrown upward from a point 20.0 meters above the earth's surface at an angle of 60 degrees with the horizontal and with a speed of 20.0 m/sec.

(a) What is its total energy?

(b) What will be its total energy when it is 15.0 m above the earth's surface?

(c) What will be its speed 15.0 m above the earth?

To compute the total energy of the stone we must first select a convenient zero of potential energy. For convenience we choose the surface of the earth.

(a) The total energy E is equal to the sum of the potential and kinetic energies at any instant.

$$E = E_K + U = \tfrac{1}{2}mv^2 + mgh$$
$$E = \tfrac{1}{2}(0.200 \text{ kg})(20.0 \text{ m/sec})^2 + (0.200 \text{ kg})(9.8 \text{ m/sec}^2)(20.0 \text{ m})$$
$$E = 40 \text{ joules} + 39.2 \text{ joules} = \underline{79.2 \text{ joules}}$$

(b) Since the *total* energy in this system is conserved (we neglect air resistance) the total energy is 79.2 joules at all points on the path.

(c) $E = E_K + U$ at 15.0 m

$$E = \tfrac{1}{2}mv^2 + mgh$$
$$v^2 = \frac{E - mgh}{\frac{1}{2}m} = \frac{(79.2 \text{ joules}) - (0.200 \text{ kg})(9.8 \text{ m/sec}^2)(15.0 \text{ m})}{\frac{1}{2}(0.200 \text{ kg})}$$
$$v^2 = 498 \text{ m}^2/\text{sec}^2$$
$$v = \underline{22.3 \text{ m/sec.}}$$

We take the positive sign because we are asked only for the speed.

Note that the answers are completely independent of the angle which the velocity of the stone makes with the horizontal. Kinetic energy depends only on speed; it does not matter whether the body is moving up, down, or sideways.

16–21

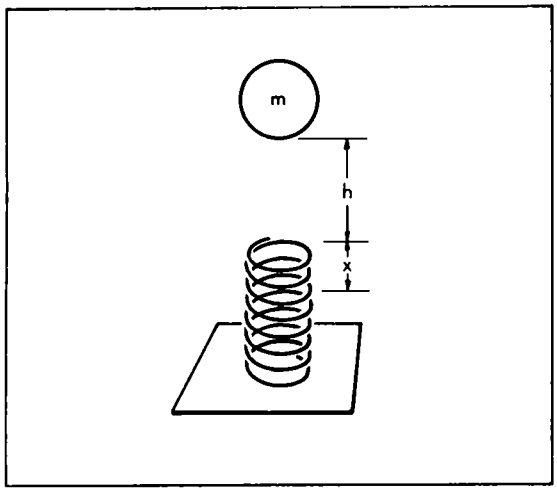
A ball of mass m drops from a height h, as shown in Fig. 16–19, and compresses the spring, of force constant k, a distance x. The mass of the spring is negligible compared with the mass of the ball.

(a) Express the maximum compression of the spring, x, in terms of m, h, and k.

(b) Evaluate x if $m = 4$ kg, $h = 3$ meters, and $k = 500$ newtons/meter.

(a) When the spring is compressed a distance x, the energy stored is the work done in compressing the spring. The force F to compress the spring varies linearly from zero to kx. The energy stored in the spring is $\frac{1}{2}kx^2$. This energy comes from the change of potential energy of the ball in the earth's gravitational field, which is $mg(h + x)$.

The positive direction of x is taken as downward with the origin at the initial position of the top of the spring. At the point of maximum compression, where the kinetic energy is zero:



$$\tfrac{1}{2}kx^2 = mg(h + x), \text{ or}$$
$$\tfrac{1}{2}kx^2 - mgx - mgh = 0.$$

So

$$x = \frac{mg}{k} \pm \frac{1}{k}\sqrt{(mg)^2 + 2\,kmgh},$$

and the + sign gives the value of maximum compression.

The first term mg/k is the distance the spring would compress to reach its equilibrium position when the ball is at rest on the spring. (See Problem 18.) The $\pm 1/k\sqrt{(mg)^2 + 2kmgh}$ is the amplitude of oscillation about the equilibrium position if the ball sticks to the end of the spring. (The − sign gives the maximum *extension* of the spring when the ball is attached.)

(b) When the values are substituted, we obtain

$$x = \frac{(4)(9.8) + \sqrt{(16)(9.8)^2 + 2(500)(4)(9.8)(3)}}{500}$$
$$= \underline{0.78 \text{ meter.}}$$

16–22

Describe the energy changes of the diver shown in the multiflash photograph in Fig. 16–20. Why does he first jump almost straight up?

The first view, at the right, shows the diver after he has made a first jump; his gravitational potential energy is decreasing and his kinetic energy increases. In the second view he is momentarily at rest in the vertical direction, his kinetic energy having been exchanged for elastic energy of the board. In the third view, he has jumped from the rising board, and now his kinetic energy is decreasing and his gravitational potential energy is increasing. In the fourth view his gravitational potential energy is maximum and his kinetic energy minimum. Finally he is once again exchanging potential energy for kinetic energy.

By springing from the depressed board, he will add the impulse given him by his spring to that given him by the board as it recovers, and hence his momentum—and kinetic energy—is increased as he leaves the board.

16–23†

(See the Appendix in the text.)

16–24

The force field between a pair of protons is repulsive. Does the potential energy increase or decrease as a pair of protons are brought together?

Since the force between the protons is repulsive, they resist being pushed together, or fly apart if released from rest. When accelerating apart, they are changing potential energy to kinetic energy. There is therefore more potential energy when they are compressed and less as they go apart.

Another way to look at the situation is to note that we transfer energy to the protons in order to push the protons together. Consequently their potential energy is increased when they are brought closer together. The situation is analogous to that of a compressed spring.

16–25*

A satellite in a circular orbit about the earth has a kinetic energy E_K. How much more energy would be required to make it escape from the earth? (Section 6.)

An additional E_K.

16–26*

A rocket of mass m ceases to climb when it has reached a distance $10r_e$ from the center of the earth. How much more energy would it require in order to escape? (Section 6.)

$GMm/(10r_e)$.

16–27*

How do the escape speeds of a 10-mg grain of sand and a 10^3-kg rocket compare? Ignore the effects of air resistance. (Section 6.)

They are equal.

16–28

What is the binding energy of the proton and electron in a hydrogen atom if the proton and electron are 0.50×10^{-10} m apart and the force of attraction between them is given by

$$F = \frac{2.3 \times 10^{-28}}{r^2} \text{ newtons,}$$

when r is in meters?

Remember, the electron will not stand still long with a force on it. Assume it is moving in a circle around the proton.

In Section 16–5 the potential energy corresponding to an attractive force GMm/r^2 was shown to be $U = -GMm/r$, if the potential energy is taken to be zero at infinite separation. The force of attraction between an electron and a proton is k/r^2. The corresponding potential energy is therefore $U = -k/r$. The total energy is then:

$$E = -k/r + \tfrac{1}{2}mv^2,$$

where m is the mass of the electron and v its speed.

Since the mass of the proton is much greater than that of an electron, we can (to a good approximation) neglect its motion just as we neglect the motion of the sun due to the force of the planets. For uniform circular motion, the acceleration toward the center (i.e., the acceleration of the electron toward the proton) is v^2/r. Equating $\frac{mv^2}{r}$ to force then gives:

$$\frac{mv^2}{r} = \frac{k}{r^2}, \text{ or } mv^2 = \frac{k}{r}.$$

Thus the total energy is:

$$E = -\frac{k}{r} + \frac{1}{2} \times \frac{k}{r} = -\frac{k}{2r}.$$

If the electron were pulled just free of the proton the total energy of the system would be 0 (0 kinetic energy is infinite separation, where $U = 0$ also). Therefore to free the electron we would need to add an amount of energy:

$$E_B = \frac{k}{2r} = \frac{2.3 \times 10^{-28}}{2 \times 0.50 \times 10}$$

$$= \underline{2.3 \times 10^{-18} \text{ joule.}}$$

16–29

Find the binding energy, to two significant figures, of:
(a) a 70-kg man to the earth.
(b) the moon to the earth.

(a) If we assume the man is initially at rest, the binding energy is

$$\frac{GMm}{r_e}$$

$$= \frac{(6.7 \times 10^{-11} \text{ m}^3/\text{kg-sec}^2)(6.0 \times 10^{24} \text{ kg})(70 \text{ kg})}{6.4 \times 10^6 \text{ m}}$$

$$= \underline{4.4 \times 10^9 \text{ joules.}}$$

Some students may be concerned about the effect of the motion of a man on the equator due to the rotation of the earth. However, since the apparent force exerted by the earth on an object at the equator is only decreased by a factor of 0.997 because of this rotation (see Section 12–11), the binding energy will not be affected within the accuracy of two significant figures. For a quantitative calculation, see Problem 30.

(b) The moon is in a circular orbit around the earth. In the case of an object in a circular orbit, the binding energy is one half the magnitude of the potential energy:

$$E_B = GMm/2r.$$

$$E_B = \frac{(6.7 \times 10^{-11} \text{ m}^3/\text{kg-sec}^2) \times (6.0 \times 10^{24} \text{ kg})(7.3 \times 10^{22} \text{ kg})}{2(3.8 \times 10^8 \text{ m})}$$

$$= \underline{3.9 \times 10^{28} \text{ joules.}}$$

16–30

Would the kinetic energy you would have at the equator due to the rotation of the earth make much difference in your binding energy to the earth?

A person rotating with the earth at speed v_e at the equator has a binding energy of $GMm/r_e - mv^2_e/2$, where

$$v_e = \frac{2\pi r_e}{T}.$$

$$v_e = \frac{2\pi(6 \times 10^6 \text{ m})}{9 \times 10^4 \text{ sec}} = 4 \times 10^2 \text{ m/sec.}$$

For a 70-kg person

$$E_K = \frac{mv_e^2}{2} = \frac{(70 \text{ kg})(4 \times 10^2 \text{ m/sec})^2}{2}$$

This is only 10^{-3} as great as the binding energy

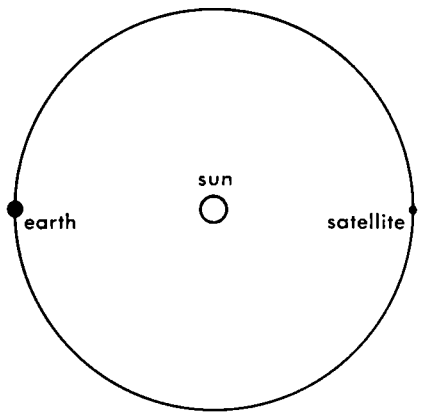
calculated in Problem 29(a), so this correction can be neglected in a two-significant-figure calculation.

16–31

(a) How much work would be required to launch a 100-kg satellite from the earth and cause it to move in the earth's orbit but on the opposite side of the sun, as shown in Fig. 16–21?

(b) What work would be required for this satellite to leave the solar system from this point?

(c) What is the ratio of the energy required to escape the solar system compared with the energy required to escape the earth, for a satellite launched from the earth?



(a) The satellite is *already* in orbit around the sun and needs only to escape the earth. When the satellite is on the opposite side of the sun, it is 5×10^4 earth radii away from the center of the earth and its potential energy due to the earth's gravitational field is only 2×10^{-5} of its value on the earth.

This energy value is much smaller than our last accurate figure of the satellite energy at the surface of the earth. Therefore, we can approximate that the satellite has zero potential energy at this point. Let r_e be the radius of the earth; then the work needed to get the satellite into the required orbit is

$$E_E \approx GMm/r_e,$$

$$\underline{E_E = 6.24 \times 10 \text{ joules.}}$$

(b) We use the sun's mass and the radius of the earth's orbit to calculate the work required for the satellite to escape from the solar system, starting at the earth's orbit. Let r_o = radius of the earth's orbit = 1.49×10^{11} meters. Because the satellite is in a circular orbit around the sun, its binding energy to the sun is

$$E_S = \frac{GMm}{r_o} - \frac{mv^2}{2}$$

$$= \frac{GMm}{r_o} - \frac{GMm}{2r_o} = \frac{GMm}{2r_o}$$

$$= \frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg-sec}^2) \times (1.98 \times 10^{30}\ \text{kg})(100\ \text{kg})}{2(1.49 \times 10^{11}\ \text{m})}$$

$$= \underline{4.43 \times 10^{10} \text{ joules.}}$$

(c) To escape out of the solar system from the earth requires enough energy to escape from both earth and solar system:

$$E_T = E_E + E_S, \quad E_T = 5.05 \times 10^{10} \text{ joules}$$

$$\frac{E_T}{E_E} \approx \underline{8}.$$

To escape from the solar system requires approximately 8 times as much energy as to escape from the earth.

16–32*

You are given a very complicated mechanism with levers and wheels in it. It is known that there is no motor inside and that there is very little friction. A string comes out of each side of the apparatus. In order to lift a body 0.10 m on the right-hand string, you must pull 6.0 m on the left-hand string. With what force do you have to pull the left-hand string in order to lift a 12-newton weight on the right-hand string? (Section 7.)

$F = 0.20$ newtons.

16–33

A drainpipe pointing slightly downhill sticks out from a retaining wall at the side of a road. A ball thrown up the pipe comes back with a speed greater than that with which it was thrown. If you observed this, would you be surprised? What would you suspect?

Whether or not one would be surprised no doubt depends on whether he realized that the returning ball has gained in total energy (the potential energy is the same as when it was thrown since it is at the same place, but the kinetic energy is larger). Let's hope the students would be surprised. The most probable explanation would be that someone had caught it at the other end of the pipe and thrown it back down.

CHAPTER 17

17–1*
By how much does the pressure of a gas in a steel cylinder increase if the temperature changes from 25°C to 100°C? (Section 1.)

P increases by a factor of $\frac{373}{298} = 1.25$.

17–2†
(See the Appendix in the text.)

17–3*
If a molecule of mass m and velocity v hits a wall head on and rebounds with velocity $-v$, what is the impulse given to the wall? (Section 2.)

The impulse given to the wall is $2\,mv$, by conservation of momentum.

17–4
A machine gun fires a stream of 10-gram bullets at the rate of 400 rounds per minute. The bullets, moving at velocity 300 m/sec, hit a wall of solid rock and stop dead. Calculate:
(a) the force on the wall.
(b) the kinetic energy of the bullets arriving at the wall in one minute.
(c) the kinetic energy of the bullets in a 1-meter length of the stream as they approach the wall. Compare twice this answer with your answer to (a).

(a) The force is the change in momentum of one bullet times the bullets per second (400 bullets/min = 6.67 bullets/sec).

$$F = (6.67/\text{sec})\,\Delta p = (6.67/\text{sec})m\,\Delta v$$

$$F = (6.67/\text{sec})(10^{-2}\text{ kg})(300\text{ m/sec}) = 20\text{ newtons}$$

(b) In one minute there are 400 bullets hitting the wall, each with $E_k = \frac{1}{2}mv^2 = 450$ joules. Therefore, the total kinetic energy is 400×450 joules $= 1.8 \times 10^5$ joules.

(c) Traveling at a speed of 300 m/sec, each bullet could travel 300 m in one second. Hence, the 6.67 bullets fired each second would be spread out over an interval of 300 meters, and the number of bullets per meter must be

$$\frac{6.67\text{ bullets}}{300\text{ meters}} = 2.22 \times 10^{-2}\text{ bullet/meter.}$$

Since each bullet has a kinetic energy of 450 joules, the kinetic energy per meter length of the stream is

$$(2.22 \times 10^{-2}\text{ bullet/meter})(450\text{ joules/bullet}) = \underline{10\text{ joules/meter.}}$$

Twice this is 20 joules/meter = 20 newtons, since 1 newton-meter equals 1 joule. This illustrates the relationship:

$$F = mv^2 A\frac{N}{V}.$$

As we found in part (a), $F = 20$ newtons. The right-hand side of this equation can be written

$$mv^2 A\frac{N}{V} = (mv^2)\frac{NA}{LA} = 2(\tfrac{1}{2}mv^2)\frac{N}{L},$$

where L is the length of path in which N particles are to be found. This expression represents just twice the kinetic energy/meter of the stream.

17–5*
How does the total kinetic energy of the center-of-mass motion of the molecules in a sample of gas depend on the number of molecules N, the volume V, and the pressure of the gas P? (Section 2.)

The total kinetic energy of the center-of-mass motion is proportional to PV/N.

17–6*
By how much does the average kinetic energy of the center-of-mass motion of the molecules of a gas change if the gas is warmed from 25°C to 100°C? (Section 3.)

E_K increases by a factor of $\frac{373}{298} = 1.25$.

17–7*
10^{20} atoms of helium at 60°C are mixed with 10^{20} atoms of argon at 30°C. What is the temperature of the mixture? (Section 3.)

45°C.

17–8*
10^{20} molecules of helium at 100°C are mixed with 10^{20} molecules of gas X at 20°C, and the final temperature of the mixture is 50°C. Can a molecule of gas X contain only one atom? (Section 3.)

No.

17–9
In a certain gas $\frac{2}{5}$ of the energy of the molecules is

tied up in motion of the atoms around each other, and $\frac{3}{5}$ in motion of the centers of mass.

(a) On the average, what is the kinetic energy of the center-of-mass motion of one such molecule when the temperature is 300°K?

(b) If the temperature is raised 1°C, what energy must be supplied to 6.02×10^{23} molecules of the gas?

(a) The temperature measures only the average kinetic energy of the center of mass of the molecules. Numerically,

$$E_K = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\,\text{joule}/°K) \times (300°K) = 6.15 \times 10^{-21}\,\text{joule}.$$

(b) We are told that the answer found in (a) is only $\frac{3}{5}$ of the actual kinetic energy. The other $\frac{2}{5}$ exists in the form of rotational kinetic energy. When we raise the temperature of the gas by 1°K, the kinetic energy of the center of mass and the energy of motion about the center of mass will increase together. For the center of mass, $E_K = \frac{3}{2}kT$, and for a temperature rise of ΔT, $\Delta E_K = \frac{3}{2}k\,\Delta T$. The total kinetic energy of one molecule will therefore be increased by:

$$\tfrac{5}{3}(\tfrac{3}{2}k\,\Delta T) = \tfrac{5}{2}k\,\Delta T.$$

To produce a 1°K temperature rise in the 6.02×10^{23} molecules of gas, the required increase of energy will be:

$$\tfrac{5}{2}(6.02 \times 10^{23})(1.38 \times 10^{-23}\,\text{joule}/°K) \times (1°K) = \underline{20.8\,\text{joules}}.$$

17–10

6.02×10^{23} molecules of an ideal monatomic gas (in practice, helium or argon) is placed in a cylinder at temperature 273°K. The gas is at atmospheric pressure, 1.02×10^5 newtons/m^2. At this pressure and temperature the gas occupies 2.24×10^{-2} m^3.

A piston in the cylinder is then pushed in to decrease the volume by 2.45×10^{-4} m^3.

(a) How much mechanical work must be done to push the piston in? (Neglect the change in pressure.)

(b) What is the final temperature of the gas if the container is completely insulated? (12.4 joules of energy raise its temperature one degree.)

(c) By what fraction of its original value does the pressure change?

(a) The amount of work done in pushing the piston in a small distance is $P\Delta V$, where ΔV is the decrease in volume. (The argument is the same as that of Problem 2 except that the work must be done *by* some external agency.)

$$P\,\Delta V = (1.02 \times 10^5\,\text{nt/m}^2)(2.45 \times 10^{-4}\,\text{m}^3) = 25.0\,\text{joules}.$$

(b) If the container is completely insulated, then all of the work done on the gas in compressing it (25.0 joules) must be transformed into kinetic energy of the gas molecules. 25 joules will raise its temperature by 25.0 joules/12.4 joules/°K, i.e., a fraction over 2°K. The final temperature will be 273° + 2° = $\underline{275°\text{K}}$.

(c) In part (b), we found that the temperature must have changed by 2°K. The equation of state for an ideal gas is $PV = NkT$. Since N is the same in the initial and final states, we can write:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \text{ or } \frac{P_2}{P_1} = \frac{V_1}{V_2} \times \frac{T_2}{T_1}.$$

Substituting the volume and temperature values, we get:

$$\frac{P_2}{P_1} = \left(\frac{2.24 \times 10^{-2}}{2.22 \times 10^{-2}}\right)\left(\frac{275}{273}\right) = 1.02.$$

Hence, the pressure increases by about $\underline{2\%}$ of its original value.

17–11

(a) Estimate the speed of oxygen molecules at room temperature from the following data: 32 grams of oxygen at room temperature (20°C) at one atmospheric pressure (1.02×10^5 newtons/m^2) occupy 2.4×10^{-2} m^3.

(b) The same volume of hydrogen at the same temperature and pressure has a mass of only 2 grams. Estimate the average speed of hydrogen molecules at room temperature.

(c) From your answer to (a), estimate the average speed of nitrogen molecules at room temperature, within 10 percent.

(d) What is the average speed of air molecules at room temperature, within 10 percent?

(e) What is the speed of oxygen molecules at room temperature and two atmospheres pressure?

(a) In Section 2, the following equation relating P, N, V, and the molecular speeds was obtained:

$$P = \tfrac{1}{3}mv^2\frac{N}{V}$$

The number of molecules, N $= \dfrac{\text{mass of sample}}{\text{mass of molecule}} = \dfrac{M}{m}$. Substituting,

$$P = \tfrac{1}{3}v^2 \frac{M}{V}$$

Solving for v,

$$v = \left(\frac{3PV}{M}\right)^{\frac{1}{2}}$$

$$v = \left(\frac{3 \times 1.02 \times 10^5 \text{ nt/m}^2 \times 2.4 \times 10^{-2} \text{ m}^3}{32 \text{ g} \times 10^{-3} \text{ kg/g}}\right)^{\frac{1}{2}}$$

$$v = (2.3 \times 10^5)^{\frac{1}{2}} \text{ m/sec}$$

$$v = 4.8 \times 10^2 \text{ m/sec.}$$

(b) The same method as in part (a) can be used to solve this problem; however, it is a little simpler to use a slightly different argument. At the same *temperature*, the average kinetic energy of the center-of-mass motion of hydrogen is the same as that of oxygen:

$$\tfrac{1}{2}m_H{v_H}^2 = \tfrac{1}{2}m_O{v_O}^2.$$

Therefore,

$$v_H/v_O = \sqrt{m_O/m_H} = \sqrt{32/2} = 4$$

$$v_H = 4v_O = 1.9 \times 10^3 \text{ m/sec.}$$

(c) From Table 3, the ratio of the mass of an oxygen molecule to a nitrogen molecule is $\dfrac{m_o}{m_n} = \dfrac{53.2 \times 10^{-24}\text{g}}{46.6 \times 10^{-24}\text{g}} = 1.14$. Using the argument of part (b),

$$\frac{V_n}{V_o} = (1.14) = 1.07$$

Thus, to within 10 percent they are the same.

(d) Air is composed predominantly of oxygen and nitrogen (in the ratio 20% : 80%). Since the velocities of their molecules are the same to within 10%, the average speed for air molecules is, to this approximation, the same as that obtained in part (a):

$$v_{av} = 4.8 \times 10^2 \text{ m/sec.}$$

(e) In part (a) we found the speed of oxygen molecules at room temperature and one atmosphere of pressure. The kinetic energy of the air molecules depends only on T, so *the average speed will be the same* even though the pressure is doubled, provided T is the same (in this case it is). In the expression used in part (a), P is doubled, so that (because of the equation of state) V is halved, and PV stays constant.

17–12*

What is the total momentum of the molecules in a gram of helium gas in a container at a temperature of 300°K? (Section 4.)

Zero, if the container is not moving.

17–13

A large bag of sand is hung from a tree by a long rope. A boy shoots a bullet into the sandbag, and the bullet stays in the bag.

(a) Describe the energy changes.

(b) Suppose a 10-gram bullet is moving at 300 m/sec when it hits the bag, and the bag has a total mass of 1990 grams. Calculate the amount of kinetic energy:
  (i) the bullet had originally.
  (ii) the bullet and bag have after collision.
  (iii) that disappears.

(c) What fraction of the original kinetic energy of the bullet goes into heat?

(a) The main parts of the energy chain are:
  (i) Kinetic energy of the bullet; this is changed to
  (iia) Heat of the bullet and the sand surrounding it after it comes to rest, and
  (iib) Kinetic energy of the bag plus bullet. The bag swings, and we get the cyclic process of kinetic energy becoming gravitational potential energy, and *vice versa*. Very slowly, because of air friction, this will cease, and the energy will become
  (iii) Heat energy of the bag, its contents, and the surrounding air.

(b) (i) Originally the kinetic energy of the bullet is:

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 0.010 \text{ kg} \times (300 \text{ m/sec})^2 = 450 \text{ joules.}$$

(ii) During the collision of the bullet with the bag, *momentum* is conserved. (So is *energy*, as long as we include heat energy, but mechanical energy of bulk motion is not.) If, after the collision, bullet and bag move with velocity v', then

$$mv = (m + M)v'$$

$$v' = \frac{mv}{m + M}.$$

The kinetic energy of the bag plus bullet is

$$\tfrac{1}{2}(m + M)v'^2 = \tfrac{1}{2}(m + M)\left(\frac{mv}{m + M}\right)^2$$
$$= \tfrac{1}{2}mv^2\left(\frac{m}{m + M}\right)$$
$$= (450 \text{ joules})(10 \text{ gm}/2000 \text{ gm})$$
$$= 2.25 \text{ joules.}$$

(iii) Before the collision, the bullet had a kinetic energy of translation of 450 joules. After the collision, only 2.25 joules remain in the form of kinetic energy of bulk motion. 448 joules have disappeared from view. This energy has gone mainly into heat of the bullet and the sand immediately surrounding it. A little goes into potential energy of separation of the sand particles disturbed by the passage of the bullet. A negligible quantity goes into sound which is quickly dissipated as heat.

(c) If we assume that all of the "lost" energy goes into heat, we get for the fractional part of the initial energy that goes into heat:

$$\tfrac{448}{450} = 99.5\%$$

17–14*

In Fig. 17–1, why is it important for the masses to fall slowly? (Section 5.)

So that the kinetic energy of the masses can be ignored in comparison with the change in gravitational potential energy and the heat which flows into the water.

17–15

In one of his most famous experiments, Joule churned water with a paddle wheel driven by two loads, each of mass 14 kg, each falling vertically about 2 meters. He had about 7 kg of water to be heated. After each churning he hauled his loads up and let them fall again. What temperature rise would you expect him to find after twenty falls?

Note: This is a reversal of the logic of Joule's great experiment. You know that 4.2 joules will always raise the temperature of a gram of water 1°C; Joule was trying to find this out.

In each fall, the loss of potential energy of the two loads is 2 mgh = 550 joules. The process is repeated twenty times, so the total loss in potential energy is 1.1×10^4 joules. Assume that this is all transformed into thermal energy of the water. The amount of energy required to raise the temperature of 7 kg of water by 1° is 7 kg $\times$ $10^3 \frac{\text{gm}}{\text{kg}} \times 4.2 \frac{\text{joules}}{\text{gm°C}} = 2.9 \times 10^4$ joules. Hence 1.1×10^4 joules supplied by the falling loads will raise the temperature by 0.38°C.

(Where did the energy come from? It came from the unfortunate laboratory attendant who had to keep lifting the loads after each fall.)

17–16*

A piece of lead is repeatedly hit with a hammer. Does its internal energy increase? Does heat flow into the lead from the outside? (Section 6.)

The internal energy does increase: mechanical energy supplied by the hammer is converted into internal energy, but heat does *not* flow in from the outside.

17–17*

A hot stone is placed in a pail of cold water. Is work done by the stone on the water? (Section 6.)

No. Energy is transferred through heat flow, not by doing mechanical work.

17–18*

If the pail in the preceding problem is thermally insulated, what happens to the total internal energy of the stone and water while the stone cools down and the water warms up? (Section 6.)

The total internal energy remains constant in this closed system.

17–19*

How many joules are required to change the temperature of 50 gm of water by 5 degrees Celsius? (Section 7.)

1×10^3 joules.

17–20

A mass of 4 grams of helium at 24°C is placed in contact with water at 26°C insulated from the rest of the world. The final temperature of both is measured to be 25°C. How much water was there? (Section 7.)

From Section 17–7, 12.4 joules are required to increase the temperature of 4 grams of helium by one degree. The amount of water that will yield 12.4 joules when cooled one degree is

$$\frac{12.4 \text{ joules}}{4.2 \text{ joules/g}} = 3.0 \text{ g}.$$

17–21

A high-speed swimmer uses 120,000 joules of energy in a half-minute race. Three quarters of the energy is wasted through heat flow; the rest is dissipated by his hands and legs, by mechanical work.

(a) In 30 seconds he swims 50 meters. Estimate the average force opposing his motion.

(b) Describe the changes of form of energy in the swimming.

(c) Where and in what form is the energy that has been released when he has finished the race?

(a) If the average force opposing the swimmer's motion through the water is F then in swimming 50 meters against this force he does work equal to $Fx = (50 \text{ meters})F$. He uses 120,000 joules, of which only a quarter, i.e., 30,000 joules, goes into useful mechanical work. Hence the average force opposing his motion is:

$F = 30{,}000 \text{ joules}/50 \text{ m} = \underline{600 \text{ newtons.}}$

(b) Chemical energy of food is turned into mechanical energy by the muscles. The swimmer's kinetic and potential energies remain constant, of course, since his velocity is roughly constant, and he swims on the surface of the water; but the energy wasted as heat flow increases. Hence all of the work he does is transferred away by heat flow. Some energy first becomes kinetic energy of turbulent motion of the water through which he passes, and as this gradually dies down, the water is left at a slightly higher temperature.

(c) This has already been answered in (b): the energy expended causes the temperature of the water in the pool to be slightly warmer. With a pool of usual dimensions (25 m × 15 m × 3 m) containing about 10^6 kg of water, the temperature rise due to the addition of 120,000 joules would be

$$\Delta t = \frac{120{,}000 \text{ joules}}{4.2 \text{ (joule/gm - degree C)} \times 10^3 \text{ gm/kg} \times 10^6 \text{ kg}} = \underline{3 \times 10^{-5} \text{ °K.}}$$

17–22*

Why do the brake drums get hot when a car goes at constant speed down a long, steep mountain road? (Section 8.)

Gravitational potential energy is converted only into internal energy of the brake drum since the car does not speed up.

17–23

When you rub your hands together vigorously, they warm up but finally reach a maximum temperature no matter how long you rub. How do you account for the warming up and the eventual stopping of the temperature rise?

The temperature of your hands will rise as long as the energy transferred into them per unit time by doing work on them (i.e., rubbing them) is greater than the energy flowing out of them per unit time as heat. As the temperature of your hands rises, the rate of heat flow rises, since it is proportional to the difference in temperature between your hands and their surroundings. You will reach a point at which heat flows out as fast as energy is put into your hands, and their temperature will rise no further.

17–24

An amateur photographer had an electric fan in his darkroom to cool him when he was working there in hot weather. He would start the fan in the *closed* room 2 hours before he started to work there. Did he act wisely?

No. A few seconds after a fan is turned on, the rotation rate of the blades becomes constant. After that, all energy fed into it through the electric current is dissipated as heat as the blades increase the kinetic energy of the air molecules they strike and the fan itself warms up due to friction.

The cooling effect felt is due to local evaporation of moisture on one's skin. The temperature of the air rises, however, when a fan is operating in a closed room.

17–25

A mountain climber can climb about 500 meters vertical rise per hour.

(a) How much energy does such a climber gain as gravitational potential energy in a five-hour climb up a mountain?

(b) The human body is an inefficient chemical-mechanical machine. At best its muscles deliver only 25 percent of the chemical energy used as useful mechanical energy. The other 75 percent or more is wasted through heat flow. Assuming this efficiency, how much chemical energy does the climber use in 5 hours?

(c) Assume that, aside from mountain climbing, he needs 2.2×10^6 small calories per 24 hours. How much total energy should he take in his daily diet if he makes that climb every morning?

(d) If the climber walks down the mountain every afternoon, he loses the potential energy he gained. Why does that not help him to reduce his diet?

(a) In five hours, the increase in height of the climber is $5 \times 500 \text{ m} = 2.5 \times 10^3 \text{ m}$. If his mass is, say 80 kg, his increase in gravitational potential energy is

$$mgh = 80 \text{ kg} \times 9.8 \text{ m/sec}^2 \times 2.5 \times 10^3 \text{ m} = \underline{2 \times 10^6 \text{ joules.}}$$

(b) The above energy is only a quarter of that expended by the climber. The chemical energy used is therefore

$$4 \times 2 \times 10^6 \text{ j} = \underline{8 \times 10^6 \text{ joules.}}$$

(c) In calories, this chemical energy is

$$\frac{8 \times 10^6 \text{ j}}{4.2 \text{ j/cal}} = 2 \times 10^6 \text{ small calories.}$$

This is about the same as his normal daily calorie requirement, so on his mountain climbing days he would need approximately $\underline{4 \times 10^6 \text{ calories.}}$

(d) The only way that the human engine accepts energy is in chemical form. It is not possible to increase its energy content by doing work on it.

17–26

(a) A cylinder containing helium gas is closed with a movable piston which has negligible friction. A man pushes on the piston and drives it in quickly, compressing the helium. The helium warms up. Why does the helium warm up? Discuss the mechanism of the warming up in terms of molecular behavior.

(b) A cylinder with a movable piston contains compressed helium. The piston is released and the helium pushes it out and cools. Explain in terms of molecular behavior how the helium cools.

(c) A large box with a good vacuum in it contains a small bottle of compressed helium. A trigger is arranged to remove the stopper of the helium bottle. When the helium is let out of the bottle, no change of temperature is observed after the release is all over. Explain, from the point of view of molecular behavior, why the helium does not change temperature when it expands into the big box.

(d) Although compressed helium shows no final change of temperature, some other gases show a noticeable cooling after they have expanded into vacuum from high compression. What does that tell you about these other compressed gases?

(a) Since work was done in pushing the piston into the cylinder against the pressure of the gas, the same amount of energy must appear somewhere else. The only place it can appear is as in an increase in internal energy of the gas, i.e., as a rise in temperature.

In greater detail, when the piston is at rest, the gas atoms continually collide with it and bounce off with the same kinetic energy as they had before they hit. (The component of momentum perpendicular to the piston is reversed, and the other components are unchanged.) When the piston is moving into the gas, the atoms hit a surface moving towards them. They recoil with *increased* kinetic energy (as does a baseball from a moving bat). That is, on the average, we are increasing the kinetic energy of the molecules, i.e., the temperature rises.

(b) The argument is similar to that of part (a). Essentially, the atoms are colliding with a piston which is retreating from them. Since their momentum relative to the piston is reversed on impact, their momentum relative to the container is decreased. Thus, the thermal energy of the gas molecules decreases. On the other hand, work is done by the gas on the mechanism connected to the piston. It can be shown that the decrease in internal energy is exactly the same as the work done by the expanding gas.

(c) This process is called "free expansion." The very important difference between it and the changes in volume produced by movement of the piston is that now the atoms never collide with a moving wall which could produce a change in their kinetic energy. The atoms will rush out of the opening in the bottle and will continue to travel until they meet the stationary wall of the box. Their kinetic energies will not change, and consequently the temperature will not change.

(d) In our previous arguments, we have neglected potential energy. For helium this is justified,

since two helium atoms are very much like billiard balls; they only interact when they are almost touching. (They do not attract each other enough to form a molecule.) However, there are other gases in which, although the molecules repel each other very strongly when very close together, there is a weak attraction between the molecules when they are relatively far apart. For rarefied gases, the "ideal" relationship is a good approximation. For denser gases, when the gas expands, the average distance between the molecules is increased, and their potential energy is increased. This results in an accompanying drop in kinetic energy in addition to the changes produced by the means discussed in part (b). Thus, even in a "free expansion," the potential energy of the molecules of some gases will increase and the kinetic energy will decrease, i.e., the gas will be cooled.

CHAPTER 18

18–1*

If two charged rods repel each other, what can be said about the sign of the charge on each rod? (Section 1.)

The sign is the same, but one cannot tell whether it is positive or negative.

18–2*

You find that object *A* repels object *B*, *A* attracts *C*, and *C* repels *D*. If you know that *D* is positively charged, what kind of charge does *B* have? (Section 1.)

Negative.

18–3

Suppose that you have electrified a plastic rod by rubbing it with wool.

(a) Do you expect the wool to become charged?

(b) How could you find out if it does?

Students' answers will depend on how much of the chapter they have read.

(a) After Section 4, they should realize that when an object is charged by friction, electric charges of one sign are transferred from one to the other of the rubbed objects. Before reading Section 4, they could have any expectation.

(b) If the wool is charged, it would affect a suspended charged rod or an electroscope.

18–4*

(a) Why can't you electrify a metal rod by rubbing it while holding it in your hand?

(b) What could you do to electrify a metal rod? (Section 1.)

(a) Charge flows through the metal rod and the body.

(b) Insulate the rod from your body.

18–5

When the gasoline tank of an airplane is being filled, the metal nozzle of the hose is always carefully connected to the metal of the airplane by a wire before the nozzle is inserted in the tank. Be prepared to explain why this procedure is followed, and to describe how it accomplishes its purpose.

Since the plane and the gasoline truck usually have rubber-tired wheels, they are insulated from the ground. Moreover the service ramps are usually concrete and/or asphalt, and are not necessarily good conductors to the earth. Therefore, in spite of grounding chains, the plane and truck could remain charged. A spark could jump when the metal nozzle was brought near the plane. The connection with the wire insures that any charge unbalance will be removed before the tanks are opened. A spark when the tanks are open might ignite the explosive mixture of gasoline and air.

When the gasoline flows, it is possible for the stream of liquid to become charged by friction with the nozzle. The charged gasoline would go into the plane and charge it oppositely to the nozzle. Unless a good conducting path is provided so the charges on the plane and nozzle could recombine, a spark could result—with disastrous consequences.

18–6

(a) When you touch a metal object like a door handle on a dry winter day, you are likely to see a spark and feel a definite shock. We usually explain this by saying that we have built up a static charge. How could you tell the sign of the charge you are carrying?

(b) If you accumulate a static charge and then touch the wooden frame of a door, you often find no spark or shock—although there would be if you touched the metal handle. Why?

(c) Sometimes, if you touch the wooden frame and

then touch the metal handle, there is no spark or shock in either case—although there would have been one if you had touched the handle first. Suggest an explanation.

(a) If you are already charged, you could charge an uncharged electroscope by touching it. The electroscope would now carry the same kind of charge that you carried. You then discharge yourself by touching ground or a grounded object. The electroscope is being used as a storage device for the charge you carried. The nature of the charge on the electroscope can be found in the following way.

Charge a plastic rod with wool. By the convention described in Section 1, this charge is negative. If, when the plastic rod is brought near the knob of the electroscope, the leaves diverge even more, the charge on the electroscope is the same as on the plastic. This is negative, so *you* were charged negatively. If the leaves collapse, the electroscope charge is the opposite of the charge on the plastic. In this case you were charged positively.

If, instead of plastic and wool, glass and silk were used, the glass rod becomes positive, and the analysis above is reversed.

(b) When you are the bearer of a sizeable charge and touch a metal object, the transfer of charge is the spark. Because metal is a conductor, the charge can distribute itself quickly throughout the conductor.

When an insulator is touched, the charge is unable to flow through or over it. Hence, little charge can be transferred and there is no spark.

(c) Sometimes the insulator is not perfect and is really simply a very poor conductor. At the instant the wood is touched, the insulator does not accept charge fast enough to cause a spark; yet the contact may last long enough to allow the charge to flow slowly away from you. When you subsequently touch the door handle, there is no spark because there is no charge left on you to be transferred.

18–7*

If you observe that the leaves of an isolated electroscope are repelling each other strongly, what can you tell about the charge on the leaves and the electroscope knob? (Section 2.)

The sign of the charge is the same on the leaves as on the electroscope knob.

18–8*

What will happen if you touch together the knobs of two identical equally charged electroscopes
(a) if both are positively charged?
(b) if both are negatively charged?
(c) if one is negative and the other positive? (Section 2.)

(a) Nothing;
(b) Nothing;
(c) Both electroscopes will discharge completely.

18–9*

As you bring a charged rod near a negatively charged electroscope, the leaves start to collapse. What is the charge on the rod? (Section 3.)

Positive.

18–10*

If you move a charged rod toward a positively charged electroscope, the leaves at first collapse and then diverge. What charge is on the rod? (Section 3.)

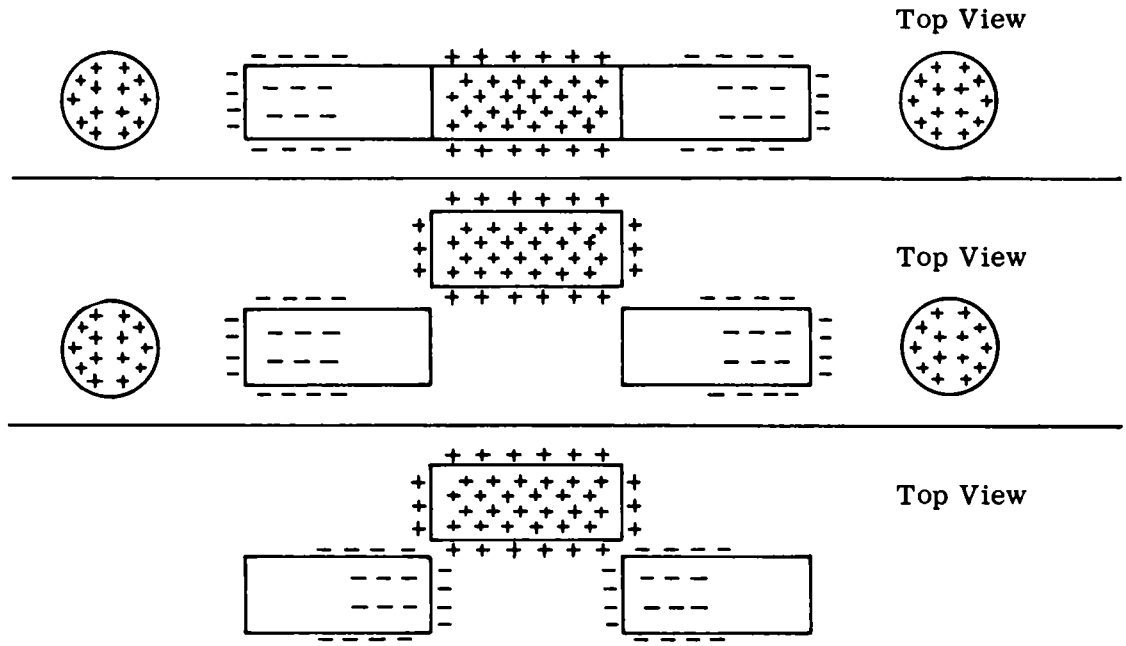
Negative.

18–11

Three metal blocks in contact are resting on a plastic tabletop. You place two objects with strong positive charges, one at each end of the line of blocks, close to but not touching the blocks. You then poke the blocks apart with an (uncharged) insulating rod, while the objects with strong positive charges are nearby. Finally you remove the two positively charged objects.
(a) What charge is now on each block?
(b) Explain how the blocks acquired these charges by describing the motion of negative particles.

(a) As shown in the diagrams, when the three blocks are separated, the two outer blocks will have negative charges and the middle block will have a positive charge.

(b) The combination of three blocks in contact is presumed to be electrically neutral at the outset. The two positively charged bodies cause a redistribution of charge. In metals, it is the negative charge that is free to wander, so the middle block loses negative charge to the two outer blocks. Thus, when the blocks are separated, the middle one is left with a



net positive charge. A return to the neutral distribution is prevented by the insulating properties of air and the table top. If the air is ionized or if the table is slightly conducting, there would be a gradual drift back to neutrality.

18–12

You have two metal spheres of the same size mounted on insulating stands, a strip of plastic, and a piece of cotton cloth. Describe what you would do to give the two spheres equal electrical charges (a) of the same sign, (b) of opposite signs.

(a) Charge the plastic strip by rubbing it with the cotton cloth. Then charge one of the two spheres, either by touching the charged plastic strip to it or, preferably, by induction. Remove the charged plastic strip and touch the two spheres together. Since they are of the same size, the charge will be divided evenly between them.

(b) This must be done by induction. Bring the two metal spheres into contact. Charge the plastic strip and hold it near, but not touching, one of the spheres. Now separate the spheres and then remove the plastic strip. The sphere farther from the plastic strip will carry a charge of the same sign as the strip; the nearer sphere a charge of the opposite sign. Since the two spheres were originally neutral, the separated charges are equal in magnitude.

18–13

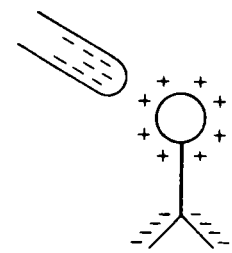
(a) Describe the steps you would take to charge an electroscope positively by induction.

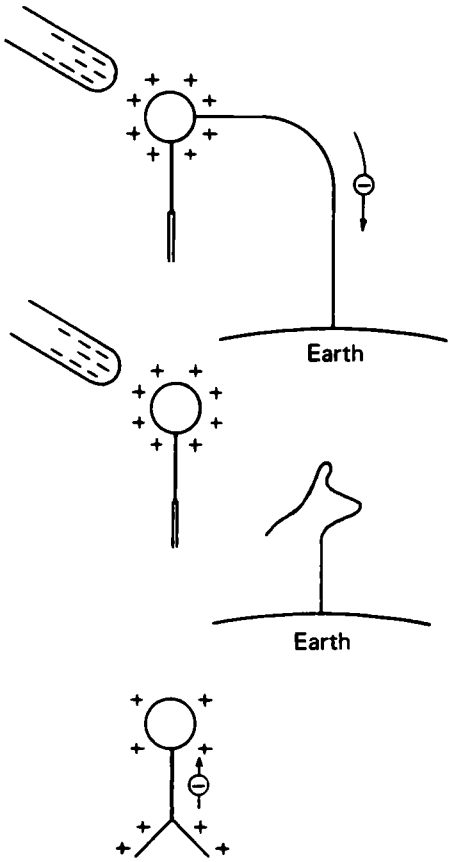
(b) Using labelled sketches, describe the movement of negative electric particles during the charging process.

(a) The first step is to obtain a negatively charged object. If you rub a plastic rod with fur, the rod will become negative.

Hold this negatively charged object near the electroscope, while the electroscope is grounded. Negative charge is repelled from the electroscope to the ground by the charge on the rod, leaving an excess of positive charge on the electroscope. The ground is removed so that the negative charge transferred to the ground cannot return; then the electroscope is left positively charged even when the negatively charged object is withdrawn.

(b)





18–14
Must all the negative charges be removed in order for a rod to become positively charged? (Section 4.)

No. The removal of any number of negative charges from a neutral object gives it a net positive charge.

18–15
How can you find out with an electroscope which terminal of a battery is positive?

Charge the electroscope with a known positively charged object. Bring the charged electroscope near one of the battery terminals. If the leaves start to diverge farther, the terminal is positively charged (see Figure 18–8). If the leaves start to collapse, the terminal is negatively charged and the other terminal must be the positive one.

18–16*
Why would an electroscope discharge more slowly in a vacuum chamber than in air? (Section 6.)

In a good vacuum there are hardly any charged ions which can remove charge from the leaves.

18–17*
Suppose the entire battery B circuit of Fig. 18–14 were replaced by a positively charged electroscope with the knob connected to the metal cylinder inside the glass envelope. What would happen when the filament was heated?

The electroscope leaves would fall.

18–18*
In Fig. 18–17, what charge would you put on the horizontal deflector nearest you to make the beam move to the left-hand side of the tube face? (Section 9.)

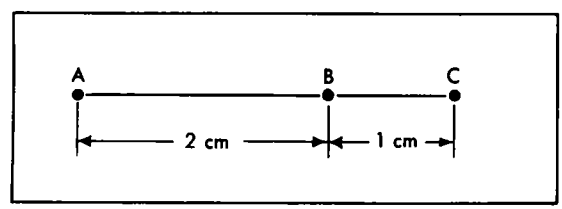
Positive.

CHAPTER 19

19–1*
Two small electrified objects A and B are separated by 0.03 meter, and repel each other with a force of 4.0×10^{-5} newton. If we move body A an additional 0.03 meter away, what is the electric force now? (Section 1.)

1.0×10^{-5} newton.

19–2
Three equally charged objects are located as shown in Fig. 19–23. The electric force exerted by A on B is 3.0×10^{-6} newton.
(a) What electric force does C exert upon B?
(b) What is the net electric force on B?



(a) Since the distance between A and B is twice that between C and B, the force of C on B must be four times that of A on B; $F_{BC} = 4 \times 3.0 \times 10^{-6}$ newton $= 1.2 \times 10^{-5}$ newton.

(b) Assuming that the three charges are of identical sign as well as magnitude, the forces are both repulsive. The force of A on B is given as 3.0×10^{-6} newton to the right. From (a), the force of C on B is 12×10^{-6} newton to the left.

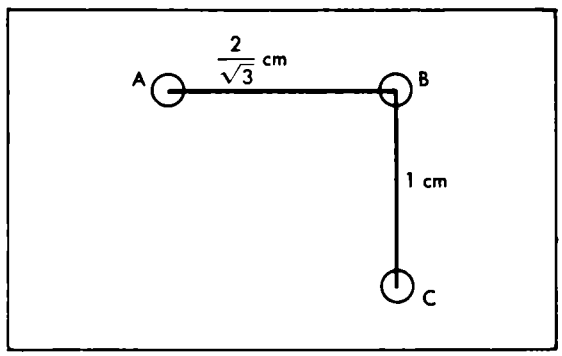
$$\therefore F_{net} = 12 \times 10^{-6} - 3 \times 10^{-6}$$
$$= \underline{9 \times 10^{-6} \text{ newton to the left.}}$$

19–3

Suppose that we place three small charged spheres, with equal charges on them, as shown in Fig. 19–24. Sphere C exerts a force of 4×10^{-6} newton on B.

(a) What force does A exert on B?

(b) What is the net force on B?



(a) Since Coulomb's law states that the force is inversely proportional to the square of the distance, then

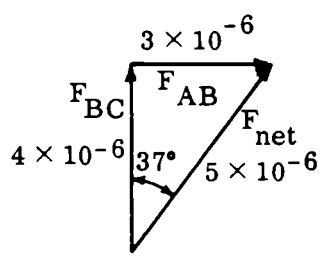
$$\frac{F_{AB}}{F_{BC}} = \frac{(1/1\text{ cm})^2}{\left(1/\frac{2}{\sqrt{3}}\text{ cm}\right)^2} = \frac{3}{4}.$$

But $F_{BC} = 4 \times 10^{-6}$ newton, so $F_{AB} = \frac{3}{4} \times 4 \times 10^{-6} = \underline{3 \times 10^{-6} \text{ newton.}}$

(b) Since the two forces acting on B are perpendicular, the magnitude of the net force will be:

$$F_{net} = \sqrt{(4 \times 10^{-6})^2 + (3 \times 10^{-6})^2}$$
$$= \underline{5 \times 10^{-6} \text{ newton.}}$$

The direction of the net force is given by the vector sum as shown.



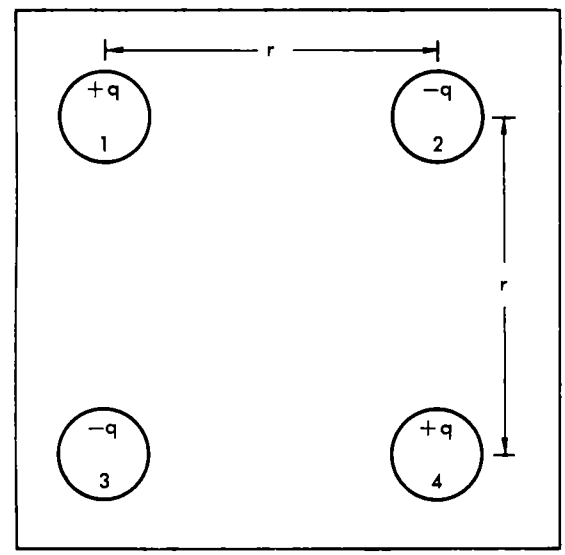
19–4

In Fig. 19–3, what is the ratio of the forces exerted on X?

The force exerted on X in each case is equal in magnitude and opposite in direction to those exerted on A and B; hence the ratio of the forces acting on X in the two cases is likewise q_A/q_B.

19–5

Four identical conducting spheres mounted on Dry Ice pucks are held on a smooth level table at the four corners of a square. Two of them carry a charge of +q and the other two carry a charge of −q each, as shown in Fig. 19–25.



(a) Make a vector diagram, to scale, showing the electric forces on sphere 1.

(b) If all the pucks are released at the same time, In what direction will each of them move?

(a) Since spheres 2 and 3 are each a distance r from sphere 1, the forces they exert on it are equal in magnitude:

$$F_{2-1} = F_{3-1} = k\frac{q^2}{r^2}.$$

Because the diagonal of a square is $\sqrt{2}$ times as long as one side,

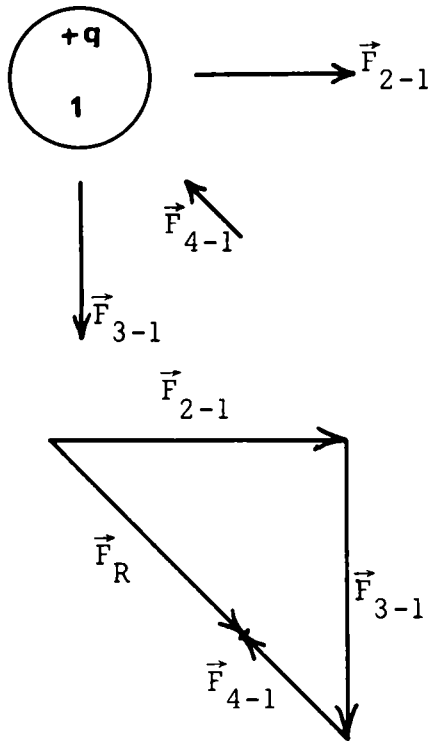
$$F_{4-1} = k\frac{q^2}{(\sqrt{2}\,r)^2} = \frac{kq^2}{2r^2}.$$

The force exerted by sphere 4 is only half as great as the force exerted by sphere 2 or 3.

The arrows on the diagram indicate the direction and relative magnitude of each force on sphere 1.

The resultant force $\vec{\mathbf{F}}_R$ equals $\vec{\mathbf{F}}_{2-1} + \vec{\mathbf{F}}_{3-1} + \vec{\mathbf{F}}_{4-1}$.

From the diagram one can see that



$$(F_R + F_{4-1})^2 = {F_{2-1}}^2 + {F_{3-1}}^2 = 2{F_{2-1}}^2.$$

Therefore,

$$F_R = \sqrt{2}\,F_{2-1} - F_{4-1}$$
$$= (\sqrt{2} - \tfrac{1}{2})F_{2-1} = \underline{0.91\,kq^2/r^2.}$$

(b) From the symmetry of the figure, one can see that the resultant force on each charge will be of the same magnitude and that each resultant will be directed toward the center of the square. When all are released at the same instant, they will move toward the center of the square.

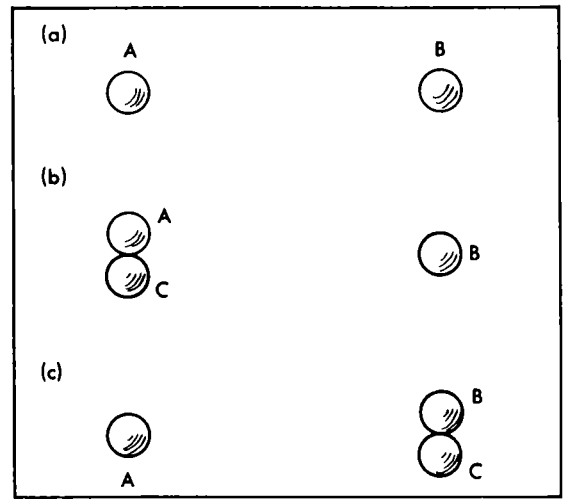
19–6

Two equally charged identical conducting spheres A and B repel each other with a force of 2.0×10^{-5} newton [Fig. 19–26(a)]. Another identical uncharged sphere C is touched to A [Fig. 19–26(b)] and then moved over right next to B [Fig. 19–26(c)].

(a) What is the electric force on A now?

(b) What is the net electric force on C (after it has touched A) when it is halfway between A and B?

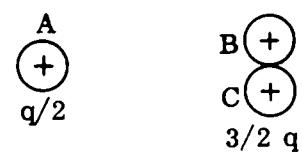
Let q be the initial charge on A and B. After C and A have touched, they will each have a charge of $q/2$.



(a) Since the force is proportional to the product of the charges,

$$\frac{F_A\ (\text{now})}{2.0 \times 10^5\ \text{nt}} = \frac{(\frac{1}{2}q)(\frac{3}{2}q)}{(q)(q)} = \frac{3}{4}.$$

$\vec{F}_A(\text{now} = \underline{1.5 \times 10^{-5}\ \text{newton to the left.}}$

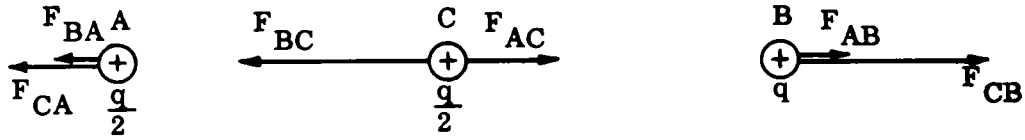


(b) If C has touched only A but not B, the charge on it is $q/2$ and we have the above situation when C is halfway between A and B.

Since the distance between A and C is half that between A and B, then the force exerted by sphere A on sphere C is:

$$\frac{F_{AC}}{2.0 \times 10^{-5}\ \text{nt}} = \frac{(q/2)(q/2)/(\frac{1}{2}\overline{AB})^2}{(q)(q)/(\overline{AB})^2} = 1.$$

$\vec{F}_{AC} = 2.0 \times 10^{-5}$ newton to the right.



Sphere B is the same distance from C as A is, but has twice the charge. Hence, the force it exerts on C is

$$\vec{F}_{BC} = 4.0 \times 10^{-5} \text{ newton to the left.}$$

The net force on sphere C due to spheres A and B is then

$$\vec{F}_{C} = \underline{2.0 \times 10^{-5} \text{ newton to the left.}}$$

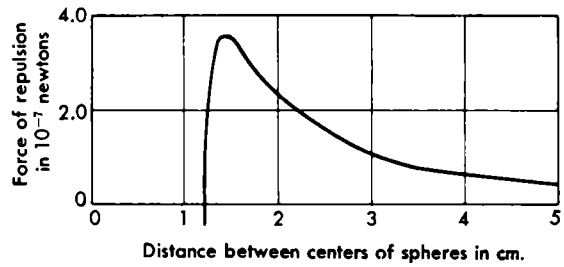
Some students may interpret the problem as meaning that C has first touched A, then touched B, and has then been moved halfway between A and B. In this case, C has shared charge twice, and the charge on it is $3q/4$, which will result in a different answer.

19–7†
(See the Appendix in the text.)

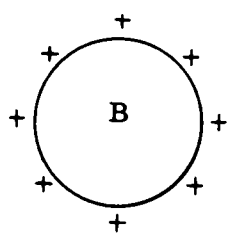
19–8
The graph in Fig. 19–28 shows the electric force of repulsion on a tiny charged conducting sphere as a function of its separation from a large conducting sphere. The large sphere has a radius of 1 cm, and has 10 times the charge of the small sphere.

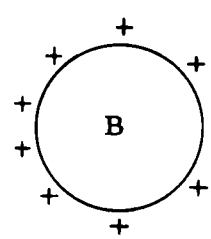
(a) How is the force changing as the separation changes from 5 cm to 3 cm?

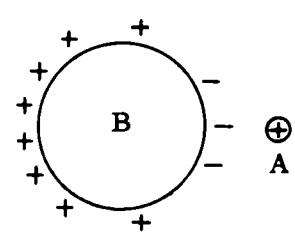
(b) Explain the behavior of the force between separations 2 cm and 1 cm.



This is an interesting problem which brings out the effects of induction in a conductor. Three special cases are shown below.







(1) Large separation: repulsion the same as between point charges.

(2) Intermediate separation: repulsion less than between point charges, because the charge on the large sphere has been effectively pushed farther away.

(3) Very close: a negative charge has been induced on the side of the large sphere. Compared to the positive charge, this negative charge is so close to the small sphere that attraction results.

(a) At 5 cm and 3 cm we measure, on Figure 19–28, ordinates of 0.40 cm and 1.1 cm respectively. Thus the ratio

$$\frac{F\,(\text{at } 3\text{ cm})}{F\,(\text{at } 5\text{ cm})} = \frac{1.1}{0.40} = \underline{2.7.}$$

The force increases by a factor of 2.7 when the separation is decreased from 5 cm to 3 cm. Had the force varied as the inverse square we would have expected a ratio

$$\frac{F\,(\text{at } 3\text{ cm})}{F\,(\text{at } 5\text{ cm})} = \frac{5^2}{3^2} = 2.8.$$

In this region, then, the force is increasing much as it would if the charges were point charges.

(b) As the spheres move closer, at 2 cm the force of repulsion between them is rising. At about 1.4 cm the force of repulsion begins to fall rapidly as a result of the induction described in (2) and (3) above. At a separation of 1.2 cm the attraction due to inductive effects equals the repulsion of the two net charges. At this separation, the net force is zero, as shown on the graph.

The graph in the text ends at a separation of 1.2 cm. As the spheres approach still more closely, the force reverses sign and becomes more strongly one of attraction. In the idealization that A is a point charge, the attractive force would become infinite. The actual attracting force would stop rising as the spheres touch. After they touch, charge will flow between them, and the character of the force changes radically. The students need only understand the role of the two effects: the repulsion of the net charges vs. the attraction due to induction.

19–9*

Match the fields shown in Fig. 19–5 with the grass-seed patterns of Fig. 19–6. (Section 3.)

19–5(a) and 19–6(a)
19–5(b) and 19–6(b)
19–5(c) and 19–6(c)
19–5(d) has no analog in 19–6.

19–10*

Can electric field lines cross one another? (Section 3.)

No. If they did, at the point they crossed the net force on a movable charge would have to be in two directions at once!

19–11

If a charged particle is free to move, will it travel along an electric field line?

Only if the line is straight and the particle is moving along the line. If the line is curved, the trajectory of the particle will, in general, be different from the direction of the force on the particle.

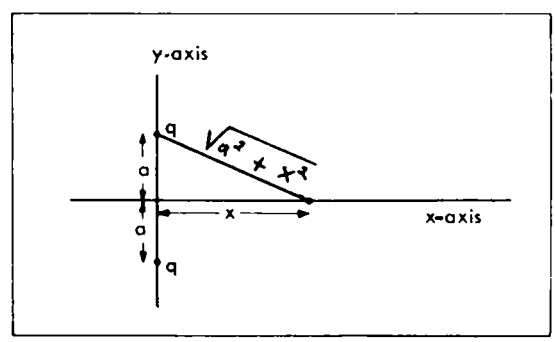
19–12*

A charge q_1 is at a distance r from a fixed charge q_2. If q_1 is allowed to move freely away under the repulsion of the Coulomb field, what will be its kinetic energy at a very large separation? (Section 4.)

The potential energy at a distance r is kq_1q_2/r. All this will be transferred to the moving charge as kinetic energy at infinity.

19–13

Two identical charges q are nailed in place on the y axis, equal distances a above and below the origin of coordinates



(a) Find an expression for the potential energy due to the interaction of a unit charge on the x axis, a distance x from the origin, with one of the nailed-down charges; with both of the nailed-down charges.

(*Note:* Measure the potential energy with the zero chosen when the unit charge is at infinity. Express your answer in terms of the constant k in Coulomb's law, the distances a and x, and the charge q.)

(b) What would be the potential energy due to the interaction of the unit charge if the two nailed-down charges were equal and opposite?

(a) The potential energy due to the Coulomb interaction of the unit charge on the x-axis with only one of the nailed-down charges on the y-axis is found from the expression $U = k(q_1q_2/r)$.

In this case $q_1 = q$

$q_2 = 1$ unit ch.

and

$$r = \sqrt{a^2 + x^2}.$$

So

$$U = k\frac{q(1 \text{ unit ch.})}{\sqrt{a^2 + x^2}}.$$

The potential energy due to the Coulomb interaction of the elementary charge with both nailed-down charges is equal to the amount of work done against the net Coulomb forces when the elementary charge is brought from infinity to its position on the x-axis. But the work done against the net force equals the (algebraic) sum of the work done by each force, or the sum of the potential energies due to the interaction with each nailed-down charge. Hence, for the interaction with both nailed-down charges:

$$U_{\text{tot}} = 2k\frac{q(1 \text{ elem.ch.})}{\sqrt{a^2 + x^2}}.$$

Note that this expression does not include a term due to the interaction of the nailed-down charges with each other. This is because we are choosing our zero of energy with the movable charge at infinity, but with the nailed-down charges in place on the y-axis.

Since the electric potential is just the electric potential energy per elementary charge, potential also can be calculated by taking the algebraic sum of the potential contributions from the individual charges that create the total electric field.

(b) If the nailed-down charges were equal and opposite, the total potential energy would be

$$U_{\text{tot}} = k\frac{(+q)(1 \text{ elem.ch.})}{\sqrt{a^2 + x^2}} + k\frac{(-q)(1 \text{ elem.ch.})}{\sqrt{a^2 + x^2}} = 0.$$

The last result, which may appear strange at first sight, comes about from the fact that the net force acting on the elementary charge is perpendicular to the x-axis so that no work is required to move the charge anywhere along the x-axis, even as far as infinity where, of course, the potential energy has been taken as zero (by definition).

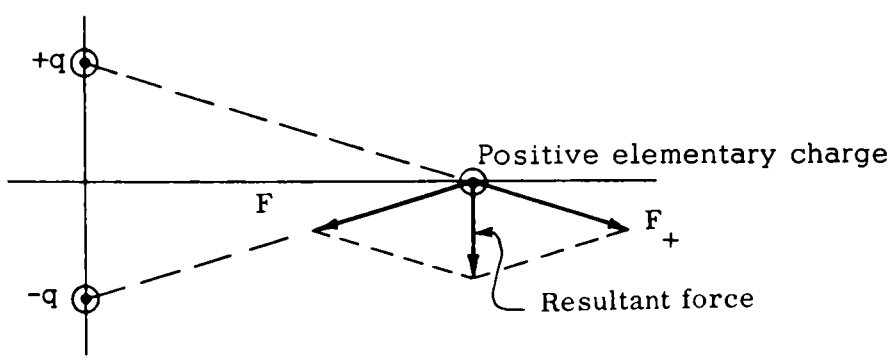
19–14

If you continue the experiment shown in Fig. 19–17, what sphere velocities will you expect to get in the situations in Fig. 19–29?

The terminal velocity of the sphere in each case is proportional to the driving force (the vector sum of the electrical and gravitational forces) on the sphere.

19–15

Suppose we measure the speed of the plastic spheres between the plates of a micro-micro-balance and find



it is three halves of their speed under gravity alone. The weight of a sphere is 2.8×10^{-14} newton.

(a) What is the electrical force on them if they are moving upward?

(b) If they are moving downward?

(a) If the sphere moves upward with a speed $\frac{3}{2}$ that of its fall under gravity alone, the driving force is upward and of magnitude $\frac{3}{2} \times$ weight $= 4.2 \times 10^{-14}$ newton. This requires that the electrical force be *upward,* and of magnitude 4.2×10^{-14} newton $+ 2.8 \times 10^{-14}$ newton $= \underline{7.0 \times 10^{-14} \text{ newton.}}$

(b) If the sphere moves downward at a speed $\frac{3}{2}$ that of its fall under gravity, the added electrical force must be *downward,* and of magnitude $(\frac{3}{2} - 1)$ (weight) $= \frac{1}{2}(2.8 \times 10^{-14}$ nt$) = \underline{1.4 \times 10^{-14} \text{ newton.}}$

19–16

There is a relationship between the charge on two parallel plates and the force they exert on a charged object between them. There is also a relationship between the number of batteries connected and the force on a charged object between the plates. What is the relationship between the number of batteries and the amount of charge the batteries put on the plates?

The force exerted on a charged object between two parallel plates is directly proportional to the charge on the plates. It is also directly proportional to the number of identical batteries connected in series to the plates. Hence, the charge deposited by the batteries must be directly proportional to the number of batteries.

19–17*

If the force on each elementary charge in a Millikan experiment was 1.4×10^{-14} newton, what would it be if we increased the distance between the plates by a factor of 10? (Section 7.)

$F = 1.4 \times 10^{-15}$ newton.

19–18

Refer to Fig. 19–20(b).

(a) Without moving the plates, how can we increase the force on the elementary charge between the plates which are a distance $2d$ apart so that it will equal the force on the plates which are a distance d apart?

(b) The plates which are a distance d apart are insulated so that when they are disconnected from their battery the number of charges on the plates remains constant. When the plates are disconnected and then separated to a distance $3d$, it is found that the force on the elementary charge has not changed. If they are reconnected to batteries, how many batteries would have to be connected in series with these plates so that force will not change?

(a) Connecting two batteries in series to the plates which are $2d$ apart will make the force the same as on the plates which are d apart.

(b) When the plates are connected to batteries, three batteries are needed at separation $3d$ to maintain the same force that one battery maintains at separation d.

19–19

Only one of several identical metal spheres is charged. The charged sphere A experiences a force $F = 1.0 \times 10^{-4}$ newton when placed midway between certain charged parallel plates.

(a) Sphere A is then touched to one of the uncharged spheres (B). What force does A now experience when placed between the same plates?

(b) A is touched to another of the uncharged spheres (C). What force does A now experience?

(c) What force is exerted on sphere B when it is between the plates? On sphere C?

This problem requires an application of two basic ideas.

1. The force which acts on a charged object placed between charged parallel plates is proportional to the charge on the object.

2. Charge is conserved, so that if a charged sphere is connected to an identical uncharged sphere, each will end up with one half of the initial charge (in the absence of induction effects by other charged bodies which could make the sharing unequal).

(a) When the charged conductor A is touched to an identical uncharged conductor B, each takes one half the initial charge on A. The force on A between the charged plates is half as large as initially, i.e., $\underline{5.0 \times 10^{-5} \text{ newton.}}$

(b) If A is touched to another identical uncharged sphere C, its charge is again divided in half, and the force is again reduced in half to $\frac{1}{4} \times 1.0 \times 10^{-4}$ newton $= \underline{2.5 \times 10^{-5} \text{ newton.}}$

(c) Since sphere B has the same charge as sphere A [in part (a)], it will be subject to the same force, 5.0×10^{-5} newton. Sphere C has the

same charge as sphere A [in part (b)]. Hence it will be subject to a force 2.5×10^{-5} newton.

19–20

Two large metal plates are separated by 0.10 m. They are connected to the terminals of a battery. A small charged ball halfway between them experiences an electric force of 3×10^{-4} newton. The plates are now moved apart until the separation is 0.15 m. The same battery is connected to them.

(a) What force now acts on the ball?

(b) If we add two more identical batteries in series at the new separation, what is the force on the ball?

The electric force field between parallel plates varies directly with the number of batteries (in series) and inversely with the separation.

(a) The separation is increased by a factor of $\frac{3}{2}$ so that the force is reduced by a factor of $\frac{2}{3}$:

$$\tfrac{2}{3} \times 3 \times 10^{-4} \text{ nt} = 2 \times 10^{-4} \text{ newton.}$$

(b) The number of batteries is tripled, resulting in a threefold increase in the force of $3 \times 2 \times 10^{-4}$ nt $= 6 \times 10^{-4}$ newton.

19–21*

What will be the force of repulsion between two small spheres 1.0 meter apart if each has an excess of 10^6 elementary charges? (Section 8.)

$F = 2.3 \times 10^{-16}$ newton.

19–22

A plastic sphere of mass 3.06×10^{-15} kg is in an electric force field which exerts an upward force of 1.00×10^{-14} newton on each positive electric particle. The electric force exerted on the sphere is sufficient to balance the force of gravity on it.

(a) What is the excess of electric particles on the sphere?
Suppose that each molecule has a mass of 3.00×10^{-26} kg.

(b) How many molecules are there in the sphere?

(c) What fraction of the molecules in the sphere have lost or gained an elementary charge?

This problem emphasizes that in macroscopic charged bodies only an exceedingly small fraction of the atoms or molecules have lost or paired a charge.

(a) Since the sphere is balanced, the electric force must be equal in magnitude to the force of gravity ($mg = 3.06 \times 10^{-15}$ kg $\times$ 9.8 m/sec^2 $= 3.0 \times 10^{-14}$ nt). The force on each elementary charge is 1.00×10^{-14} newton, so there is an excess of 3 positive electric particles.

(b) $$\text{Number of molecules} = \frac{\text{mass of sphere}}{\text{mass of mol.}} = \frac{3.06 \times 10^{-15} \text{ kg}}{3.00 \times 10^{-26} \text{ kg}}.$$

$$\text{Number of molecules} = 1.02 \times 10^{11}.$$

(c) The fraction of molecules which have gained or lost an elementary charge is

$$\frac{3}{1.02 \times 10^{11}} = 2.94 \times 10^{-11},$$ a rather small fraction!

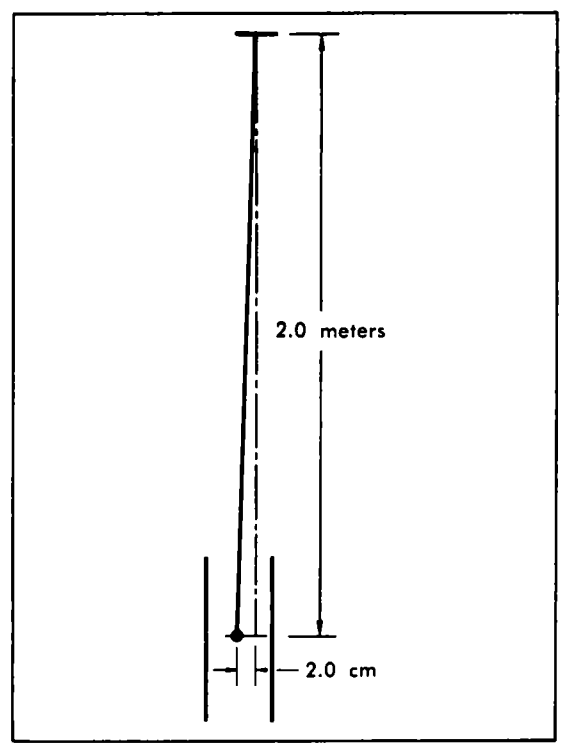
19–23

A sphere of mass 4.5×10^{-3} kg is hanging on a string 2.0 meters long between two oppositely charged parallel plates as shown in Fig. 19–30. At equilibrium the ball has been pulled 2.0 cm from its original position.

(a) What is the magnitude of the electric force on the sphere?

The electric force on each elementary charge between the plates is 3.0×10^{-14} newton.

(b) What is the excess of electric particles on the ball?



This problem involves the quantitative evaluation of a moderately large charge on an electrified object by measurement of the electrical force exerted on it by a set of fixed charges, i.e., the charges on parallel plates, the force on a unit charge being known. The electrical force on the body is measured by comparing it with its weight in a kind of balance.

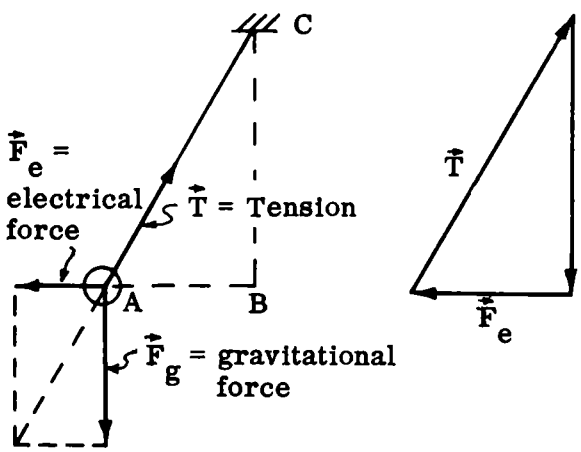
$$AB = 0.020\text{ m}$$
$$AC = 2.0\text{ m}$$
$$BC = \sqrt{(2.0)^2 - (0.02)^2} \approx 2.0\text{ m},$$

as shown in the figure.

(a) By similar triangles

$$\frac{F_e}{F_g} = \frac{AB}{BC} = \frac{0.020}{2.0} = 0.01.$$
$$F = mg = 4.5 \times 10^{-3}\text{ kg} \times 9.8\text{ m/sec}^2$$
$$= 4.4 \times 10^{-2}\text{ newton}.$$
$$F_e = 0.01(4.4 \times 10^{-2}\text{ nt}) = \underline{4.4 \times 10^{-4}\text{ nt}}.$$

The use of a pendulum balance is a simple and neat way of measuring a force which is rather small in comparison to available weights.

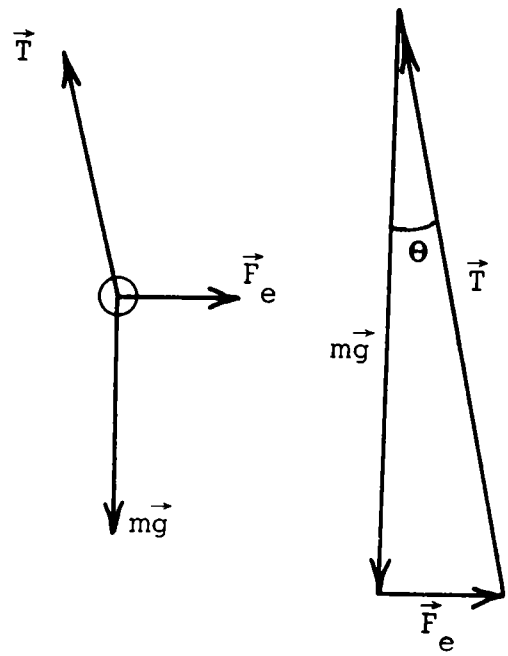


(b) Given that the electrical force on an elementary charge is 3.0×10^{-14} newton, the observed force here is 4.4×10^{-4} nt/3.0×10^{-14} nt $= 1.5 \times 10^{10}$ times larger. Hence, there is an excess of about $\underline{1.5 \times 10^{10}}$ elementary charges on the ball.

19–24

In an experiment two small conducting spheres (120 milligrams each) were suspended on two fine insulating fibers 82 cm long. When equal charges were placed on the two spheres, they separated and came to rest with a distance of 10 cm between centers. What was the number of elementary charges on each sphere?

When each sphere remains at rest, the net force on it is zero. The vectors representing the three forces—gravity, tension, and electric repulsion—must form a closed triangle.



The number of elementary charges, q, on each sphere is given by

$$F_e = \frac{kq^2}{r^2}, \quad \text{or} \quad q = r\sqrt{F_e/k}.$$

From the force diagram, $F_e/mg = \tan\theta$ and $F_e = mg \tan\theta$. From the displacement diagram, $\sin\theta = \frac{5}{82} = 0.061$. We see that the angle must be very small and $\sin\theta \approx \theta$ (in radians) $\approx \tan\theta$. Thus,

$$q = r\sqrt{mg \tan\theta/k}$$
$$= 0.10\text{ m}\sqrt{\frac{(1.2 \times 10^{-4}\text{ kg})(9.8\text{ m/sec}^2)}{2.3 \times 10^{-28}\text{ nt-m}^2/(\text{elem ch})^2}\,(6.1 \times 10^{-2})}$$
$$= \underline{5.6 \times 10^{10}\text{ elementary charges}}.$$

19–25*

An excess of 1×10^{10} negative elementary charges is placed on a small object. What would be the electric

field and electric potential at a point 0.5 m away? (Section .)

$\vec{E} = 9 \times 10^{-18}$ newton/elementary charge, pointing toward the object.
$V = -5 \times 10^{-18}$ joule/elementary charge.

19–26

(a) Find the electric and the gravitational forces on an electron and on a proton between two large parallel metal plates, 20 cm apart. The potential difference between the plates is maintained by 20,000 volts' worth of battery. The plates are horizontal; consider the upper plate positive.

(b) Move the particles up 1 cm. What are the forces now?

(c) What are the changes in gravitational potential energy and in electric potential energy in this motion?

(a) The gravitational forces on both electron and proton are directed downward and given by mass × gravitational field strength.

For the proton: gravitational force
$= (1.67 \times 10^{-27}\ \text{kg}) \times (9.8\ \text{newtons/kg})$
$= \underline{1.6 \times 10^{-26}\ \text{newton}}.$

For the electron: gravitational force
$= (9.1 \times 10^{-31}\ \text{kg}) \times (9.8\ \text{newtons/kg})$
$= \underline{8.9 \times 10^{-30}\ \text{newton}}.$

Since the charge of both electron and proton is one elementary charge, the magnitude of the electrical forces is the same for both, but is directed up for the negatively charged electron and down for the positively charged proton. The electric force can be found by comparing the voltage and plate separation in this problem with those of the Millikan experiment in Section 19–8. In the Millikan experiment, the voltage = three 90-volt batteries = 2.7×10^2 volts; the plate separation = 3.1×10^{-3} m, and the force on one elementary charge = 1.4×10^{-14} newton. Since the force varies directly with the voltage and inversely with the plate separation,

$$F = 1.4 \times 10^{-14}\ \frac{\text{nt}}{\text{el. chg.}}\ \frac{\dfrac{2 \times 10^4\ \text{volts}}{2.7 \times 10^2\ \text{volts}}}{\dfrac{2.0 \times 10^{-1}\ \text{m}}{3.1 \times 10^{-3}\ \text{m}}}$$

$$= 1.4 \times 10^{-14}\ \frac{\text{nt}}{\text{el. chg.}}\ \frac{.74 \times 10^2}{.65 \times 10^2}$$

$$F = 1.6 \times 10^{-14}\ \frac{\text{newton}}{\text{el. chg.}}.$$

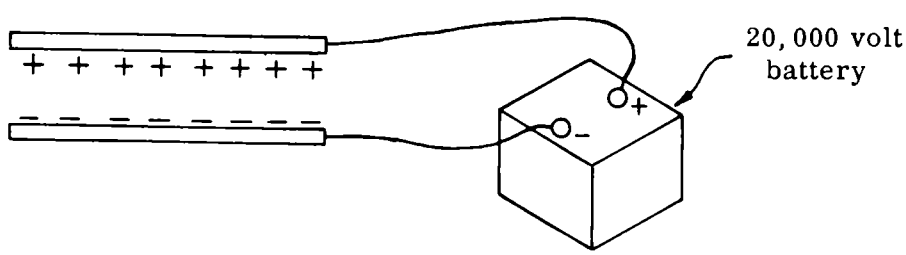
This is enormous compared to the gravitational forces.

(b) The electrical force field between parallel plates is independent of position; the gravitational force field of the earth is also essentially unchanged when one moves an additional distance of only 1 cm from the center of the earth. This means that both electrical and gravitational forces remain constant.

(c) This part of the problem probably can be discussed most usefully by leaning on the analogy between the uniform gravitational field and the uniform electrical field.

To move the proton up 0.01 meter against the downward gravitational force of 1.6×10^{-26} newton requires that work be done by an applied upward force in the amount of 1.6×10^{-26} newton $\times$ 0.01 meter $= 1.6 \times 10^{-28}$ joule. Similarly, to move the proton up 0.01 meter against the downward electrical force of 1.6×10^{-14} newton requires that work be done by an applied upward force in the amount of 1.6×10^{-14} newton $\times 10^{-2}$ meter $= 1.6 \times 10^{-16}$ joule. Therefore corresponding energy changes for the proton are an increase of $\underline{1.6 \times 10^{-28}\ \text{joule}}$ in the gravitational potential energy, and an increase of $\underline{1.6 \times 10^{-16}\ \text{joule}}$ in the electrical potential energy.

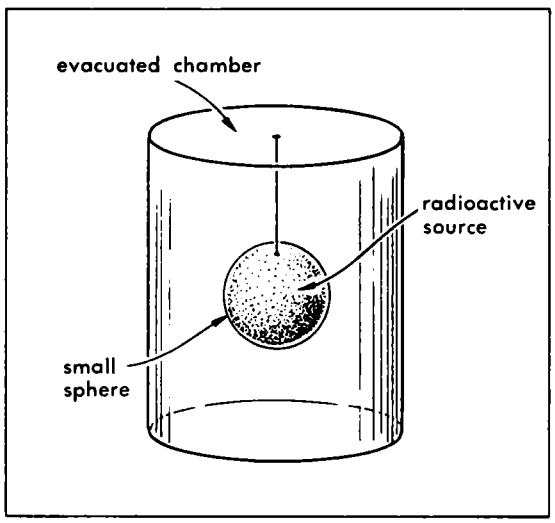
The energy changes for the electron can be obtained in the same way. The gravitational potential energy is increased by 0.89×10^{-29} newton $\times 10^{-2}$ meter, or $\underline{8.9 \times 10^{-32}\ \text{joule}}$, while the electrical potential



energy is *decreased by* 1.6×10^{-16} joule. This is a decrease because the electron moved in the same direction as the electrical force.

19–27

A thin layer of radioactive material emitting beta particles is painted on a small conducting sphere. The sphere is then hung on an insulating thread in an evacuated chamber (Fig. 19–31).



After 50 days we measure the charge on the sphere and find it to be 4.32×10^{10} positive elementary charges. (The sphere has been kept in an evacuated chamber, so very few charges have leaked off.) In a separate experiment with a Geiger counter we find beta particles shooting out at the average rate of 1.00×10^4 per second.

(a) What is the charge of one beta particle?

(b) What do you think beta particles may be?

As the sphere is charged positively, the beta particles must carry a negative charge.

(a) The number of beta particles

$$= \text{rate of emission} \times \text{time}$$

$$= 1.00 \times 10^4 \frac{\text{beta particles}}{\text{sec}} \times \left(50 \text{ days} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{3600 \text{ sec}}{\text{hour}}\right)$$

$$= 4.32 \times 10^{10} \text{ beta particles emitted in 50 days.}$$

The beta particles carried off a total of 4.32×10^{10} negative elementary charges. Therefore, each beta particle must carry one negative elementary charge, just as an electron does.

(b) Before we jump to a conclusion identifying beta particles with electrons, other properties (e.g., mass) will have to be compared, but this experiment gives us one reason to entertain this hypothesis.

CHAPTER 20

20–1*

Suppose we use two 90-volt batteries in the apparatus of Fig. 20–1, and use doubly charged ions; what energy will each ion have when it hits the negative plate? (Section 1.)

5.6×10^{-17} joule.

20–2

Two large parallel metal plates, 6.2 cm apart in a vacuum tube, are connected to the terminals of two 90-volt batteries in series. A doubly charged oxygen ion starts from rest at the surface of one plate and is accelerated across to the other plate.

(a) With what kinetic energy $\frac{1}{2}mv^2$ does the oxygen ion crash into the other plate?

(b) If the ion starts halfway between the plates, with what kinetic energy does it hit the negative plate?

This straightforward problem illustrates the application of conservation of energy to the motion of charged particles in a uniform electric field. One approach is the following.

(a) We first calculate the force on an oxygen ion, using the results of the Millikan experiment discussed in Section 19–8. There it was found that with three 90-volt batteries in series connected to plates 0.31 cm apart, the force on an elementary charge was 1.4×10^{-14} newton. Since the force exerted by charged parallel plates on an ion between them varies directly with the number of batteries in series and with the charge of the ion, and inversely with the separation of the plates, the force on a doubly charged oxygen ion exerted by plates 6.2 cm apart connected to two 90-volt batteries is

$$F = (1.4 \times 10^{-14} \text{ newton}) \times \tfrac{2}{3} \times 2 \times \left(\frac{0.31 \text{ cm}}{6.2 \text{ cm}}\right) = 9.3 \times 10^{-16} \text{ newton.}$$

If an oxygen ion moves the full distance of 6.2 cm between plates, the work done by the electrical force will be

$$9.3 \times 10^{-16} \text{ newton} \times 0.062 \text{ meter} = \underline{5.8 \times 10^{-17} \text{ joule.}}$$

Since the ion starts with no kinetic energy, this will be its final kinetic energy as it reaches the plate.

(b) If the ion starts from rest halfway between the plates, it travels only one half the distance, i.e., 3.1 cm. The work done by the electric force, and therefore the kinetic energy acquired by the ion will be

$$9.3 \times 10^{-16} \text{ newton} \times 0.031 \text{ meter} = \underline{2.9 \times 10^{-17} \text{ joule.}}$$

Note: After the chapter has been completed, the problem may be done by the following alternate method.

With a potential difference of 180 volts between the two plates, one elementary charge would acquire a kinetic energy of 180 electron volts in traveling from one plate to the other.

(a) Thus a doubly charged oxygen ion will gain 360 electron volts in going from one plate to the other.

(b) Since the electric field is uniform between two such plates, the potential difference will be uniformly divided. Thus in traveling one half the total distance, the doubly charged oxygen ion will gain 180 electron volts of energy.

20–3*

If singly charged helium ions were put through the apparatus of Fig. 20–4, what would be the time of flight? (Section 1.)

7.64×10^{-6} sec.

20–4

In a time-of-flight experiment like that described in this chapter, you have adjusted the sweep circuit of the oscilloscope so that the beam moves across the tube in 1.0×10^{-6} sec. The total length of the trace made in that time is 10 cm. The distance between the deflecting plates and the detector is 1.2 m. If the spikes on the oscilloscope trace are 8.0 cm apart, what is the speed of the ions?

If the beam moves 10 cm in 1.0×10^{-6} sec, it would move 8.0 cm, the distance between the spikes, in 8×10^{-7} sec.

Since the ions travel 1.2 m during this time interval, their speed is

$$v = 1.2 \text{ m}/8 \times 10^{-7} \text{ sec} = 1.5 \times 10^{6} \text{ m/sec.}$$

20–5

How does the mass of a hydrogen ion compare with the mass of a hydrogen atom?

In Section 20–1, the mass of a hydrogen ion was determined to be 1.7×10^{-27} kg, and the mass of an electron to be 0.91×10^{-30} kg. The mass of the hydrogen atom would be the sum of these two masses: $1.7 \times 10^{-27} + 0.00091 \times 10^{-27}$ kg. Evidently the mass of the electron is much less than the uncertainty in the two-digit value given for the mass of the hydrogen ion; the addition of these two masses would be meaningless. To two significant digits, the mass of the hydrogen ion and the mass of the hydrogen atom are the same.

20–6*

If 2×10^{16} pairs of singly charged ions are produced per second and reach the plates of Fig. 20–8 without recombining, what is the current in the battery connected to the plates? (Section 2.)

$I = 2 \times 10^{16}$ elementary charges/second.

20–7*

A poorly made electron gun has considerable residual gas left inside. When the filament is heated and a battery connected between filament and cylinder (Section 18–15), there is a current consisting of both electrons and positive ions. Consider a plane close to the filament. In each second, 3.0×10^{16} electrons are emitted by the filament and move across the plane toward the cylinder. Also, 0.6×10^{16} singly charged positive ions per second pass through this surface in the opposite direction. What is the current through the plane? (Section 2.)

$I = 3.6 \times 10^{16}$ elementary charges/second.

20–8

You ionize some gas between parallel metal plates, making equal numbers of negative and positive ions in every small volume between the plates. There is a battery and a current meter connected in series with the plates, and a steady current is flowing through the circuit. The positive plate is on the left, and the negative on the right.

(a) In what direction do you expect the electric current to go between the plates?

(b) If the current meter measures 10^{16} elementary

charges per second, how many positive elementary charges are brought per second by the ions coming to the negative plate?

(c) How many negative elementary charges are coming per second to the negative plate?

(d) In one second, how many positive ions cross to the right through a plane halfway between the plates?

This problem illustrates that the total current at any point in a circuit is given by the sum of the flow of positive charges (in the direction of the current) and the negative charges (flowing in the opposite direction) per unit of time.

In this problem we are concerned only with the steady-state behavior of the circuit. In the steady state, positive and negative ions are created in equal numbers per unit time by the X-ray beam. The positive ions will move toward the negative plate on the right, and the negative ions toward the positive plate on the left. If there were no recombination, all of the charges would contribute to the current, which would then be equal to the number of ion pairs produced per second. With recombination, the total current will be less, but the number of positive charges reaching the right-hand plate must still equal the number of negative charges reaching the left-hand plate per second. The current will in any event be given by the number of charges striking either plate per second.

(a) The direction of the current is the direction of motion of the positive charges, hence from left to right.

(b) The current must be equal to the number of positive charges striking the negative plate per second, and must be the same everywhere in the circuit. Hence the number of positive charges striking the negative plate must be 10^{16} elementary charges/second.

(c) There are *no* negative charges coming to the negative plate *from within the tube,* but the external circuit must supply negative charges equal to the positive charges reaching the plate per second. Hence, the flow of negative charge to the negative plate, *from the external circuit,* is 10^{16} elementary charges/second.

(d) At a plane halfway between the plates, the current must consist of equal numbers of positive and negative charges flowing in opposite directions. Thus the number of positive charges crossing to the right at this point must be half the current, or 5×10^{15} elementary charges/second. If each ion carries n elementary charges, the positive-ion current must be $(5 \times 10^{15})/n$ per second.

20–9

You are given some metal wire, batteries, and a calibrated current meter. How can you calibrate a second current meter?

When two meters are connected in series in a circuit, the same current must flow through both. The current on the calibrated meter can be read, and the scale of the other meter can be marked accordingly. By varying the number of batteries connected in series to the circuit, one can vary the current, so a number of values can be marked on the second meter. If the scale of the second meter appears to be linear, the other values can easily be interpolated.

In this problem we are not concerned with the possibility that if the wires are good conductors, the current might exceed the capacity of the meter. We can assume that the meters operate within the range of the batteries and wires we are given.

20–10†

(See the Appendix in the text.)

20–11*

When 3.00×10^{22} atoms of aluminum are deposited in the apparatus of Fig. 20–10, how many atoms of copper will be deposited? (Section 3.)

4.50×10^{22} atoms of copper.

20–12

A current meter has been built and you want to calibrate its scale. It is connected in series with an electrolytic cell containing a solution of silver nitrate ($Ag^+NO_3^-$) and a battery. A steady current which deflects the meter to the upper part of the scale flows for half an hour. At the end of this period, it is found by weighing that the negative electrode has 1.92 gm of silver deposited on it.

(a) How many elementary electric charges per second passed through the circuit? (*Note:* A silver atom has a mass of 1.80×10^{-25} kg.)

(b) What would you do to complete the calibration of the instrument?

(a) Each silver ion carries one elementary charge. To find the number of elementary charges per second passing through the cir-

cuit, we divide the total number of silver atoms deposited by the time in which they were deposited:

$$1\frac{\text{el.chg.}}{\text{atom}}\left(\frac{1.92\text{ g}}{1.08\times10^{-25}\frac{\text{kg}}{\text{atom}}\times10^{3}\frac{\text{g}}{\text{kg}}}\right)\Big/1800\text{ sec}$$

or $\underline{5.94\times10^{18}\text{ elementary charges/second.}}$

(b) The zero point of the meter must be marked when there is no flow of charge, and several other points within its range must be established by the above method to determine whether the meter movement is linear. If it is, the scale can be made with equal divisions up to the full range of the meter, using the calibrated points to determine the size of each division.

20–13*

A doubly charged ion is accelerated from rest by a force of 3.10×10^{-15} newton over a distance of 9.3 millimeters. How much kinetic energy does the ion gain? (Section 4.)

2.9×10^{-17} joule.

20–14*

Other measurements on the ion of the previous problem, after it had been accelerated, show its momentum to be 1.24×10^{-21} kg meters/second. What is the mass of the ion? (Section 4.)

$m = 2.7\times10^{-26}$ kg.

20–15

In Fig. 20–13, electrons emitted at F are accelerated toward B.

(a) How much energy will the electrons have which hit plate B?

(b) What is the electric field between A and B and between B and C?

(c) Some will travel through the hole in B. Where will they go?

(a) The energy supplied to an electron moving through a potential difference of 100 volts is

$$\begin{aligned} q\,\Delta V &= (1\text{ elem.ch.})(100\text{ volts}) \\ &= 100\text{ electron volts} \\ &= \underline{1.6\times10^{-17}\text{ joule.}} \end{aligned}$$

(b) The electric field is the force per elementary charge, or F/q. Since the energy acquired by a charge equals the force F on it times the distance d it travels, $q\Delta V = Fd$, the electric field can be expressed as

$$E = \frac{F}{q} = \frac{(q\,\Delta V/d)}{q} = \frac{\Delta V}{d}.$$

The electric field in the region between A and B is

$$E_{AB} = \frac{100\text{ volts}}{1.0\times10^{-2}\text{ m}} \times\left(\frac{1.6\times10^{-19}\text{ joule/elementary charge}}{1\text{ volt}}\right)$$

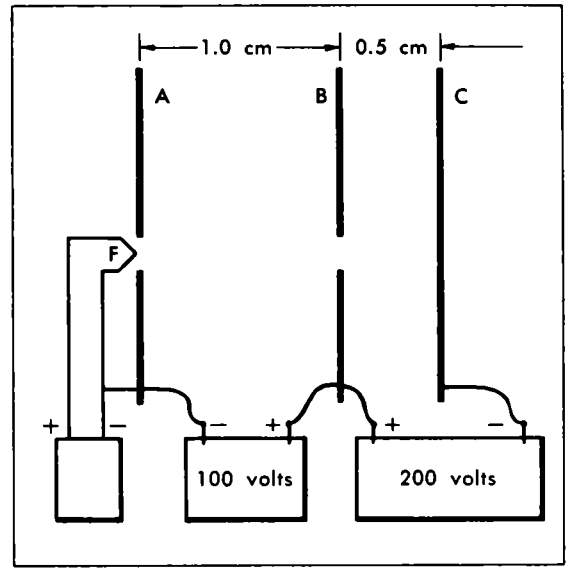
$$= \underline{1.6\times10^{-15}\text{ newton/elementary charge.}}$$

The direction of the field is from the positive plate, B, to the negative plate, A. The field between B and C is

$$E_{BC} = \frac{200\text{ volts}}{5\times10^{-3}\text{ m}} \times\left(\frac{1.6\times10^{-19}\text{ joule/elementary charge}}{1\text{ volt}}\right)$$

$$= \underline{6.4\times10^{-15}\text{ newton/elementary charge.}}$$

The direction of the field is from plate B to plate C.



(c) Since the electric field is uniform everywhere between the parallel plates, the potential halfway between B and C is 100 volts less than at B. When the electrons reach the halfway mark, they will have given up the entire 100 electron volts of kinetic energy they acquired between A and B, and they will be

attracted back to B. If they hit B, on this trip or a later one, they will give up as heat the 100 ev they will have regained.

20–16

With the same apparatus as in Problem 2, how many doubly charged oxygen ions would have to go from the positive plate to the negative plate before that plate is heated up by one joule by the dissipation of kinetic energy of the ions?

Since one 90-volt battery supplies 1.4×10^{-17} joule to each elementary charge, the energy supplied by two 90-volt batteries to a doubly charged ion is

$$2(2)(1.4 \times 10^{-17}\,\text{joule}) = 5.6 \times 10^{-17}\,\text{joule}.$$

(This is approximately the same answer we calculated by a different method in Problem 2.) If each ion transfers its entire energy to the plate when it crashes into it, the number of ions necessary to supply one joule of energy is

$$N = \frac{1\ \text{joule}}{5.6 \times 10^{-17}\,\text{joule/ion}} = \underline{1.8 \times 10^{16}\ \text{ions.}}$$

The most common oxygen ions are negatively charged, but from their direction of travel these must be positively charged.

20–17

Suppose the doubly charged ions in the last problem were made by ripping electrons off oxygen atoms located halfway between the plates. Then the oxygen ions are accelerated in one direction and the electrons are accelerated in the opposite direction from the midpoint between the plates. How much energy will the electrons have dissipated heating the positive plate in the time that one joule is dissipated at the negative plate?

The magnitude of the force on each elementary charge depends only on the setup of the plates and battery, and the distance traveled in the direction of the force is the same for the electrons and ions. Since the same number of elementary charges go in each direction, the energy the electrons transfer to the positive plate is the same as that transferred to the negative plate by the positive ions. In this particular instance, both equal 1 joule.

20–18

What evidence do you have that all elementary charges are the same? Be prepared to discuss it in class.

This question provides a good review of Sections 1–4.

All of the evidence—from the Millikan experiment, from time-of-flight measurements, and from electrolytic cells—shows that the force acting on any charged particle in a given electric field is an exact integral multiple of some least value. This can only be true if all elementary charges are identical, regardless of the mass or any other property of the charged particle, and without regard to the way in which the charge is produced.

Other sources of evidence include the neutrality of atoms and molecules, and the argument of Section 20–4 which can also be done with just a wire heater, a battery calibrated by a Millikan experiment and a current meter calibrated electrolytically.

20–19*

Three 90-volt batteries are connected in series across two parallel plates 1.00 mm apart. What is the force on an elementary charge between the plates? (Section 5.)

$F = 4.2 \times 10^{-14}$ newton.

20–20*

A battery with an EMF of 2.9×10^{-17} joule/elementary charge is transporting charge at the rate of 6.0×10^{16} elementary charges/second. What is the power supplied by the battery? (Section 5.)

1.7 joules/sec = 1.7 watts.

20–21*

A battery connected to an electric heater supplies 3.12×10^{19} elementary charges per second and produces an output of 60 joules per second (watts). What is the EMF of the battery in joules per elementary charge? In volts? (Section 6.)

EMF $= 1.9 \times 10^{-18}$ joule/elementary charge $=$ 12 volts.

20–22*

What is the current in amperes when 3.12×10^{19} elementary charges per second flow in a circuit? (Section 6.)

$I = 5.00$ amp.

20–23*

In Problem 28 of Chapter 16, we found that the binding energy of the electron to the proton was 2.3×10^{-18} joule. How many electron volts is this? (Section 6.)

14 ev.

CHAPTER 21

21–1*
What can you conclude about the field in a current-carrying wire from the fact that the current is always in the direction of the wire? (Section 1.)

The electric field is also in the direction of the wire.

21–2*
A uniform high-resistance wire, 10 meters long, is connected to a battery having an EMF of 30 volts as in Fig. 21–3. What is the electric field in this wire? (Section 1.)

E = 3.0 volts/meter = 3.0 newtons/coulomb.

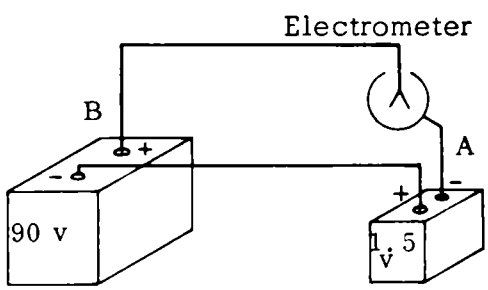
21–3*
Assume that a second wire, identical to the first, is connected in parallel with the wire of the previous problem. What changes will occur in the electric field? (Section 1.)

The electric field in the first wire will be the same as before, since the difference in potential from one end of the wire to the other still equals the EMF of the battery, no matter what else has been connected in parallel.

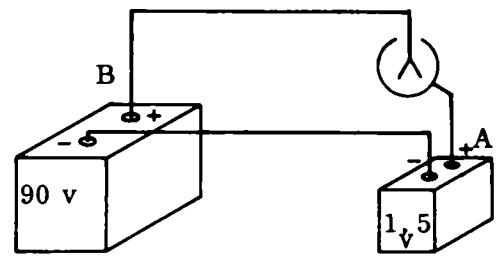
21–4*
If a uniform high-resistance wire, 20 meters long, is connected to a 30-volt battery, what is the potential difference between two points 3 and 15 meters from one end? (Section 1.)

V = 18 volts.

21–5
(a) How would you connect a 90-volt "B" battery and a 1.5-volt flashlight cell to obtain the maximum potential difference?
(b) How big is that potential difference?
(c) Suppose you disconnect the 90-volt battery, turn it end for end, and then reconnect it. What potential difference do you get?

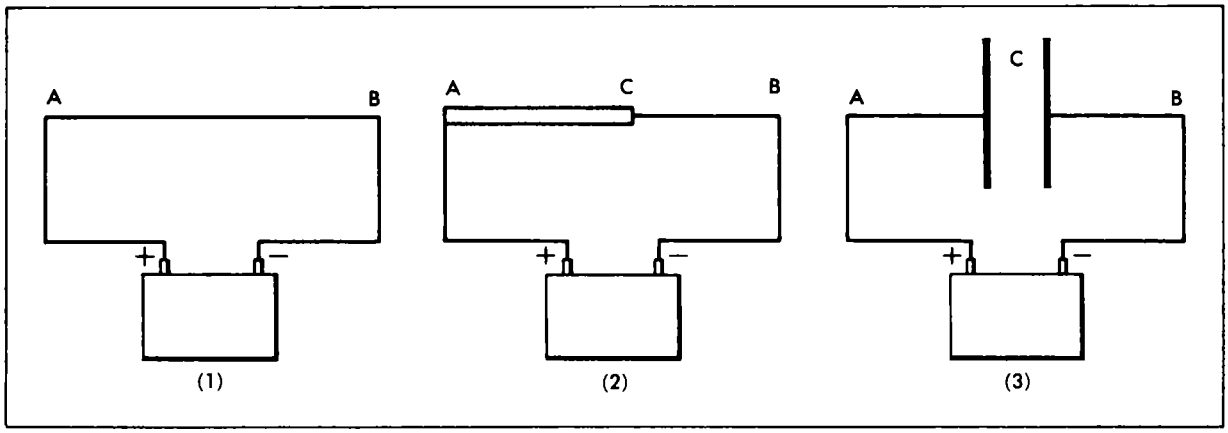


(a) and (b) If the batteries are connected *in series*, + *to* −, the potential difference between A and B is the sum of the potential differences supplied by each battery, 91.5 volts.
(c) If the connections to either of the batteries are reversed, the potential difference from A to B would be less. In this case, the potential difference would be 88.5 volts.



21–6
(a) Plot the magnitude of the electric field against position along AB for the cases shown in Fig. 21–16.
The circuits in (2) and (3) are the same as in (1), except that in (2) AC consists of two parallel lengths of the same uniform wire and in (3) there are two large parallel plates at C. The EMF of the battery is the same in every case.
(b) Plot the difference in potential between B and each intermediate point against position along AB for the three cases.

(a) The electric field is the same everywhere inside a uniform straight wire, so E must be a constant for case (1). In case (2), the sum of the currents through the two parallel wires is the same as the current passing from C to B, since no charge accumulates at C. Hence, the current through each of the parallel wires is only half as great as the current from C to B, and the electric field inside the parallel wires must be only half as great as the electric field between C and B. In case (3), the electric field between the parallel charged plates is the same at every point. When the plates are fully charged, there is no electric field between the battery and the plates: if there were, charges would be moving onto the plates or off the plates, contradicting our hypothesis that the plates are fully charged.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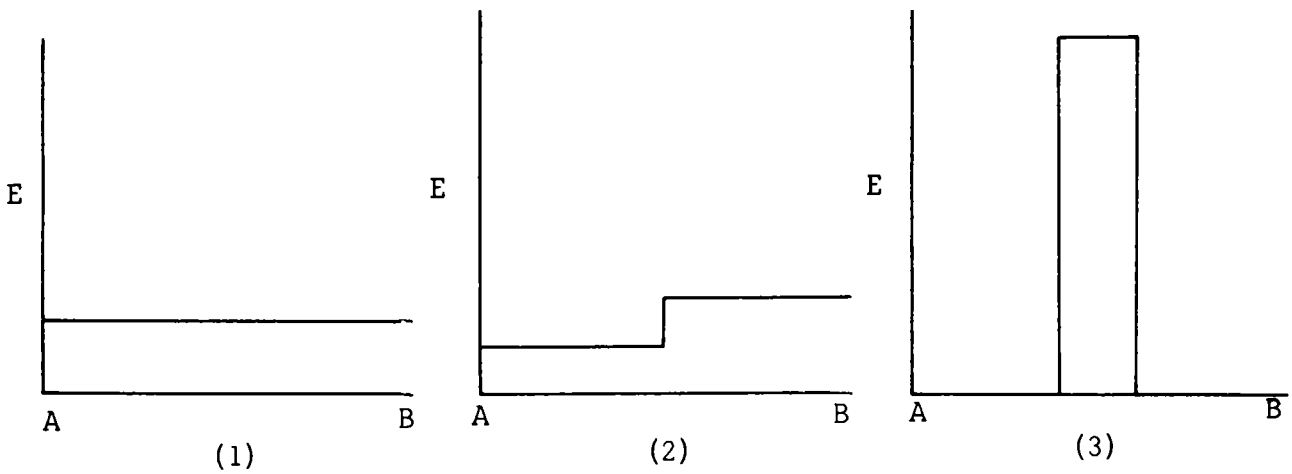
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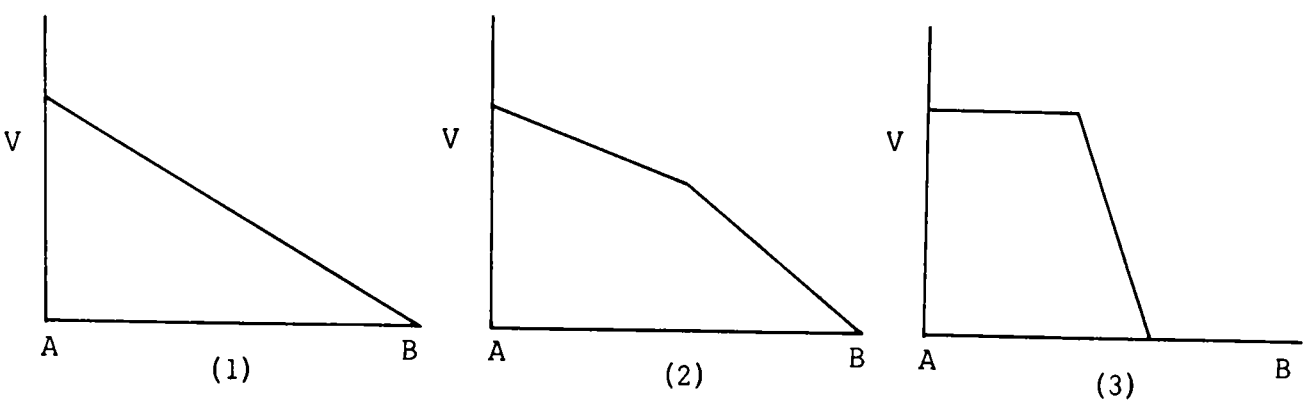
B
(3)


V
A
B
(1)
V
A
B
(2)
V
A
B
(3)

Notice that the total area under the curve is the same in every case. If we had plotted the force on a positive elementary charge against position, the area under the curve would have been the work done by the electric field in moving it there, and the work done represents potential energy spent:

$$\text{force} \times \text{distance} = \text{work} = \text{change in potential energy.}$$

Then,

$$\frac{\text{force}}{\text{elementary charge}} \times \text{distance} = \frac{\text{change in potential energy}}{\text{elementary charge}},$$

or

$$\text{electric field intensity} \times \text{distance} = \text{change in potential,}$$

and the difference in potential between A and B is the same in every case.

(b) Since the difference in potential between any point and B is just the electric field intensity at that point times its distance from B, we can plot the area under the graphs in part (a) from B to each point in order to obtain the graphs of potential difference with respect to B as a function of position.

21–7*

Under what condition can you use a current-type voltmeter to measure potential difference between two points in a closed circuit? (Section 2.)

The current in the voltmeter must be small compared with the current in the circuit.

21–8*

Assume that the battery shown connected to the source of electrons in Fig. 21–6 is one of 45 volts. What will be the kinetic energy of the electrons passing through the hole in the curved plate? (Section 3.)

45 electron volts = 7.6×10^{-18} joule.

21–9*

Assume that the electrons in Fig. 21–6 are accelerated by a 45-volt battery and that the potential divider and two batteries across the parallel plates are replaced by a 30-volt storage battery. Describe what happens to the kinetic energy of an electron which passes through the hole in the plate at the left and strikes the plate at the right. (Section 3.)

If the negative terminal is connected to the right-hand plate, as in Fig. 21–6, the electron will give up 30 electron volts of its kinetic energy in order to do work against the repulsive force. This energy is transferred to the 30-volt battery. When the electron hits the plate, it will give up the other 15 electron volts as heat.

21–10*

Assume that the diode whose current vs. potential curves are shown in Fig. 21–10 is connected, in series with a variable resistor, to a 90-volt battery. If the filament is heated to 2100°K, what resistance in the resistor will limit the current to 1.2×10^{-3} ampere? (Section 4.)

$R = 4.4 \times 10^4$ ohms. The potential difference across the resistor is (90–37) volts.

21–11

An electron gun shoots a stream of electrons between two deflecting plates of length $l = 6.0$ cm (Fig. 21–17). The electron mass is 0.91×10^{-30} kg. The potential difference between the filament and plate of the gun is 2500 volts ($2.5 \times 10^3 \times 1.6 \times 10^{-19}$ joule per elementary charge). The electric field E between the deflecting plates is 1000 newtons per coulomb (1000 newtons per 6.25×10^{18} elementary charges).

(a) If the deflecting plates are separated by 0.20 cm, what is the potential difference from one deflecting plate to the other?

(b) What is the speed of the electrons leaving the gun?

the length l?

(d) As an electron passes between the deflecting plates, it picks up a sidewise velocity because of the electric force exerted on it. How big is this sidewise velocity when the electron leaves the deflecting plates?

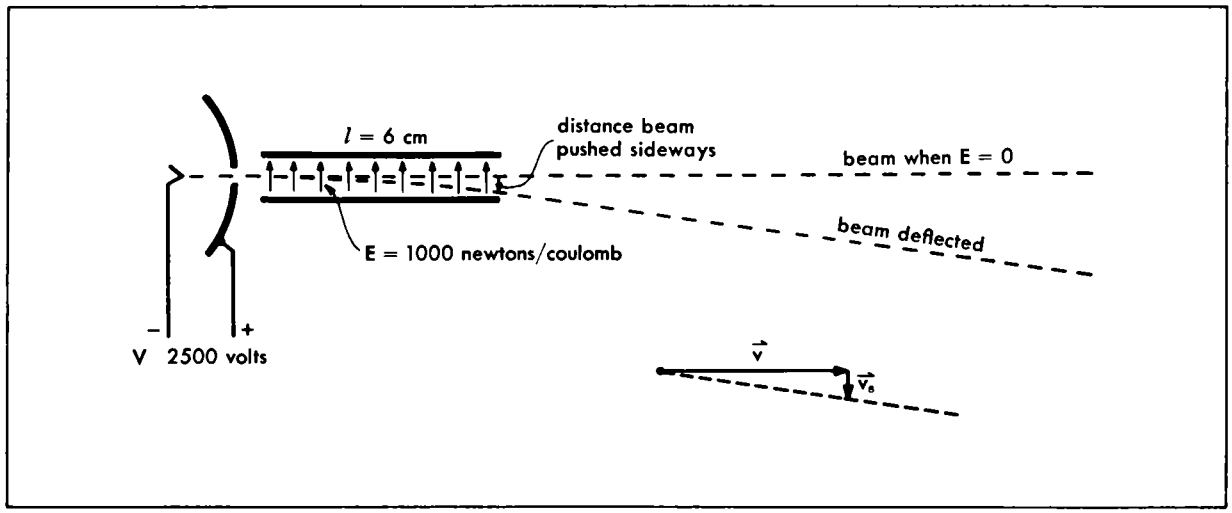
(e) How far is the electron pushed sidewise in passing between the plates?

(a) Since the energy per charge = force per charge × distance, then the potential difference from one plate to the other is

$$V = Ed$$
$$= (1000 \text{ nt/coul})(2.0 \times 10^{-3} \text{ m})$$
$$= 2.0 \text{ joules/coul} = \underline{2.0 \text{ volts.}}$$

(b) The accelerating potential of 2500 volts between filament and plate gives each electron a kinetic energy of 2500 electron volts, or

$$\tfrac{1}{2}mv^2 = E_K$$
$$= (2500 \text{ ev})(1.60 \times 10^{-19} \text{ joule/ev})$$
$$= 4.00 \times 10^{-16} \text{ joule.}$$



Hence,

$$v^2 = 2E_K/m,$$

and

$$v = \sqrt{\frac{2(4.00 \times 10^{-16}\ \text{joule})}{0.91 \times 10^{-30}\ \text{kg}}}$$

$$= \underline{3.0 \times 10^7\ \text{m/sec.}}$$

(c) The experimental setup presumably has been arranged so that there is no electric field component outside the gun in the direction of the undeflected beam. (Call this direction x.) This will be the case if the accelerating plate of the gun is at a potential intermediate between that of the two deflecting plates. If so, the x-component of velocity all the way across the deflecting plates is the same as when the electron left the gun, i.e., 3.0×10^7 meters/second. The time required for the electrons to traverse the distance $l = 0.060$ meter will be:

$$t = \frac{l}{v_x} = \frac{0.060\ \text{meter}}{3.0 \times 10^7\ \text{meters/sec}}$$

$$= \underline{2.0 \times 10^{-9}\ \text{second.}}$$

(d) The electric field between the plates exerts a sidewise force $F_s = qE$ on any charge q between the plates. This force, acting for a time interval t, gives the charged particle a momentum mv_s:

$$F_s t = mv_s.$$

Hence, $$v_s = F_s t/m = qEt/m.$$

For the electron,

$$v_s = \frac{(1.60 \times 10^{-19}\ \text{coul})(1.00 \times 10^3\ \text{nt/coul})(2.0 \times 10^{-9}\ \text{sec})}{9.1 \times 10^{-31}\ \text{kg}}$$

$$= \underline{3.5 \times 10^5\ \text{m/sec.}}$$

(e) Since the acceleration is constant, and the initial sidewise velocity is zero, the average sidewise velocity is half the final sidewise velocity. The sidewise displacement s is

$$s = v_{s_{ave}} t$$

$$= (2.0 \times 10^{-9}\ \text{sec})\left(\frac{3.5 \times 10^5\ \text{m/sec}}{2}\right)$$

$$= \underline{3.5 \times 10^{-4}\ \text{m}},\quad \text{or} \quad 0.35\ \text{millimeter.}$$

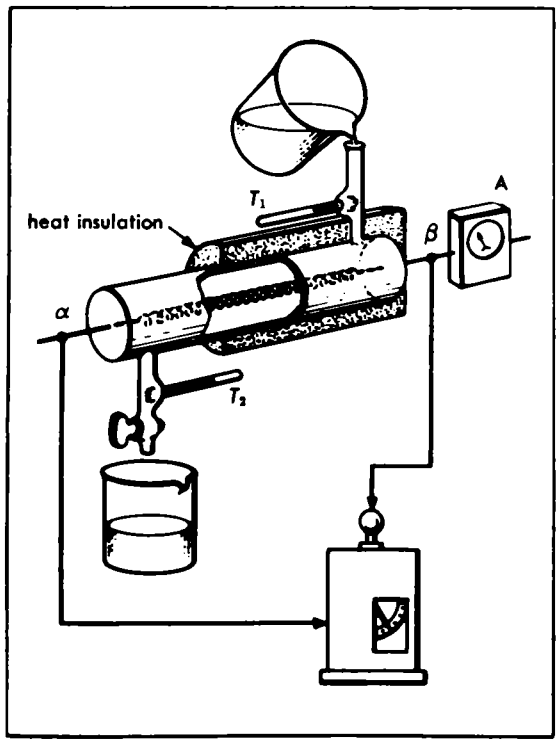
21–12†

(See the Appendix in the text.)

21–13

Figure 21–18 shows a device for calibrating the ammeter A. It is an insulated can with a coil of wire in it. Water flows in at the top of the diagram and passes thermometer T_1. Then, after flowing past the coil of wire, it flows out at the bottom of the diagram past thermometer T_2. The difference of potential across the coil in the calorimeter is measured, using an electrometer, so that we know the potential difference between the points α and β. In an experiment (i) the flow of water is 1 gram per second, (ii) thermometer T_2 reads a temperature 1°C higher than thermometer T_1 and (iii) the electrometer reads a potential difference of 2.09 volts.

(a) What is the current through the ammeter?
(b) How would you calibrate an ammeter with this device?



As the electrons flow through the coil they gain energy in an amount equal to the work done by the electric force. However, they almost immediately transfer the energy acquired to the atoms of the metal. When the current is first turned on, the wire heats up. The wire soon becomes hotter than the surrounding water and loses heat to the water. This exchange quite rapidly reaches a steady state in which the temperature of the wire is constant and the energy transferred to the electrons by the electric field gives rise to an equivalent amount of heat transferred to the water. Consider what happens in one second (after a steady state has been reached).

(a) In one second, 1 gram of water passes through. The temperature of the gram of water is increased by 1°C, so the energy transferred to the water is 1 calorie = 4.18 joules. The work done by the electric field on a charge q is $qV = 4.18$ joules, so the charge q passing in one second must have been:

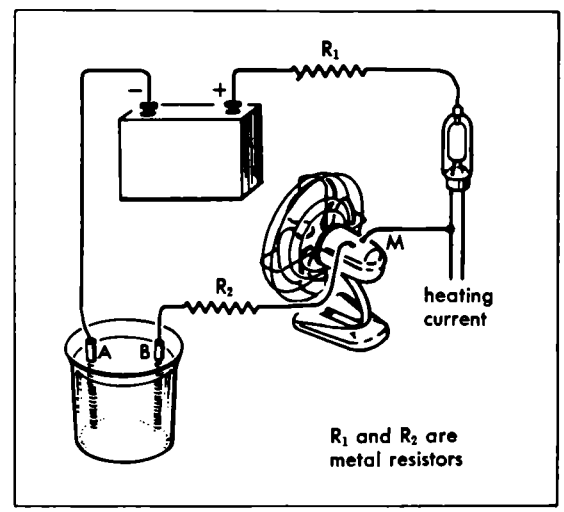
$$q = \frac{4.18 \text{ joules}}{2.09 \text{ volts}} = 2.00 \text{ coulombs.}$$

A flow of 2.00 coulombs in 1 second is a current of 2.00 amperes.

(b) To calibrate an ammeter we connect it in series as shown in the figure. The ammeter will carry the same current as the coil. By varying the applied EMF (not shown) which causes the current to flow, we can vary the potential difference between α and β. For each V we measure with the electrometer, we can calculate the current.

21–14

In the circuit shown in Fig. 21–19, two resistors, a diode (vacuum tube), an electric motor M, and an electrolytic cell are connected in series with a battery.



The difference of potential across each of these circuit elements is measured and it is found that they are all the same. A certain amount of heat is dissipated per second in the first resistor. Compare this with the amount of heat dissipated per second:

(a) In the second resistor.
(b) In the diode.
(c) In the motor which runs a fan.
(d) If there is actually less heat dissipated in the electrolytic cell than in resistor R_1, which terminal, A or B, would be positive when the cell is used as a source of potential difference?

The principle of energy conservation enables us to make useful qualitative statements about the behavior of each of the elements of the circuit. The same current must flow through each of the elements, since they are connected in series, and we are told that the drop in electrical *potential* is

the same across each element. We can conclude that the same amount of *energy* is supplied to each element per unit time. For each element, the amount of heat dissipated will then depend on what else is done with the energy supplied.

(a) The resistors are simply long metal wires. They can do nothing with the energy they receive except dissipate it as heat. Since they both receive the same energy per second, they must dissipate the *same* amount of heat per second.

(b) In the diode, as in part (a), the only place the electrical energy can go is into heat. All of it will be converted into heat, and consequently the same amount of heat is dissipated as in the resistors. The conversion of electrical energy into heat is a two-step process: the potential across the diode accelerates the electrons from the filament to the plate, changing electrical potential energy into translational kinetic energy of the electron. This kinetic energy is then converted to heat when the electrons collide with the plate.

In the above, we have assumed that the diode requires no other source of energy to perform its function. This would be true for a crystal diode, for example. For a vacuum tube diode such as is shown in the figure, however, an auxiliary source of current is required to heat the filament so that the electrons can escape more easily and travel to the plate. Heat will be dissipated at the filament due to the passage of this auxiliary current, but the extra heat due to the current in the main circuit will still be the same as that dissipated in the resistors.

(c) The motor is not an isolated system; it is turning a fan which moves air, and this requires energy. If the motor is efficient, only a small part of the energy supplied to it will be dissipated as heat within the motor itself. Most of the energy will go into increasing the kinetic energy of the air molecules, thus warming up the air.

(d) The passage of a current in the electrolytic cell causes chemical changes in it, i.e., the *chemical energy* of the cell changes. It can increase or decrease, depending on which way the cell is connected in the circuit. If it *increases,* then some of the electrical energy provided by the battery is used in accomplishing this change, so that only a part will be dissipated in heat. In this case *less* heat will be dissipated than in the resistor. On the other hand, if the chemical energy *decreases,* then this amount of energy plus the energy from the battery must all be dissipated in heat. Then the amount will be *greater* than in the resistors.

Since, in this problem, we are told that the cell dissipates less heat than the resistor, the cell is connected so as to increase its chemical energy. This means that passing current from terminal B to terminal A through the cell increases the internal energy of the cell. If the cell is now placed in a circuit in which it is the only source of energy, its internal energy will tend to decrease, which will mean that current will flow from A to B through the cell. In the circuit, the current will thus flow from B to A. Thus B is the positive terminal.

A circuit in which all of the circuit elements have an equal potential drop is a very special one. In practice, each of these elements—resistors, diodes, motors, and cell—can have widely varying characteristics in regard to potential drop and heat dissipated.

21–15

(a) In Problem 13, what is the resistance in ohms of the coil?

(b) Show that the power dissipation from the coil is given by I^2R.

(a) The current in the coil was calculated to be 2.00 amperes, and the potential difference across it is 2.09 volts. The resistance is given by Ohm's law, $V = RI$:

$$R = \frac{V}{I} = \frac{2.09 \text{ v}}{2.00 \text{ amp}} = \underline{1.05 \text{ ohms.}}$$

(b) The power dissipation is the rate at which energy is dissipated as heat. From Problem 13, this is 4.18 joules per second. But

$$I^2R = (2.00 \text{ coul/sec})^2(1.05 \text{ volts/amp})$$
$$= \underline{4.20 \text{ joules/sec.,}}$$

which is essentially the same. This is no surprise, since we used the rate of heat transfer to calculate the current in the first place.

21–16

If the conduction electrons in a metal move freely through it like the molecules of a gas, what is the mean thermal speed of the electrons at 27°C? (See Section 17–4.)

The temperature of a gas is related to the average kinetic energy of the center-of-mass motion of the gas molecules by

$$E_K = \tfrac{1}{2}mv^2 = \tfrac{3}{2}kT,$$

where k is the Boltzmann constant and T is the absolute temperature. Hence, for the electrons in a metal,

$$v^2 = 3kT/m$$

and

$$v = \sqrt{\frac{3(1.38 \times 10^{-23}\,\text{joule/°K})(300\text{°K})}{9.11 \times 10^{-31}\,\text{kg}}}$$

$$= \underline{1.17 \times 10^5\,\text{m/sec}}.$$

21–17

(a) A copper wire has n conduction electrons per unit volume. The wire is carrying a current I produced by a systematic drift of the conduction electrons parallel to its length with an average velocity v. The area of cross section of the wire is A. How many electrons pass a marked point on the wire during the time Δt?

(b) If each electron carries a charge q, what is the current in the wire?

(c) Suppose that each copper atom in the wire contributes one conduction electron. Calculate the number of conduction electrons per cubic centimeter, taking the atomic mass of copper as 63 and its density as 8.9 gm/cm³.

(d) A common size of wire has an area of cross section of 0.83 mm². A reasonable current for this wire is 5 amperes. What is the systematic drift velocity of the electrons along it?

(a) The electrons which are able to pass the marked point during Δt are just those which are within a distance $v\Delta t$ behind the mark at the start of the time interval. The volume they occupy is thus $Av\,\Delta t$ and the number of electrons within this volume is $\underline{nAv\,\Delta t}$.

(b) The current equals the total charge passing per unit time:

$$I = \frac{Q}{\Delta t} = q\frac{nAv\,\Delta t}{\Delta t} = \underline{qnAv}.$$

(c) Density = mass per volume
= (number of atoms per volume) × (mass per atom).

Hence, n = density/mass per atom.
Since the atomic mass of copper is 63 amu, Avogadro's number of copper atoms has a total mass of 63 grams and the mass per atom is (63 grams)/N_0. Hence,

$$n = \frac{8.9\,\text{gm/cm}^3\,(6.02 \times 10^{23})}{63\,\text{gm}}$$

$$= \underline{8.5 \times 10^{22}\,\text{electrons/cm}^3}.$$

(d) From part (b) the drift velocity is

$$= \frac{5\,\text{coul/sec}}{(1.6 \times 10^{-19}\,\text{coul})(8.5 \times 10^{22}/\text{cm}^3) \times (8.3 \times 10^{-3}\,\text{cm}^2)}$$

$$= \underline{4.4 \times 10^{-2}\,\text{cm/sec}} \text{ or } 0.44\,\text{mm/sec}.$$

This result should be compared with the random velocity of thermal motion calculated in Problem 16. Because of the enormous number of conduction electrons in copper, even large currents produce only very slight disturbances in their ordinary thermal motion. The assumption of one conduction electron per atom is reasonable, but if it should be wrong by a factor of 2 or 5, the contrast in velocities will not really be affected.

21–18*

If the scale divisions in Fig. 21–11 refer to volts and amperes and we connect the two circuit elements in series with a battery, what voltage would be needed to give a current of 2.0 amperes? (Section 5.)

$V = 4.9$ volts

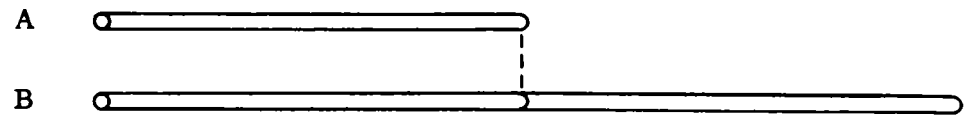
21–19

We have two lengths of wire made of exactly the same metal and of exactly the same cross-sectional area. One wire is twice as long as the other.

(a) When the same difference of potential is imposed on the ends of both wires, what is the ratio of the currents that will pass through the wires?

(b) How does the resistance of a wire depend on its length?

Problems 19 and 20 together give the dependence of the resistance of a wire on its length (Problem



19) and its cross-sectional area (Problem 20).

(a) The drop in potential between the ends of wire B is the same as that between the ends of wire A. Since in B the ends are twice as far apart as in A, the electric field E in B is only half of the electric field in A. Because the electric current is the result of a small drift velocity of the electrons superposed on their rapid thermal motions, and the drift velocity is itself proportional to the electric field, then the drift velocity of the electrons in B is therefore only half of that in A, i.e., the current is halved. The *resistance* of the wire is defined by Ohm's law:

$$V = IR;\ R = V/I.$$

Since V is the same for both wires, the *resistance* of B is *twice* that of A.

(b) By using the argument in (a), it can be seen that the resistance of a wire is directly proportional to its length.

21–20

There are two wires of identical material of exactly the same length and the same cross-sectional area.

(a) If the same difference of potential is imposed across their ends, what is the ratio of the current that will pass through them?

(b) If the wires side by side are viewed as a single conductor, what is the ratio of the current through this combined conductor to the current through one of the single wires with the same potential difference across its ends?

(c) If we have a wire of twice the cross-sectional area and the same length and material as each of the original ones, what current do you expect will flow through it compared to one of the original wires, with the same potential difference across its ends?

(d) What do you expect to be the dependence of the resistance of a wire on its cross-sectional area?

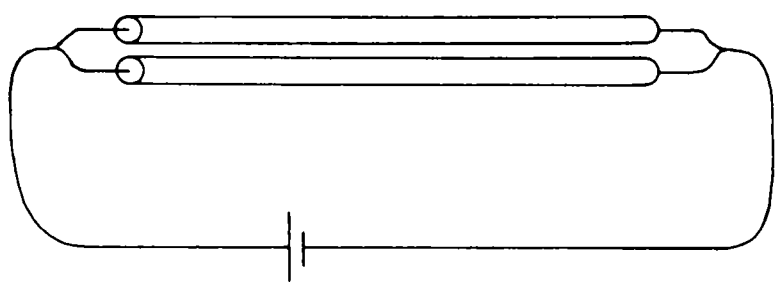
(a) If two identical wires have equal potential differences across their ends, the currents induced must be identical. It does not affect the result if one source of potential is used for both wires simultaneously. As far as the electrons in the wires are concerned, the electric force they experience is just the same.

(b) The total current passed by the two identical wires is thus twice the current in either of them.

(c) The current flow is along the length of the wires, in the direction of the electric field. There is no flow of current across the cylindrical surfaces of the wires. Hence it does not matter if they touch along their length. Thus we could touch the two wires of part (b) together along their length and regard the pair as one wire. The current in this "new" wire would be twice as great as in either of the original parts, and the cross-sectional area would have been doubled.

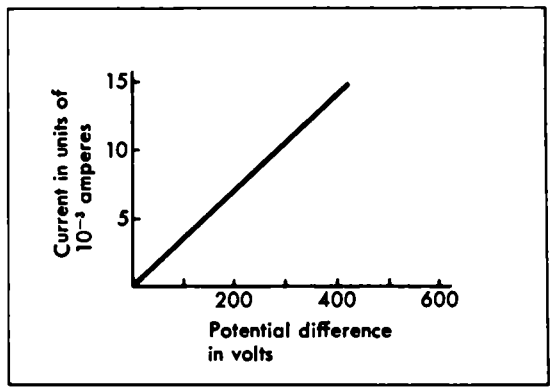
Whether we have two identical wires or a single wire of equivalent cross-sectional area, a given potential difference will force twice as much current through them (or it) as it will through a single one of the original wires. By Ohm's law, $V = RI$. Since I is doubled and V is the same, the resistance must be halved.

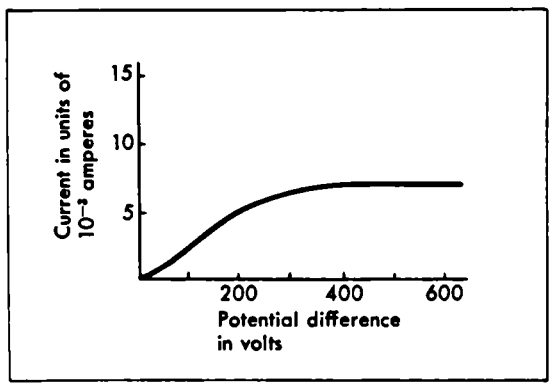
(d) The argument of part (c) leads us to believe that the resistance is inversely proportional to the cross-sectional area.



21–21

In Fig. 21–20(a), the current through a metal circuit is plotted as a function of the potential difference across its ends. In Fig. 21–20(b), we plot the current through a vacuum tube as a function of the potential difference across it.





(a) If a current of 5×10^{-3} ampere passes through both circuit elements in series, what is the potential difference from one end to the other of the resistor? What is the potential difference across the vacuum tube?

(b) If the vacuum tube and the resistor are connected in parallel as in Fig. 21–20(c), and the potential difference across the vacuum tube is 100 volts, what is the current through each of the circuit elements?

(c) What then is the total current passing point A in the figure?

(d) What is the resistance in ohms of the resistor?

(a) From Figure 21–20(a) we read that when $I = 5 \times 10^{-3}$ ampere, $V = 140$ volts. The same current in the vacuum tube (Figure 21–20(b)) is produced by a potential difference across the vacuum tube of 200 volts.

(b) In circuits of this kind (Figure 21–20(c)) one assumes that the wires themselves have negligible resistance. Thus the drop in potential between the positive terminal of the battery and the adjacent end of the resistor is negligible. Similarly the corresponding ends of the resistor and vacuum tube effectively are at the same potential, so they have equal potential differences across them.

When the potential difference across the vacuum tube is 100 volts, the current it passes is about 2×10^{-3} ampere. We have argued above that the same potential difference will appear across the resistor, and from the graph we can find that 100 volts drives a current of 3×10^{-3} ampere through the resistor.

(c) The total current at A is the sum of the currents in the resistor and in the vacuum tube, i.e., $(2 + 3) \times 10^{-3}$ ampere $= 5 \times 10^{-3}$ ampere.

(d) The resistance of the resistor is, by Ohm's law, V/I. This is the reciprocal of the slope of the straight line in Figure 21–20(a). If the axes of the graph of Figure 21–20(a) were interchanged, then the resistance would be simply the slope of the straight line. We read on the graph that $V = 0$ when $I = 0$, and $V = 430$ volts when $I = 15 \times 10^{-3}$ ampere. Then R is given by:

$$R = \frac{430 \text{ volts} - 0 \text{ volts}}{15 \times 10^{-3} \text{ amp} - 0 \text{ amps}}$$

$$= 2.9 \times 10^{4} \text{ ohms.}$$

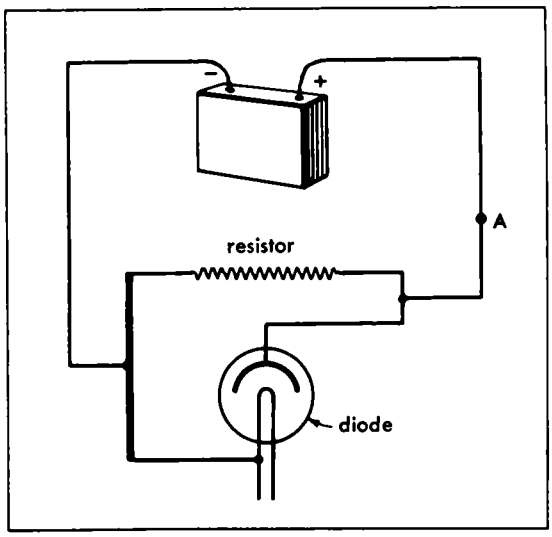
21–22

The same vacuum tube and resistor as were used in Problem 21 are connected in series. The potential difference from one end of the circuit to the other is 1000 volts.

(a) What current flows through the circuit?

(b) What is the potential difference between one end and the other of the resistor?

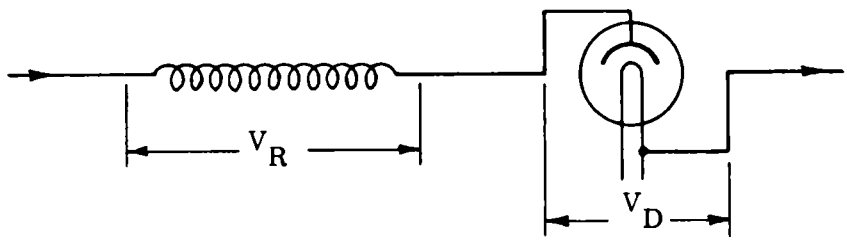
(a) Since the behavior of the vacuum tube cannot be expressed simply in terms of Ohm's law, but must be read from the graph (Figure 21–20), the general problem would have to be solved graphically. For this particular case, however, the potential difference is so high that we can give the answer without actually drawing a graph.

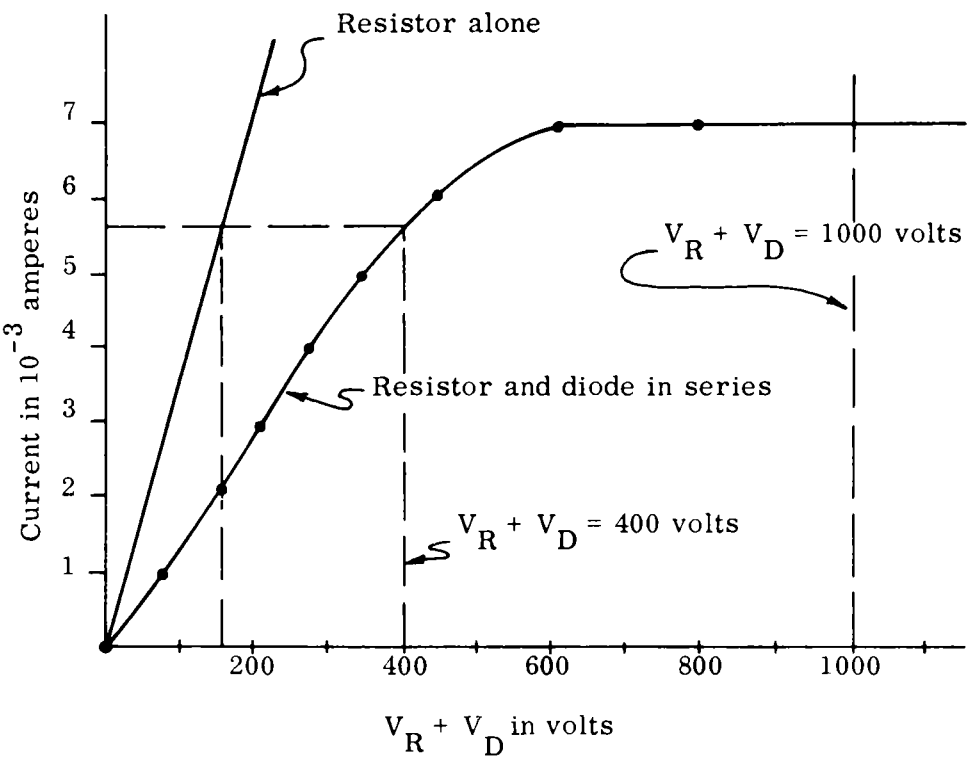


We are given the total potential difference across both vacuum tube and resistor, and it must be the sum of the potential differences across each:

$$V_R + V_D = 1000 \text{ volts.}$$

The same current flows through each. The maximum current we can get is limited by the vacuum tube, and from Figure 21–20(b) we read it to be 7×10^{-3} ampere. We shall get this provided V_D is greater than about 400 volts. We can now read from Figure 21–20(a) that a current of this magnitude requires that $V_R = 200$ volts. Then V_D must in fact be (1000 − 200) volts = 800 volts, which is greater than the potential required to give the saturation current in the vacuum





tube. This completes the circle of our argument, and justifies our guess that the current is in fact the saturation current of the vacuum tube, 7×10^{-3} ampere.

(b) We have already calculated that the potential difference across the resistor is 200 volts. The general way to solve this problem for any applied potential is to consider the reverse situation, that the current is known, and calculate the total potential difference $V_R + V_D$. We can then draw a graph of $V_R + V_D$ vs. I, and interpolate graphically to find I for a given $V_R + V_D$. Reading from Figures 21–20(a) and 21–20(b), we can make up the table above. From this we then draw the graph on the previous page.

CURRENT (IN 10^{-3} AMPERE)	v_R (IN VOLTS)	v_D (IN VOLTS)	$v_R + v_D$ (IN VOLTS)
0	0	0	0
1	30	40	70
2	60	90	150
3	90	120	210
4	110	160	270
5	140	200	340
6	170	270	440
7	200	and greater	and greater

The solution to the original problem can now be read from this graph. We see that it lies in the high-voltage region, where the behavior is very simple. Suppose the total potential difference is maintained at 400 volts. We can then read from the graph that the current will be approximately 5.6×10^{-3} ampere, and that the potential difference across the resistor alone is 160 volts.

21–23

Connect a 270-volt battery in series with a 1,000,000-ohm and a 2,000,000-ohm resistor.

(a) What potential difference do you expect across the megohm resistor?

(b) What potential difference do you expect across the 2,000,000-ohm resistor?

Now suppose you measure the potential differences with a voltmeter.

(c) If the voltmeter has a resistance of 10 megohms, what potential difference would you expect across the 2,000,000-ohm resistor?

(d) What potential difference would you expect to read across the megohm resistor?

This is a straightforward problem in the application of Ohm's law to a simple series-parallel circuit. If you discuss the problem in class, it is best to work through the solution following the method outlined in the text, without stressing "formulas" for circuits.

(a) We see that the sum of the potential differences across the two resistors is

$$V_1 + V_2 = 270 \text{ volts.}$$

Since the current is the same everywhere in the circuit,

$$V_1 = IR_1, \text{ and } V_2 = IR_2.$$

Thus,

$$V_1 + V_2 = I(R_1 + R_2),$$
$$I = V_1 + V_2/R_1 + R_2.$$

The potential difference across the megohm resistor is

$$V_1 = IR_1 = \frac{(V_1 + V_2)R_1}{R_1 + R_2}$$
$$= \frac{270\,v \times 1 \text{ meg}}{3 \text{ meg}} = \underline{90 \text{ volts.}}$$

(b) The potential difference across the 2-megohm resistor is

$$V_2 = IR_2 = \frac{(V_1 + V_2)R_2}{R_1 + R_2}$$
$$= \frac{270\,v \times 2 \text{ meg}}{3 \text{ meg}} = \underline{180 \text{ volts.}}$$

(c) With the meter placed in the circuit across the 2-megohm resistor, we now have the equivalent of a 10-megohm resistor in parallel with R_2. As in (a),

$$V_1 + V_2 = 270 \text{ volts.}$$
$$V_1 = IR_1$$
$$V_2 = I_2R_2 + I_mR_m.$$

Since the currents I_2 and I_m must equal the total current,

$$I = I_2 + I_m = \frac{V_2}{R_2} + \frac{V_2}{R_m} = \frac{V_1}{R_1}.$$

But $V_1 = 270 \text{ volts} - V_2$,

$$\frac{V_2}{R_2} + \frac{V_2}{R_m} = \frac{(270 \text{ volts} - V_2)}{R_1}.$$

Since we know the values of all the resistances, and are trying to find V_2,

$$\frac{V_2}{2\text{meg}} + \frac{V_2}{10\text{ meg}} = \frac{(270\text{ volts} - V_2)}{1\text{ meg}}.$$

Multiplying both sides of the equation by 10 meg,

$$\begin{aligned} 5V_2 + 1V_2 &= 2700\text{ volts} - 10V_2 \\ 16V_2 &= 2700\text{ volts} \\ V_2 &= 170\text{ volts}. \end{aligned}$$

It may be interesting to solve for the other voltages and currents:

$$V_1 = 270\text{ volts} - V_2 = 100\text{ volts};$$
$$I = \frac{V_1}{R_1} = \frac{100\text{ volts}}{10^6\text{ ohms}} = 10^{-4}\text{ amp}$$
$$I_2 = V_2/R_2 = 170\text{ volts}/(2 \times 10^6\text{ ohms}) = 0.85 \times 10^{-4}\text{ amp}$$
$$I_m = I - I_2 = 0.15 \times 10^{-4}\text{ amp}.$$

(d) Using the same method as in (c),

$$I = I_m + I_1$$
$$\frac{V_2}{R_2} = \frac{V_1}{R_m} + \frac{V_1}{R_1}.$$

Now, $V_2 = 270 - V_1$

$$\frac{270 - V_1}{2\text{ meg}} = \frac{V_1}{10\text{ meg}} + \frac{V_1}{1\text{ meg}}$$
$$V_1 = 84\text{ volts}.$$

If they don't notice it, it is worthwhile to point out to students that the solutions to (c) and (d) do not add up to 270 volts, even though the voltmeter would read 270 volts when placed across both resistances.

CHAPTER 22

22–1*
If a long straight wire carrying a current were placed flat on a paper and iron filings were sprinkled on the paper, what would you expect the iron filings to do? (Section 2.)

The filings would try to stand up at right angles to the paper.

22–2*
Do magnetic field lines have a beginning and ending? (Section 2.)

No; they are all closed loops of various shapes.

22–3*
What is the direction of the magnetic field produced by positive charges moving directly away from you? Negative charges moving toward you? (Section 2.)

Clockwise, in both cases.

22–4*
Two identical circular loops carry equal currents, have a common center, and are at right angles to each other. How does the magnitude of the field they produce compare with the magnitude of the field from just one of them? (Section 3.)

The magnitude of the resultant field is $\sqrt{2}$ times the magnitude of the field from one.

22–5*
How does the field produced by a current-carrying wire depend on the magnitude of the current? (Section 3.)

The field is proportional to the magnitude of the current.

22–6
A horizontal uniform magnetic field $\vec{\mathbf{B}}$ is perpendicular to the horizontal component $\vec{\mathbf{B}}_e$ of the earth's magnetic field and points east.

(a) The ratio $B/B_e = \sqrt{3}$; in which direction will a compass needle point?
(*Note:* the compass needle rotates in a horizontal plane.)

(b) If the compass needle points northeast, what is the magnitude of $\vec{\mathbf{B}}$ now?

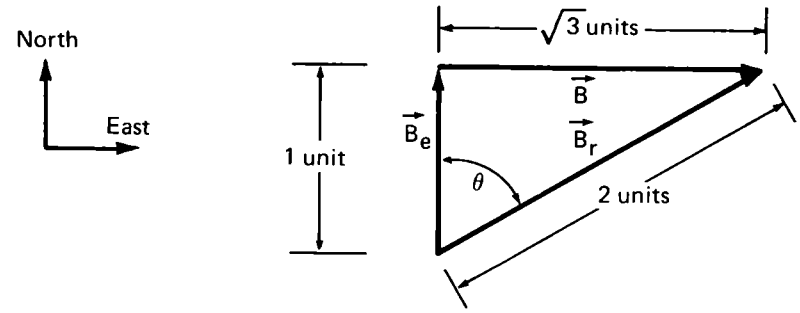
(a) Since we know the directions and relative magnitude of both fields, we can draw a vector diagram to scale, and construct the resultant magnetic field vector $\vec{\mathbf{B}}_r$. From it we can find both the magnitude of $\vec{\mathbf{B}}_r$ relative to that of $\vec{\mathbf{B}}_e$ and the direction of $\vec{\mathbf{B}}_r$.

Some students will recognize a triangle whose sides are in the ratio of $1:\sqrt{3}:2$ as a $30°-60°-90°$ triangle, or we can find the angle θ from the diagram or by trigonometry:

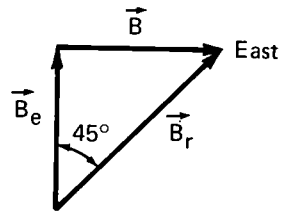
$$\tan\theta = \frac{B}{B_e} = \sqrt{3}, \text{ therefore } \theta = 60°.$$

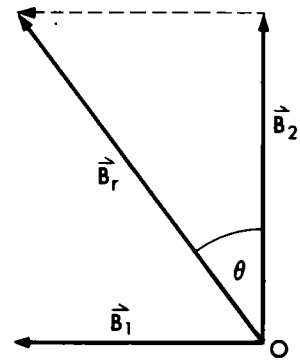
The compass needle will point 60° east of north.

(b) If we continue to assume that $\vec{\mathbf{B}}$ points east we can find its magnitude by constructing the

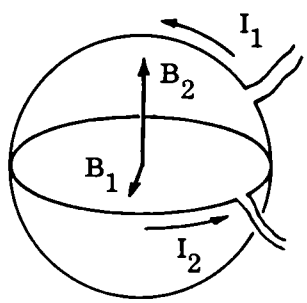


vector addition triangle of $\vec{B}$, $\vec{B}_e$, and $\vec{B}_r$. These three vectors form a 45° right triangle, so $B = B_e$.





22–7
Two circular wire loops of equal radius are set up at right angles to each other with a common center (Fig. 22–33). If $I_1 = 3.0$ amperes and $I_2 = 4.0$ amperes, what is the direction of the magnetic field at the center, 0?



The direction of the field produced by I_1 is directly out of the page, by the "right-hand rule"; the direction of the field produced by I_2 is vertically up (see drawing). Since the fields are proportional to the currents producing them, we can make a scale drawing to find the direction of the resultant. The angle θ between the resultant $\vec{B}_r$ and the field $\vec{B}_2$ produced by I_2 is 37°. This answer can also be obtained from trigonometry, since

$$\tan\theta = B_1/B_2 = 3.0/4.0 = 0.75.$$

22–8
Suppose two loops are fixed at right angles and you can run a current in either direction through either loop. Neglecting the earth's field, how many different magnetic fields could you produce if a current of 2 amps flows whenever a loop is connected?

If we run a current through one loop at a time, there are four possible fields we can produce, corresponding to a current in each direction in each loop. If we run currents through both loops simultaneously, there are four other possible fields, corresponding to all possible combinations. Hence, the total number of different fields is eight.

22–9*
In what direction can an electric charge move through a magnetic field without having a force exerted on it? (Section 4.)

Parallel to the field, in either the same or opposite direction as the field.

22–10*
Suppose a wire carries a current from left to right through a magnetic field whose direction is toward you. In what direction is the magnetic force on the wire? (Section 4.)

Downward.

22–11*

What is the force on a wire 0.20 meter long, carrying a current of 3.0 amps, which is placed within a uniform magnetic field of 5.0 newtons/amp-m at an angle of 45° to the field? (Section 4.)

$F = 2.1$ newtons.

22–12

The magnetic field inside a long solenoid is uniform and parallel to the axis of the solenoid.

(a) What is the force on a current-carrying wire inside the solenoid and parallel to the axis?

(b) What is the force on the wire shown in Fig. 22–34 which carries a current of 1.0 ampere, if the field B in the solenoid is 1.0×10^{-2} newton/amp-m and the length CD is 2 cm?

(c) How much mass m should be placed on the other end of the balance to keep it from tilting?

(d) Describe how you could use this setup as an ammeter.

(a) The current in a wire parallel to the axis of a solenoid is also parallel to the field of the solenoid; hence there is no component of $\vec{\mathbf{B}}$ perpendicular to the current. The force on such a wire is zero. (The effects near the end of the solenoid, where $\vec{\mathbf{B}}$ is not quite parallel to the current, are negligible.)

(b) As stated in (a), the wires carrying the current to and from the segment CD are parallel to the field; there is no force on them. The current in CD is perpendicular to the field. By the right-hand rule of Section 4 this force is down. The magnitude of the force is BIl:

$$F = 1.0 \times 10^{-2} \text{ nt/amp-m} \times 1.0 \text{ amp} \times 0.02 \text{ m}$$
$$= 2 \times 10^{-4} \text{ newton, downward.}$$

(c) Assuming that the balance is an equal-arm balance, the force found in (b) can be counterbalanced by a mass, m, whose weight $w = mg$) must be equal to 2×10^{-4} newton. Solving for m ($g = 9.8$ m/sec^2),

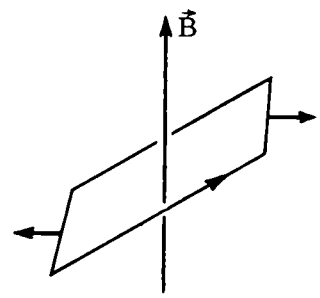
$$m = \frac{2 \times 10^{-4} \text{ nt}}{9.8 \text{ m/sec}^2} = 2 \times 10^{-5} \text{ kg}.$$

(d) The weight of the mass could be replaced by the force of an extended spring. The extension of a spring is approximately linear with the stretching force, and the magnetic force is linear with the current, so the scale of the spring balance could be marked in a linear way to read amperes directly.

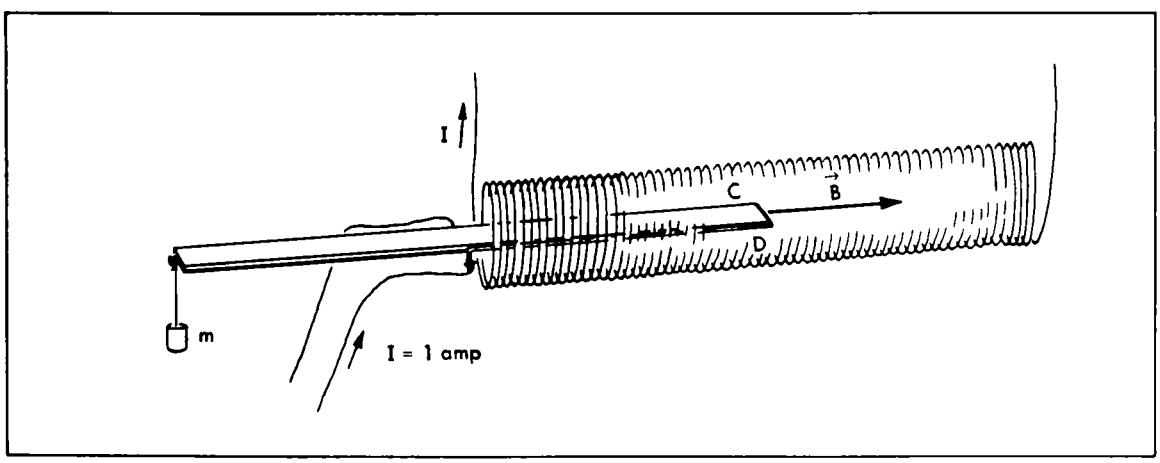
22–13

(a) Show that a square loop of wire carrying a current will always tend to align itself in a magnetic field so that the plane of the loop is perpendicular to the field.

(b) What do the magnetic forces tend to do to the loop in this position? (Hint: draw a side view.)

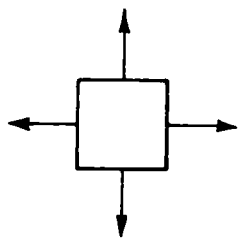


(a) Suppose the plane of the loop is not perpendicular to the direction of the magnetic field. In the position shown, the current in



the loop will experience forces tending to pull the loop into the plane perpendicular to the magnetic field. More complicated orientations give more complicated forces, but they will still tend to rotate the loop into the plane perpendicular to the field.

(b) When the loop is perpendicular to the field, the forces are as shown in the diagram. Because the sides are all equal in length, and because the magnitude of the current is the same in all the sides, the forces are all equal in magnitude. Therefore, there is no net translational force on the loop. The forces tend to pull the loop apart, and they tend to keep the loop perpendicular to the field.



Notes on Problem 13.

(1) If the field were nonuniform the loop would still tend to rotate to a position where the plane of the loop would be perpendicular to the field in some average sense. (It would be possible to find fields which would not rotate the loop.) After this rotation there might be a net translational force on the loop. This unbalanced force can be resolved into two components: one parallel to the plane of the loop and the other perpendicular to the plane of the loop. It is easy to show that both of these components are in directions such as to translate the loop toward the region where the magnetic field is strongest.

(2) If we remember that a bar magnet is equivalent to some configuration of current loops with planes perpendicular to the magnet, we can use the converse of this fact to see the solution of part (a). Replace the square current loop with a proper configuration of bar magnets lying along the direction $\vec{N}$. They, we know, will tend to line up with the field. Thus the current loop will have its plane perpendicular to the field. Also, there is another position of equilibrium for the current loop. This is the inverted position. In that position the force on each side is inward instead of outward, and any small deflection from the exactly perpendicular position will lead to a flipping over of the coil by the forces acting on it. This kind of an equilibrium is therefore called unstable equilibrium. With the forces acting outward, the equilibrium is called stable, because the action of the forces will restore that position if any small deflection occurs.

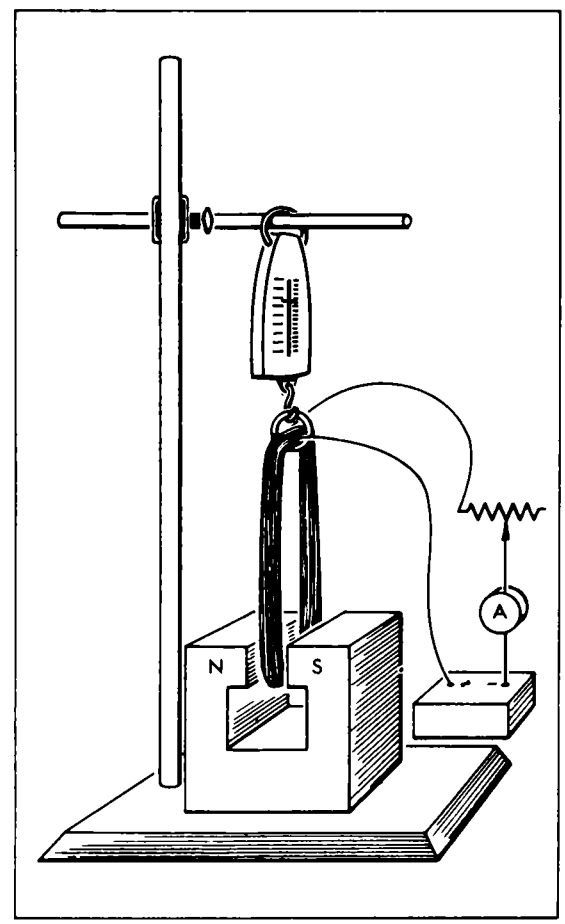
22–14*

How does the force that opposes the magnetic force in a moving-coil ammeter depend on the angle of rotation of the pointer on the ammeter's scale? (Section 5.)

The force is directly proportional to the angle of rotation.

22–15

In the moving-coil ammeter of Fig. 22–16, the magnetic force on the coil is balanced by a torsion spring. An ammeter can also be constructed as shown in Fig. 22–35, using a small spring balance to measure



the magnetic force on the coil. By adjustment of the position of the support, the conductors can be kept in the same position in the magnetic field for different currents.

One such coil, 4.0 cm wide by 25 cm high, was constructed of 100 turns and weighed 2.4 newtons. The spring balance had a scale of 0 to 5 newtons.

(a) When supported in a magnetic field, as shown, the spring balance changed from 2.4 to 2.7 newtons as the current was increased from 0 to 0.5 ampere. What is the maximum current that could be measured on this spring balance?

(b) What is the magnetic field in the air gap of the magnet?

(a) The useful range of our spring balance is from 2.4 to 5.0 newtons, an interval of 2.6 newtons. At 0.5 ampere per 0.3 newton, it would give us a maximum range of

$$(2.6\text{ nt})(1.7\text{ amp/nt}) = \underline{4.4\text{ amperes.}}$$

(b) With 100 turns of wire, each contributing 4.0 cm of wire perpendicular to the field which will produce a downward force, the length is 4.0 meters.

$$B = \frac{F}{Il} = \frac{0.3\text{ nt}}{(0.5\text{ amp})(4.0\text{ m})}$$

$$= \underline{0.15\text{ newton/ampere-meter.}}$$

22–16*

How could the direction of rotation of the coil in Fig. 22–17 be reversed? (Section 5.)

Switch the connections to the battery terminals, or reverse the direction of the magnetic field—but not both.

22–17*

What happens to the radius of the circle in Fig. 22–18(b) if the accelerating potential inside the electron gun is raised? (Section 6.)

The radius increases as the square root of the accelerating potential.

22–18*

Upon what does the magnetic force on a moving electric charge depend? (Section 6.)

The velocity of the charge, the sign and magnitude of the charge, and the component of magnetic field perpendicular to the velocity.

22–19

How do the momenta of a proton and an electron compare if both are moving perpendicular to the same magnetic field in circles of the same radius?

The magnetic force F on a moving charged particle is $F = qB_\perp v$. Since it always acts perpendicular to the velocity of the particle, it causes the particle to move in a circle whose radius r is related to the magnetic force by $F = mv^2/r$. Hence, the momentum p of the particle is

$$p = mv = qB_\perp r.$$

Since a proton and an electron each have one elementary charge, then if they move in circles of the same radius perpendicular to the same magnetic field, their momenta must be equal in magnitude.

Note that the momentum vectors of the proton and of the electron rotate in the opposite sense; if for one particle they rotate in the clockwise sense, for the other particle they rotate in the counterclockwise sense. In order to determine which particle is rotating in which sense, we must know the direction of the magnetic field.

22–20†

(See also the Appendix in the text.)

A number of questions may arise from discussion of the solution to this problem as given in the text. If your students wish to continue the discussion you may find these additional points helpful.

For the lightest element, hydrogen ($m = 1$ amu), this difference will be

$$\Delta\alpha = c\left(1 - \frac{1}{\sqrt{2}}\right) = 0.293c, \text{ where } c = \frac{1}{\sqrt{k}}.$$

For very heavy elements $\frac{1}{\sqrt{m+1}}$ will be very nearly equal to $\frac{1}{\sqrt{m}}$ and will be very small compared to $0.293c$. In fact, for the two isotopes of uranium, U^{234} and U^{235}, the difference $\Delta\alpha$ in the angle of deflection will be

$$\Delta\alpha = c\left(\frac{1}{\sqrt{234}} - \frac{1}{\sqrt{235}}\right) = c(0.0654 - 0.0653).$$

Slide-rule calculations give zero for the answer, since the difference inside the parentheses is zero to three significant figures. Five-place tables of logarithms could be used to evaluate $\Delta\alpha$, but it is

not necessary to do this to answer the question. It is quite clear that $(\Delta\alpha)_H$ is at least 3×10^3 $(\Delta\alpha)_U$.

Since the ion beam has some width, it is highly probable that a mass spectograph that can separate H^1 from H^2 to the two ends of the available distance across the photographic-plate detector will not show any separation at all between U^{234} and U^{235} ions of the same charge.

To separate U^{234} from U^{235} ions a different mass spectrograph from that used for light elements like H^1 would probably be used. Such a spectrograph would employ a much stronger magnetic field to give a reasonable deflection for heavy ions.

In principle, it is also possible to increase the angular deflection $\Delta\alpha$ by decreasing the accelerating potential V. That this is true can be seen from the equation

$$m = \frac{B^2q}{2V}r^2$$

$$r = \sqrt{\frac{2Vm}{B^2q}}$$

$$= k\sqrt{v}$$

for a given ion and a fixed magnetic field $\vec{\mathbf{B}}$. Since $r \propto \frac{1}{\alpha}$, the deflection α and the accelerating potential are related by

$$\alpha \propto \frac{1}{\sqrt{v}}.$$

However, if the accelerating potential is too low the beam broadens because the random thermal velocity of the ions becomes a significant fraction of the velocity given them by the accelerating potential.

Although it is easy to see that $\Delta\alpha = c\left(\frac{1}{\sqrt{234}} - \frac{1}{\sqrt{235}}\right)$ is much smaller than $\Delta\alpha$ for hydrogen. For uranium, $\Delta\alpha$ can be evaluated using a slide rule after a little algebraic manipulation as follows:

$$\Delta\alpha = c\left(\frac{1}{\sqrt{m}} - \frac{1}{\sqrt{m+1}}\right) = \frac{\sqrt{m+1} - \sqrt{m}}{\sqrt{m}\sqrt{m+1}}$$

multiplying numerator and divisor by $\sqrt{m+1} + \sqrt{m}$ gives

$$\Delta\alpha = c\left(\frac{(\sqrt{m+1})^2 - (\sqrt{m})^2}{\sqrt{m}\sqrt{m+1}\sqrt{m+1} + \sqrt{m}}\right)$$

$$= \frac{c}{\sqrt{m}(m+1) + m\sqrt{m+1}}.$$

Thus for U^{234} and U^{235}

$$\Delta\alpha = \frac{c}{\sqrt{234}(235) + 234\sqrt{235}}$$

$$= 1.40 \times 10^{-4}c$$

and

$$\frac{(\Delta\alpha)H}{(\Delta\alpha)U} = \frac{10.293c}{1.40 \times 10^{-4}c} = 2.08 \times 10^3.$$

22–21

A beam of protons of kinetic energy E_K is injected into a region of uniform magnetic field $\vec{\mathbf{B}}$ at right angles to the direction of the beam.

(a) Find expressions for the radius and the period of the resulting circular motion in terms of the magnetic field strength.

(b) How would your answers to (a) be affected if the incident energy were four times as great?

(c) Protons of energy 4.60×10^5 electron volts are injected into a proton synchrotron in which the magnetic field strength is 2.10×10^{-2} newton/amp-m. Calculate the period and the radius of the circular motion.

(a) Let m be the proton mass, q the charge on the proton, and v its speed. The radius of the circular motion at right angles to a magnetic field of strength B is

$$r = \frac{mv}{qB}.$$

In terms of the kinetic energy, the momentum is

$$p = mv = (2mE_K)^{1/2}.$$

Then the radius can be expressed in terms of the known quantities m, E_K, q, and B as

$$r = \frac{(2mE_K)^{1/2}}{qB}.$$

The expression for the period is simpler. The period of a particle moving in a circle of radius r at constant speed v is $T = 2\pi r/v$. From our first expression for the radius, in terms of v, we see that $r/v = m/qB$. Hence,

$$T = \frac{2\pi m}{qB}.$$

(b) If E_K were four times as great, the radius would be only twice as great, since the radius varies as the square root of the kinetic energy.

The period does not depend on the kinetic energy at all, and would be unchanged.

(c) Using the expressions found in (a), we have

$$T = \frac{2\pi m}{qB}$$

$$= \frac{2(3.14)(1.67 \times 10^{-27}\ \text{kg})}{(1.60 \times 10^{-19}\ \text{coul})(2.10 \times 10^{-2}\ \text{nt/amp-m})}$$

$$= 3.12 \times 10^{-6}\ \text{second}$$

$$= \underline{3.12\ \text{microseconds.}}$$

The radius is

$$r = (2mE_K)^{1/2}/qB$$

$$= \frac{\sqrt{2(1.67 \times 10^{-27}\ \text{kg})(4.60 \times 10^{5}\ \text{ev}) \times (1.60 \times 10^{-19}\ \text{joule/ev})}}{(1.60 \times 10^{-19}\ \text{coul})(2.10 \times 10^{-2}\ \text{nt/amp-m})}$$

$$= \underline{4.67\ \text{meters.}}$$

22–22*

What is the kinetic energy of alpha particles accelerated by 180 volts? (Section 7.)

360 electron volts $= 5.76 \times 10^{-17}$ joule.

22–23

Charged particles are accelerated by a uniform electric field and then deflected by a magnetic field. What quantities must be measured to determine the mass of the particles from this experiment?

The quantities which must be measured are the potential difference through which the particle moves, the charge of the particle (these two quantities determine the kinetic energy of the particle as it enters the magnetic field), the magnetic field, and the radius of curvature of the path of the particle in the magnetic field (these last two quantities, together with the charge, tell us the momentum of the particle). From the kinetic energy and the momentum, we can then determine the mass of the particle.

22–24

When helium gas is fed into the apparatus shown in Fig. 22–21, mounted in a chamber from which the gas can be pumped to keep the pressure low, He^+ and He^{++} ions are produced. These two ions form two spots on the screen below the position of the beam with no field. The face of the magnet is 3.0 cm by 1.5 cm and the center of the face is 10 cm from the screen. The strength of the magnetic field is 1.5×10^{-2} newton/amp-m. The ions are accelerated by 90 volts.

(a) Compute the radius of the circular part of the path of each ion while it is in the magnetic field.

(b) Make a scale drawing and determine the position of each spot on the screen.

(c) What would be the path of a He^{++} ion which gains one electron while in the electric field? Describe it qualitatively.

(d) What would be the path of He^{++} ion which gains one electron while in the magnetic field? Describe it qualitatively.

(a) The radius of the path of a charged particle moving in a magnetic field perpendicular to the path is $r = mv/qB$. Since $mv = \sqrt{2mE_K}$ and $E_K = qV$, we have

$$r = \frac{mv}{qB} = \frac{\sqrt{2mqV}}{qB} = \left(\frac{1}{B}\right)\sqrt{\frac{2mV}{q}}.$$

For the He^+ ion,

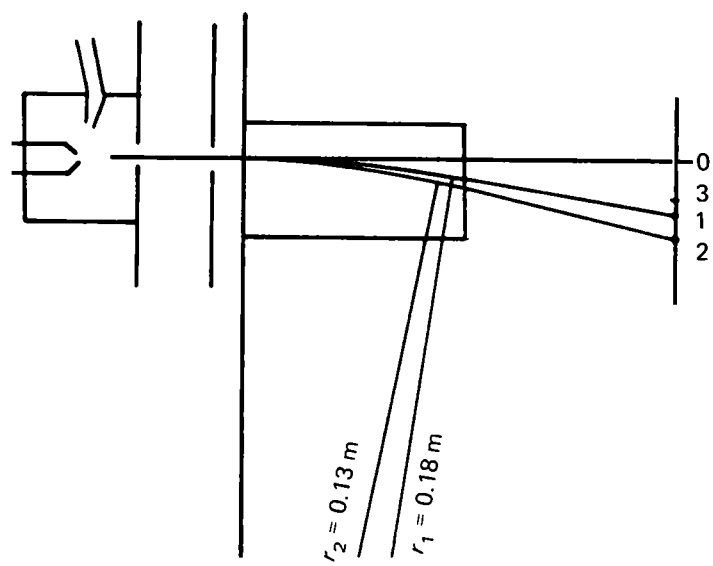
$$r^+ = \sqrt{\frac{2(6.6 \times 10^{-27}\ \text{kg})(90\ \text{volts})}{1.60 \times 10^{-19}\ \text{coul}}} \times \left(\frac{1}{1.5 \times 10^{-2}\ \text{nt/amp-m}}\right)$$

$$= \underline{0.18\ \text{meter.}}$$

For the He^{++} ion, all quantities are the same except the charge, which is twice as great, so

$$r^{++} = r_1/\sqrt{2} = \underline{0.13\ \text{meter.}}$$

(b)



(c) If an electron is gained in the electric field, the energy of the ion will be between ev and 2 ev. Its speed v as it enters the magnetic

field will be between v_1 and v_2. Let's look at the solutions at the limiting values. If the electron is gained right at the beginning, $v = v_1$ and $R = R_1$. If the electron is gained just as the ion leaves the electric field, $v = v_2$ and $R = 2R_2$, which puts a spot above 1 in the figure about halfway from 0 to 2, or at 3. Therefore, gaining an electron in the electric field causes R to vary between R_1 and $2R_2$, and the spot to vary from position 1 to 3.

(d) If an electron is gained in the magnetic field, the speed of the ion will be v_2. We have already worked out the limiting cases. If the ion picks up an electron between the two fields, R will equal $2R_2$. If the electron is picked up at the end of the field, the radius of curvature will be R_2. The ion will travel on a path of radius R_2 until the electron is picked up and then it will travel on a path of radius $2R_2$ and will hit the screen somewhere between 2 and 3.

22–25

A nearly uniform electric field is set up parallel to the surface of a glass-topped table by charging two large parallel, vertical metal plates. The plates are 0.50 m apart and the potential difference between them is 20 kv. A small Dry Ice puck with a total mass of 60 grams carries a copper-coated Ping-Pong ball which has a charge of 2.2×10^{-8} coulomb.

(a) What is the electric force on the ball when it is placed between the plates?

(b) What would be the shape of its path if it were given a brief push perpendicular to the field lines?

(c) Even in a restricted region, the largest magnetic field that can be obtained fairly easily is 0.50 newton/amp-m. If you projected the puck through such a field perpendicular to the field at a speed of one meter/second, what would be the magnetic force on the puck?

(a) The electric force on the ball is

$$F = qE = qV/d$$
$$= \frac{(2.2 \times 10^{-8}\ \text{coul})(2.0 \times 10^{4}\ \text{volts})}{0.50\ \text{m}}$$
$$= \underline{8.8 \times 10^{-4}\ \text{newton.}}$$

(b) The path would be a parabola.

(c) The magnetic force is

$$F_{\text{mag}} = qvB$$
$$= (2.2 \times 10^{-8}\ \text{coul})(1\ \text{m/sec})(0.50\ \text{nt/amp-m})$$
$$= \underline{1.1 \times 10^{-8}\ \text{newton.}}$$

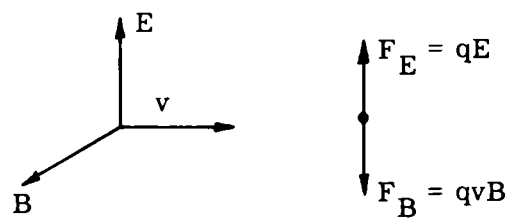
We see that in this case the electric force is 8×10^4 times as large as the magnetic force.

22–26

A beam of singly charged ions moves into a region of space where there is a uniform electric field $E = 1.0 \times 10^3$ newton per coulomb, and a uniform magnetic field $B = 2.0 \times 10^{-2}$ newton/amp-m. The electric and magnetic fields are at right angles to each other and both are perpendicular to the beam so that the electric and magnetic forces on an ion oppose each other. What is the speed of those ions which move *undeflected* through these crossed electric and magnetic fields?

We must find the speed at which the magnetic force is equal and opposite to the electric force. The magnetic force and electric force are oppositely directed so that we may write $qE = qvB$. Cancelling q and solving for v, we have

$$v = \frac{E}{B} = \frac{1.0 \times 10^3}{2.0 \times 10^{-2}}$$
$$= \underline{5.0 \times 10^4\ \text{m/sec.}}$$

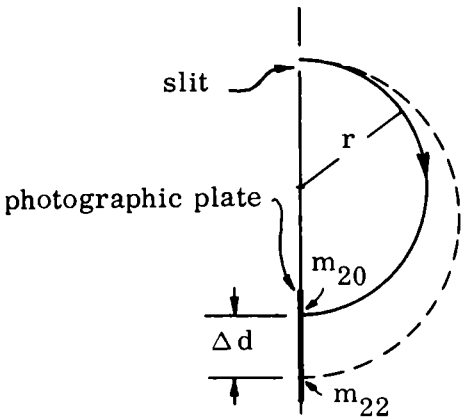


22–27

The undeflected ions of Problem 26 are passed through a slit and move into a region where there is a uniform magnetic field $B = 0.09$ newton/amp-m at right angles to their motion. If the ions are a mixture of the neon ions of mass 20 and 22 atomic mass units, how far apart will these ions land on a photographic plate after they have moved through a semicircle?

(In this view, the direction of the field $\vec{\mathbf{B}}$ is out from the page toward the viewer.)

The spot where the neon ions strike the plate will be a diameter away from the slit. We can calculate the radius of the circular path by the method described in Section 7. The magnetic force qvB produces a centripetal acceleration v^2/r. Thus, by Newton's law,



$$qvB = m\frac{v^2}{r},$$

$$mv = qBr, \text{ and}$$

$$r = mv/qB.$$

The ions have the same speed and the same charge, so the difference in the diameters of their paths is due only to the mass difference:

$$\Delta d = d_{22} - d_{20} = 2(m_{22} - m_{20})\frac{v}{qB}.$$

Since 1 amu $= 1.7 \times 10^{-27}$ kg,

$$\Delta d = \frac{2(2 \text{ amu})(1.7 \times 10^{-27} \text{ kg/amu}) \times (5.0 \times 10^{4} \text{ m/sec})}{(1.6 \times 10^{-19} \text{ coul})(9 \times 10^{-2} \text{ nt/amp-m})}$$

$$= \underline{0.02 \text{ meter.}}$$

22–28*

A wire carrying a current I threads a square loop 2 cm on a side. What is the circulation around the square? (Express your answer in terms of K.) (Section 10.)

$2\pi KI$.

22–29*

What is the circulation around the loop shown in Fig. 22–36? (Section 10.)

Zero.

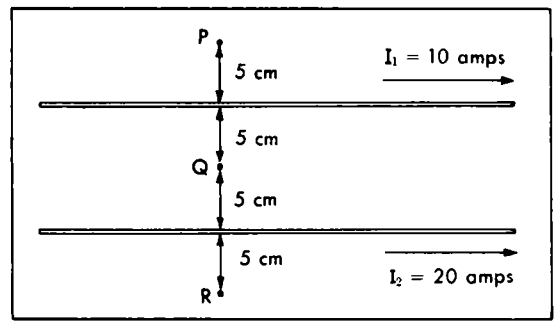
22–30*

What is the field in the center of a long solenoid of 10 turns/cm carrying 5 amp? (Section 11.)

$B = 2\pi \times 10^{-3}$ newton/amp-m $=$
6×10^{-3} newton/amp-m.

22–31

Two very long straight parallel wires, one carrying 10 amperes and the other 20 amperes, are separated by a distance of 10 cm (Fig. 22–37). Find the magnitude and direction of $\vec{\mathbf{B}}$ at the points P, Q, and R shown in the figure.



The magnetic field of each wire is perpendicular to the page at the points P, Q, and R. By the right-hand rule we can tell whether each field points into or out of the page at these points. The magnitude of each field at a distance r from the wire producing it is $B = KI/r$, where $K = 2 \times 10^{-7}$ nt/(amp)^2.

At point P:

$$\vec{\mathbf{B}}_1 = \frac{(2 \times 10^{-7})(10)}{5 \times 10^{-2}} = 4 \times 10^{-5} \text{ nt/amp-m, out.}$$

$$\vec{\mathbf{B}}_2 = \frac{(2 \times 10^{-7})(20)}{15 \times 10^{-2}} = 3 \times 10^{-5} \text{ nt/amp-m, out.}$$

Hence, the resultant field is

$$\vec{\mathbf{B}} = \underline{7 \times 10^{-5} \text{ nt/amp-m, out.}}$$

At point Q: $\vec{\mathbf{B}}_1$ has the same magnitude as at P but points *into the page;* $\vec{\mathbf{B}}_2$ points outward and has twice the magnitude of $\vec{\mathbf{B}}_1$, since the current producing it is twice as great. Hence, the resultant field is

$$\vec{\mathbf{B}} = \underline{4 \times 10^{-5} \text{ nt/amp-m, out.}}$$

At point R: $\vec{\mathbf{B}}_1$ points in and is only a third as strong as at point P:

$$\vec{\mathbf{B}}_1 = 1 \times 10^{-5} \text{ nt/amp-m, in.}$$

$\vec{\mathbf{B}}_2$ points in also and is as strong as at Q:

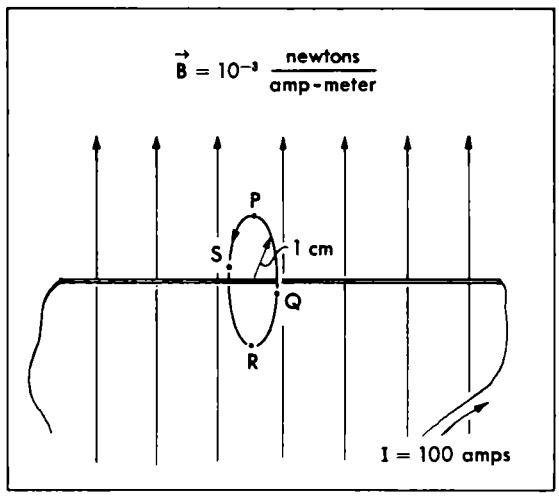
$$\vec{\mathbf{B}}_2 = 8 \times 10^{-5} \text{ nt/amp-m, in.}$$

The resultant is therefore

$$\vec{\mathbf{B}} = \underline{9 \times 10^{-5} \text{ nt/amp-m, in.}}$$

22–32

A long straight wire carrying a current $I = 100$ amperes is perpendicular to a uniform magnetic field of strength $B = 1.0 \times 10^{-3}$ newton/amp-m (Fig. 22–38). What is the strength and direction of the resultant $\vec{\mathbf{B}}$ field at the points P, Q, R, S on a circle of radius 1 cm around the wire?



We find the field $\vec{\mathbf{B}}_w$ produced at each point by the 100 ampere current in wire and add it vectorially to the field $\vec{\mathbf{B}}$ to obtain the resultant field $\vec{B}_r$ at each point. The magnitude of $\vec{\mathbf{B}}_w$ is

$$B_w = 2 \times 10^{-7} \frac{\text{newton}}{\text{amperes}^2} \times \frac{10^2 \text{ amperes}}{10^{-2} \text{ meter}}$$

$$= 2 \times 10^{-3} \frac{\text{newton}}{\text{ampere-meter}}.$$

The direction of $\vec{\mathbf{B}}_w$ at each point is shown in the diagram.

At P or R, $B_r = \sqrt{B^2 + B_w{}^2} = \sqrt{1^2 + 2^2} \times 10^{-3}$

$$= \underline{2.2 \times 10^{-3} \text{ newton/amp-meter.}}$$

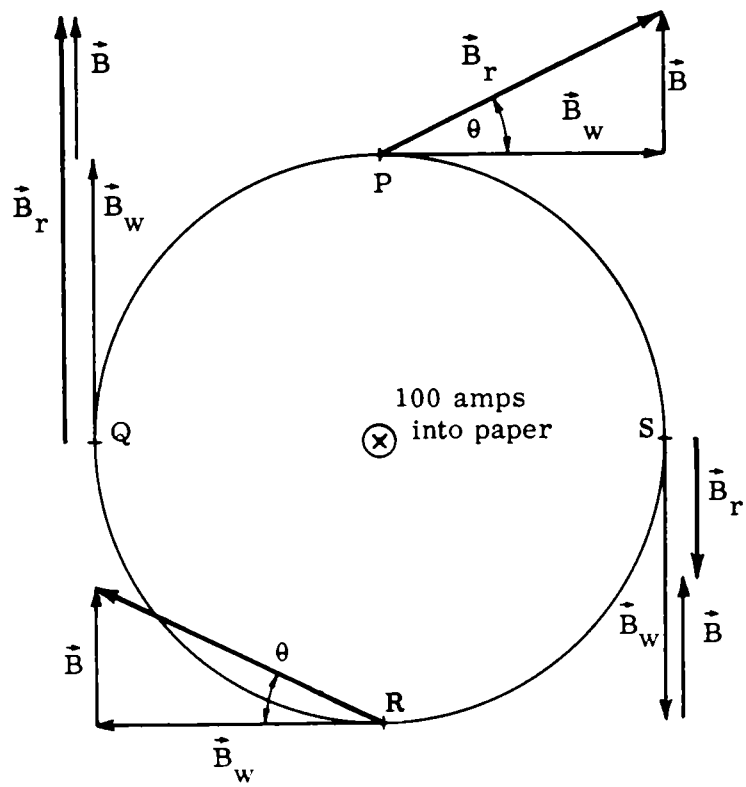
At Q, $B_r = B_w + B = (2 + 1) \times 10^{-3}$

$$= \underline{3 \times 10^{-3} \text{ newton/amp-meter.}}$$

At S, $B_r = B_w - B = (2 - 1) \times 10^{-3}$

$$= \underline{1 \times 10^{-3} \text{ newton/amp-meter.}}$$

The directions of B_r are as shown in the diagram. For the angle θ,



$$\tan\theta = \frac{B}{B_w} = \frac{1 \times 10^{-3}}{2 \times 10^{-3}} = 0.5$$
$$\underline{\theta = 26.5°.}$$

22–33

Suppose we have a beam of electrons moving at a speed $v = 3.00 \times 10^6$m/sec and carrying a current of 1.00 microampere.

(a) How many electrons per second pass a given point?

(b) How many electrons are in 1.00 meter of the beam?

(c) What magnetic field does the beam produce at 1.00 m distance?

(d) What is the total force on all of the electrons in 1.00 m of the beam if it passes through a field of 0.10 newton/amp-m?

(e) What is the force on a single electron if you assume that each electron experiences the same force?

(a) 1 microampere $= 10^{-6}$ ampere. This is 10^{-6} coulomb passing a given point every second. One coulomb is equivalent to 6.25×10^{18} elementary charges, and since a microampere is a microcoulomb per second,

$$1.00 \text{ microamp} = \frac{1.00 \text{ microcoulomb}}{\text{sec}}$$
$$= 1.00 \times 10^{-6} \text{ coul/sec} \times 6.25 \times 10^{18} \text{ elem.ch./coul}$$
$$= \underline{6.25 \times 10^{12} \text{ electrons/sec.}}$$

(b) Since the speed of the electrons is 3.00×10^6 m/sec, 3.00×10^6 meters of beam will pass a given point every second. From (a) we know that this length of beam contains 6.25×10^{12} electrons. If the electrons are distributed uniformly along 3.00×10^6 meters of beam, then in 1.00 meter there are

$$\frac{6.25 \times 10^{12} \text{ electrons}}{3.00 \times 10^6 \text{ m}} = \underline{2.08 \times 10^6 \text{ electrons.}}$$

(c) A current, I, in a straight wire produces a magnetic field $B = K(I/d)$ at a distance d from the wire. It is current that produces a field; a wire is simply a convenient container for charges in motion. Thus the electron beam constitutes a current and gives rise to a field:

$$B = (2 \times 10^{-7} \text{ nt/amp}^2) \times \left(\frac{1.00 \times 10^{-6} \text{ amp}}{1.00 \text{ m}}\right)$$
$$= \underline{2.00 \times 10^{-13} \text{ nt/amp-m.}}$$

(d) A length l of current I will experience a force $F = BIl$ in a field of strength B. For one meter of the beam in a field of 0.10 newton/ampere-meter, the total force is

$$F = 0.10 \text{ nt/amp-m} \times 1.00 \times 10^{-6} \text{ amp} \times 1.00 \text{ m}$$
$$= \underline{1.0 \times 10^{-7} \text{ newton.}}$$

(e) From (b), one meter of beam contains 2.08×10^6 electrons. If the force is uniform for each electron, then the total force, 1.0×10^{-7} newton, divided by the number of electrons, 2.08×10^6, will give the force on each electron:

$$F = \frac{1.0 \times 10^{-7} \text{ nt}}{2.08 \times 10^6 \text{ electrons}} = \underline{4.8 \times 10^{-14} \text{ newton.}}$$

This result follows from the same argument used in Section 6; we could have used the end formula for F:

$$F = qvb = (1.6 \times 10^{-19} \text{ coul})(3.00 \times 10^6 \text{ m/sec}) \times (0.10 \text{ nt/amp-m})$$
$$= \underline{4.8 \times 10^{-14} \text{ newton.}}$$

22–34

Point A is 2 cm from each of 3 currents I_1, I_2, and I_3, as shown in Fig. 22–39. All three currents are carried by long wires perpendicular to the paper.

(a) If $I_1 = I_2 = I_3 = 10$ amperes, and all three currents are directed out of the paper, what is the magnitude and direction of the magnetic field at A due to the currents in the wires?

(b) If $I_1 = I_2 = I_3 = 10$ amperes, but I_2 is into the paper while I_1 and I_3 are out, what is the magnitude and direction of the magnetic field at A due to the currents in the wires?

(a) Since A is equidistant from I_2 and I_3 on the line joining them and the two currents are parallel and both out from the paper, their contributions to the field at A cancel exactly. Thus, the net field at A results only from the current I_1:

$$B = B_1 = K(I/d) = (2 \times 10^{-7} \text{ nt/amp}^2) \times \left(\frac{10 \text{ amp}}{2 \times 10^{-2} \text{ m}}\right) = \underline{1 \times 10^{-4} \text{ nt/amp-m.}}$$

The direction of the field is horizontal, *from left to right.*

(b) The fields $\vec{\mathbf{B}}_2$ and $\vec{\mathbf{B}}_3$ at A are now in the same direction, down. Since

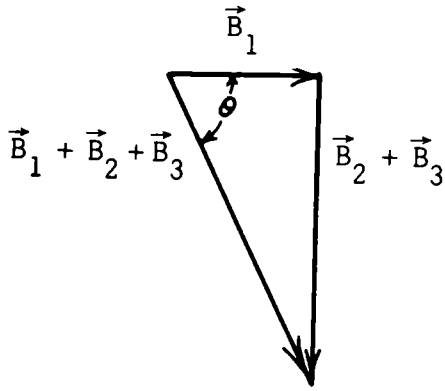
$$B_1 = B_2 = B_3 = 1 \times 10^{-4} \text{ nt/amp-m,}$$

the resultant field strength is

$$B = \sqrt{B_1^2 + (B_2 + B_3)^2}$$
$$= \underline{2.2 \times 10^{-4} \text{ nt/amp-m.}}$$

The resultant field makes an angle θ with B_1 given by

$$\tan \theta = 2$$
$$\theta = \underline{63°.}$$



CHAPTER 23

23–1*
Where will the charges shown in Fig. 23–1 be located if the wire is moved in the opposite direction? (Section 1.)

The positive charges will be at Q, the negative charges at P.

23–2*
(a) In which direction is the force (due to motion through the magnetic field) on charges in the side arms of the loop of Fig. 23–2?
(b) Does this force help push the charges around the loop? (Section 1.)

(a) The force on positive charges is directed outward from the page; the force on negative charges is directed into the page.

(b) Yes.

23–3
(a) Analyze the situation of Fig. 23–5 entirely from the point of view of the forces on charges in the wire as it moves through the magnetic field, and show why there is no induced current.
(b) Continue this line of argument to show why there is induced current as one side of the loop gets into the weaker field at the edge of the magnet.

(a) Since $\vec{\mathbf{v}}$ is to the left and $\vec{\mathbf{B}}$ points down, the force on all positive charges in the wire is toward the front. Since the entire wire is in a region of uniform magnetic field, the magnitude of the force is the same everywhere. Positive charges between P and Q are pushed toward P, but positive charges between R and S are pushed toward S with the same force, and there is no net current. The charges between Q and R and between S and P are pushed perpendicular to the wire and cause no current.
(b) When RS reaches a region of weaker field, the force on positive charges within it is still toward S but is not as great as the force pushing positive charges from Q toward P. Hence, positive charges can move from Q to P to S to R, creating a current which is clockwise from our point of view.

23–4*
In Fig. 23–5, would you expect induced current if the magnet were moved while the loop was held stationary? (Section 3.)

There would be no current as long as the entire loop was in the region of uniform field.

23–5*
In Fig. 23–12, as the loop of wire rotates, at what orientation (a) is the maximum current induced, (b) does the induced current reverse direction? (Section 3.)

(a) When the plane of the loop is parallel to the magnetic field.
(b) When the plane of the loop is perpendicular to the magnetic field.

23–6*
Why can't you induce a current in a loop of wire by moving it parallel to itself in a uniform field? (Section 3.)

The flux through the loop will not change.

23–7*
A loop of wire with an area of 0.5 m^2 is in a uniform field of 2×10^{-2} newton/amp-m. What is the flux through the loop if the plane of the loop is perpendicular to the field? (Section 3.)

$\phi_B = 1 \times 10^{-2}$ newton-m/amp.

23–8
What would be the magnetic flux inside a long solenoid 4.0 cm in diameter with 10 turns per cm of

length when the current is 1.0 amp?

From Section 22–11, the magnetic field B inside a solenoid carrying a current I, where N is the number of windings per unit length, is $B = 2\pi KNI$. The cross-sectional area of a solenoid of diameter d is $\pi d^2/4$. Hence the flux is

$$\phi_B = BA = (2\pi KNI)(\pi d^2/4) = \pi^2 KNI d^2/2.$$

For the given solenoid,

$$\begin{aligned}\phi_B &= (3.14)^2(2 \times 10^{-7}\ \text{nt/amp}^2) \\ &\quad \times (1.0 \times 10^3\ \text{turns/meter})(1.0\ \text{amp}) \\ &\quad \times (4.0 \times 10^{-2}\ \text{m})^2/2 \\ &= \underline{1.6 \times 10^{-6}\ \text{nt-m/amp.}}\end{aligned}$$

23–9*

If you were pulling the loop in Fig. 23–2 at a fixed speed, would you find it easier to pull it if its electrical resistance were increased? (Section 4.)

Yes. The force you must exert on the loop is proportional to the current, which would be reduced if the electrical resistance of the loop were increased.

23–10*

Does it take more work to pull a coil out of a magnetic field at high speed or at low speed? (Section 4.)

At high speed, since the pulling force would have to be greater.

23–11*

What EMF is induced in a 5-cm-long wire moving with a speed of 10 cm/sec across a field of 0.02 newton/amp-m? (Section 4.)

$\mathcal{E} = 1 \times 10^{-4}$ volt.

23–12*

What EMF is induced in a loop of wire if the flux through the loop changes at a rate of 0.5 newton-m/amp-sec? (Section 4.)

$\mathcal{E} = 0.5$ volt.

23–13

Suppose that in Fig. 23–2 the magnetic field strength is 0.5 newton/amp-m, l is 10 cm, the resistance of the loop is 0.5 ohm, and the loop is moved at a constant speed of 2 m/sec.

(a) What is the EMF generated in the wire?
(b) What is the current in the circuit?
(c) What is the magnetic force acting on the wire?

(a) The EMF is

$$\begin{aligned}\mathcal{E} &= vB_\perp l \\ &= (2\text{m/sec})(0.5\ \text{nt/amp-m})(0.1\ \text{m}) \\ &= 0.1\ \text{joule/amp-sec} = \underline{0.1\ \text{volt.}}\end{aligned}$$

(b) $I = \mathcal{E}/R = 0.1\ \text{volt}/0.05\ \text{ohm} = \underline{2\ \text{amps.}}$

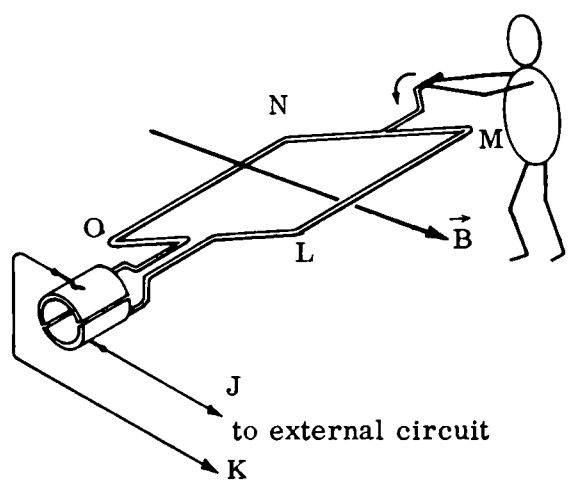
(c) $$\begin{aligned}F_{\text{mag}} &= B_\perp I l \\ &= (0.5\ \text{newton/amp-m})(2\ \text{amp})(0.1\ \text{m}) \\ &= \underline{0.1\ \text{newton.}}\end{aligned}$$

23–14

Figure 22–17 is a diagram of an electric motor.

(a) Explain how you could use the same arrangement of magnetic field and wire loop to make a generator that would produce current in only one direction in an external circuit.
(b) How should the wire loop be oriented in the magnetic field when the commutator interchanges the connections between the ends of the loop and the wires of the outside circuit?

(a) Some students may give a short answer which is correct but which they have not checked properly. There should be three steps in a complete answer:
  (1) Arrange to rotate the coil with an external mechanical force. (The fact that an external mechanical force will be needed follows from Section 4, but probably can be "guessed" before.)
  (2) Put the external circuit in place of the battery.
  (3) Check the polarity to be sure that the current goes in only one direction. This check essentially incorporates the answer to part (b).



When the plane of the coil is parallel to the field, and the right-hand side is moving upward while the left-hand side is moving down, the flux through the coil is momentarily zero, but the flux is changing with time and there is an induced EMF. The direction of the induced current can be found by considering the force exerted by the magnetic field on a moving positive charge. A positive charge along LM is pushed from L to M, according to the right-hand rule, and a positive charge along NO is pushed from N to O. The current therefore flows from K, through the external circuit, and back to J.

As the coil continues turning, the flux through the coil keeps changing until the coil is perpendicular to the field. At this point, the flux reaches its extreme value, and further turning causes a reverse change in the flux. If the flux was increasing, it is now decreasing. (See Figs. 23–12, 23–13, and 23–14.) Hence, the current through the coil reverses, and is momentarily zero as it changes direction. From the point of view of the force on a positive charge, the velocity of the charge is parallel to the field, so there is no force when the coil is perpendicular to the field. Note that the commutator changes the connections between the external circuit and the coil just at this point. Hence the current through the external circuit remains in its previous direction. This line of argument can be extended for further orientations of the coil.

(b) The plane of the wire loop should be perpendicular to $\vec{\mathbf{B}}$ when the commutator interchanges connections because the induced EMF and current are then zero. See part (a) above.

23–15

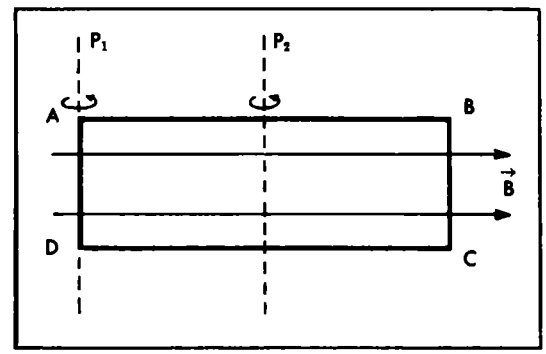
The rectangular wire loop shown in Fig. 23–29 is in a uniform magnetic field.

(a) In what direction (clockwise or counter-clockwise) is the induced current when the loop is in the plane of the paper and BC is moving into the page about P_1 as an axis? About P_2?

(b) For a given speed of rotation, how would the induced currents compare when the loop is rotated about P_1 and then about P_2?

(c) For a given speed of rotation, how does the induced EMF depend on the area of the loop?

(d) If the magnetic field has a magnitude of 1.5 newtons/amp-m, $AB = 10$ cm, and $BC = 4$ cm, what is the maximum induced EMF (in volts) when the loop is rotating about P_1 at 19.1 revolutions per second $\left(\frac{120}{2\pi}\text{rps}\right)$?



This is a straightforward problem about induced EMF, but it may prove difficult because it involves circular motion. It emphasizes the importance of flux. This problem is probably better suited to class discussion than a home assignment.

(a) The direction of current can be found by determining the direction of the force on a positive charge. Use the right-hand rule. Consider the length BC, and consider the rotation about P_1.

Point the thumb in the direction of $\vec{\mathbf{v}}$, into the page. Point the forefingers along the magnetic field. The open palm shows the direction of the force on a positive charge, from B to C. There is no force along AB or DC. AD is not moving. The induced current is clockwise in the loop.

For the rotation about P_2, the force in BC is in the same direction. There is also a force in AD. In this case, AD is moving out of the page. The thumb points out of the page. The fingers point toward the magnetic field. The palm is up; the force on a positive charge goes from D to A. This contributes to the clockwise induced current.

(b) The speed of rotation determines the rate at which the loop rotates, and hence the rate at which the component of the magnetic field perpendicular to the plane of the loop changes with time. Hence, when the loop is in the plane of the paper, the rate of change of flux must be the same for rotation about either

axis, since the area of the loop itself does not change. We conclude that the current is the same in both cases.

(c) The induced EMF is proportional to the rate of change of flux, which is proportional to the area of the loop. Hence, the induced EMF is itself proportional to the area of the loop.

(d) The maximum EMF is induced when the plane of the loop is parallel to the field, as in the figure, since at this orientation the flux is changing at the greatest rate. To calculate the maximum EMF, we recall from part (a) that the only side along which an EMF is induced is BC. The EMF along a wire of length l moving perpendicular to a field B at speed v is $\mathcal{E} = vBl$. In one complete revolution at frequency f, side BC covers a distance $2\pi \times$ (length AB) in time $T = 1/f$, so its speed is

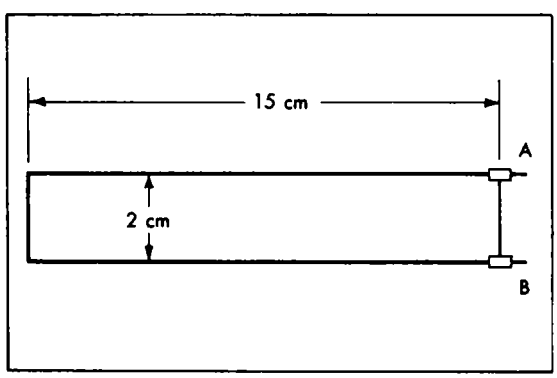
$$v = \frac{2\pi \times (\text{length } AB)}{T} = 2\pi f \times (\text{length } AB).$$

Hence, the EMF induced along BC when the loop is parallel to the field is

$$\begin{aligned}\mathcal{E} &= 2\pi f B \times (\text{length } AB) \times (\text{length } BC)\\ &= 2\pi\left(\frac{120}{2\pi}\text{ cycles/sec}\right)(1.5\text{ nt/amp-m})\\ &\qquad \times (0.10\text{ m})(0.04\text{ m})\\ &= 0.72\text{ nt-m/amp-sec} = 0.72\text{ joule/coul}\\ &= \underline{0.72\text{ volt}}.\end{aligned}$$

23–16

A U-shaped wire (Fig. 23–30) has a movable wire AB connected to it. This arrangement is in a uniform field perpendicular to, and into, the page.



(a) If the magnetic field strength is 40 newtons/amp-m, what is the induced EMF (in volts) when AB is in the position shown and moving at 20 cm/sec? Calculate this first from the rate of flux change and then from the magnetic force on the elementary charges in the wire.

(b) What is the induced EMF when AB is 5.0 cm from the left end and moving at 20 cm/sec? At 10 cm/sec?

(c) At what rate is energy fed into the loop when AB moves at 20 cm/sec and the induced current is 2.0 amperes?

(d) If the induced current is 2.0 amperes, what force is needed to move AB at 20 cm/sec?

(a) To calculate the induced EMF from the flux change we use

$$\mathcal{E} = \frac{\Delta\phi_{\mathcal{E}}}{\Delta t}.$$

(We ignore the minus sign in the equation because we are not told whether AB is moving to the right or left, so we shall be concerned only with magnitudes.) Now, the change in flux is due to the change in area $\Delta A = l\,v\,\Delta t$, where v is the speed of the movable wire AB and l is the length of AB. Hence,

$$\begin{aligned}\mathcal{E} &= \frac{\Delta\phi_{\mathcal{E}}}{\Delta t} = \frac{B\,\Delta A}{\Delta t} = \frac{Blv\,\Delta t}{\Delta t} = Bvl\\ &= (40\text{ nt/amp-m})(0.20\text{ m/sec})(0.02\text{ m})\\ &= \underline{0.16\text{ volt}}.\end{aligned}$$

Alternate Solution

Although Section 23–4 stresses the relation between flux and induced EMF, some students may prefer to work the problem from the point of view of forces. The force on an elementary charge q moving at a velocity $\vec{\mathbf{v}}$ perpendicular to a magnetic field $\vec{\mathbf{B}}$ is $F = qvB$. The EMF is the energy per charge needed to move a charge the entire distance between A and B. Since work is force times distance, the work done per charge when a charge is moved the distance l is

$$\mathcal{E} = (F/q)l = vBl.$$

This is identical to the expression we derived by considering the change in flux, so again $\mathcal{E} = \underline{0.16\text{ volt}}$.

(b) The induced EMF depends on the time rate of flux change. In this example, the flux change is due to a change in area. Because the rate of change of area depends on the speed of AB but not on the area itself, the

induced voltage is 0.16 volt is $v = 20$ cm/sec as in part (a), even though 15 cm has been changed to 5 cm and the area is different.

(c) By conservation of energy, the energy fed into the loop is the energy given to the charges which move through a potential difference given by the EMF. Since the EMF is the energy per charge, the total energy fed in is equal to the EMF times the total charge, and the *rate* at which energy is fed in is equal to the EMF times the rate of flow of charge, or the EMF times the current. In this case, using the value of $\mathcal{E}$ found in (a),

$$\mathcal{E}I = (0.16 \text{ volt})(2.0 \text{ amps})$$
$$= 0.32 \text{ joule/sec} = \underline{0.32 \text{ watt.}}$$

(d) The force needed is equal in magnitude and opposite in direction to the magnetic force on the wire. Hence, the magnitude of the force is

$$F_{\text{wire}} = lIB = (2 \times 10^{-2} \text{ m})(2.0 \text{ amps}) \times (40 \text{ nt/amp-m})$$
$$= \underline{1.6 \text{ newtons.}}$$

Alternatively, the rate at which energy is supplied, or the power, is equal to the applied force times the speed of the wire:

$$F_{\text{wire}}v = \text{power, or}$$
$$F_{\text{wire}} = \text{power}/v = 0.32 \text{ watt}/0.20 \text{ m/sec}$$
$$= 1.6 \text{ joules/m}$$
$$= \underline{1.6 \text{ newtons.}}$$

23–17

Suppose that the moving circuit of Problem 13 has a mass of 200 grams, and that the external force is suddenly removed when it is moving at 2 m/sec. What will its motion be like afterwards?

This problem can be discussed at several different levels, depending on the time available and the abilities, interest, and preparation of the class.

Most simply, as long as the wire is moving there is an EMF induced in it, and this EMF produces a current in the circuit. While there is a current in the wire there is a magnetic force on it, and the right-hand rule shows that this force is opposite in direction to the motion of the wire. When the external force is removed, the wire begins to slow down.

When the wire moves more slowly, the induced EMF decreases, and the magnetic force decreases in proportion with it. When the force decreases, so does the acceleration of the wire, so the wire loses speed at a decreasing rate.

23–18*

If we applied a constant force to the loop in Fig. 23–2, would the loop accelerate uniformly as long as the segment of length l was in the magnetic field? (Section 5.)

No; the magnetic force on the charges in the moving loop, which is proportional to the speed of the loop, would increase until it exactly cancelled the applied force, and the loop would then move with constant velocity.

23–19

A loop of wire *JKL* lies between the pole pieces of a magnet with the plane of the loop parallel to the pole pieces, as shown in Fig. 23–31(a). It is connected to a sensitive ammeter whose pointer stands at the center of the scale when there is no current, and which is so constructed that the pointer swings toward the terminal (*P* or *Q*) at which the current enters.

(a) Which way does the pointer swing when the loop is pulled out of the field of the magnet? When it is put back as it is in Fig. 23–31(a)?

(b) Which way does the pointer swing when the loop is put back as it is in Fig. 23–31(b)?

(c) How does the pointer move when the loop is turned over from the first position to the second without being taken out of the field of the magnet?

(a) If we apply the right-hand rule to the portion of the loop at *K*, we find that positive charges are moved toward us, and go around the loop in the sense of *LKJ*. The current enters the ammeter at *P*, and the pointer swings toward *P*.

When the loop is put back, the original action is reversed, the current flows away from us at *K*, and the pointer swings toward *Q*.

(b) If the loop is put back in the position of the second drawing, the current flows away from us at *K*; so it goes around the loop in the sense of *LKJ*. The pointer swings toward *P*.

(c) Turning the loop from its first position through 90°, so that its plane lies parallel to the magnetic field, brings it into such a position that there is no flux through it. This is equivalent to drawing it out of the field. The

pointer swings toward *P*. Turning it through another 90° in the same direction to bring it to the second position is equivalent to putting it back as we did in part (b). The pointer still swings toward *P*.

23–20

(a) Will the earth's magnetic field induce currents in an artificial satellite with a metal surface that is in orbit around the equator? Around the poles?

(b) If so, how would these currents affect the motion of the satellite?

This is a qualitative problem but is difficult because it requires students to consider the flux through a satellite in the magnetic field of the earth. Students find it hard to visualize currents in a spherical shell.

(a) An orbit around a circle of constant magnetic latitude provides a case in which the flux through the satellite does not change. The orbit about the equator is the only possible such orbit. (The center of the earth must lie in the plane of the orbit.) Thus there will be no induced currents in the metal of a satellite which is orbiting around an equatorial circle. (Strictly these statements apply only to the magnetic equator which does not coincide with the geographical equator.) In other orbits, since $\vec{\mathbf{B}}$ will be changing in magnitude and direction, the flux will be changing, and there will be induced currents.

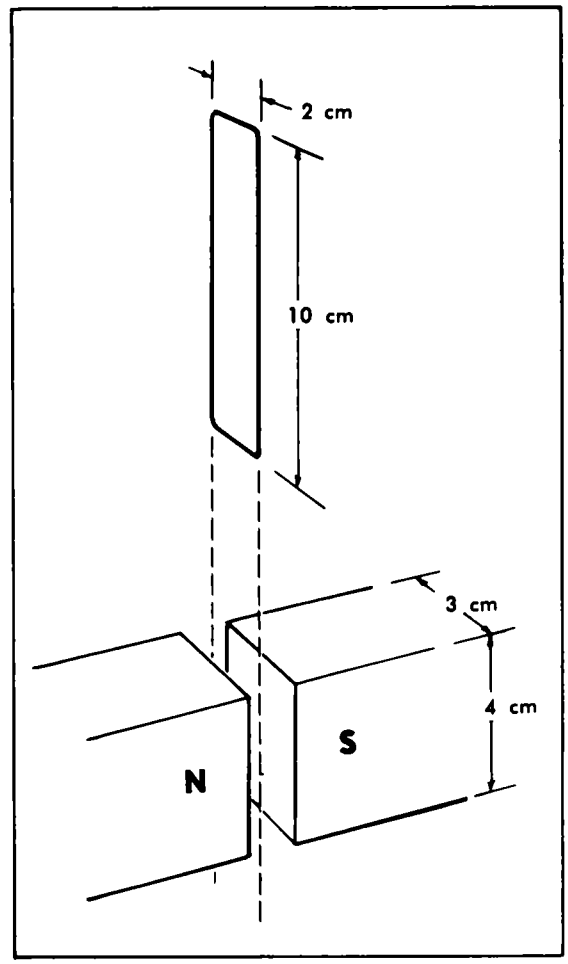
(b) The currents produce a magnetic field, and therefore the satellite will itself act like a small magnet. It will experience a small magnetic interaction with the earth so that the force is not purely gravitational. There will therefore be deflection from the gravitational path the satellite would take if it moved in the gravitational field alone. Part of the effect of the interaction must be to cause the satellite to lose energy dissipated in heat by the induced currents. Paradoxically, if it loses energy it will speed up. See Section 16–6. It is hard to estimate the magnitude of these forces.

23–21†

(See the Appendix in the text.)

23–22

A rectangular loop of copper wire is dropped between the poles of a magnet as shown in Fig. 23–33. Describe qualitatively the motion of the loop. Neglect air friction.



While the loop is above the magnetic field, it will fall freely with constant acceleration.

When the lower side of the loop enters the magnetic field, and as long as it stays in the field, the magnetic flux through the loop will be increasing. This changing flux in the loop induces an EMF which causes a current as shown in diagram (a). The magnetic force $\vec{\mathbf{F}}_m$ on the lower side of the loop decreases the acceleration of the loop.

After the bottom side of the loop leaves the field the loop will fall freely with the acceleration of gravity until the top side of the loop enters the field. During this time the flux in the loop is constant as can be seen in diagram (b).

As the top side of the loop enters the magnetic field the magnetic flux in the loop begins to decrease. This induces an EMF in the loop which

Figure (a)

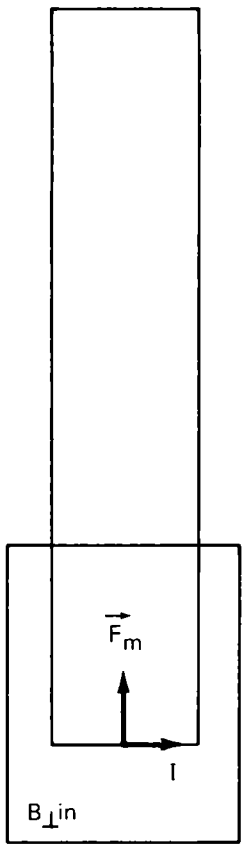


Figure (b)

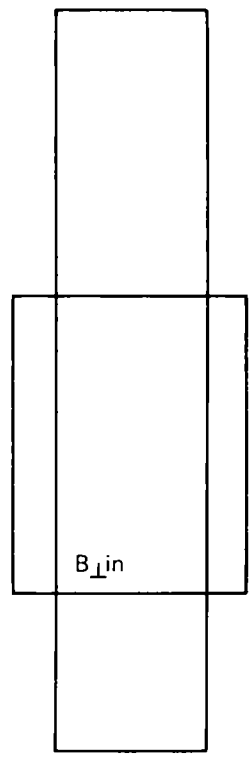
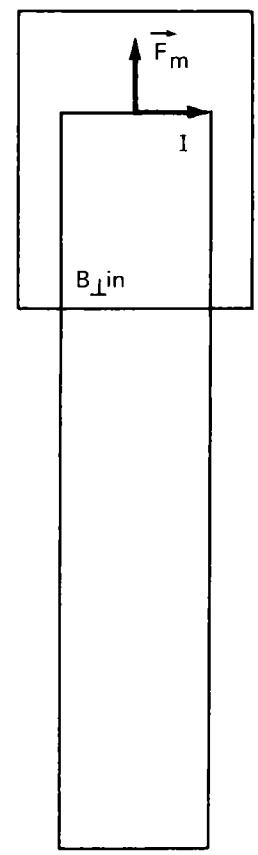


Figure (c)



causes a current as shown in diagram (c). The magnetic force, $\vec{F}_m$, on the current again opposes the motion and decreases the acceleration of the loop.

As soon as the loop is entirely out of the magnetic field, it falls freely once more with constant acceleration.

23–23*

Do the lines of an electric field induced by a change in magnetic flux have a beginning and an end? (Section 6.)

No; the lines are closed loops, since there are no free charges on which they can begin or end.

23–24

In a betatron electrons are accelerated to very high energies by a changing magnetic flux. The electrons move in a circle around the inside of an evacuated glass "doughtnut" and speed up when the flux through the hole in the doughnut increases. A separate magnetic field passes through the doughnut perpendicular to its plane to keep the electrons moving in a circle.

An electron is traveling in a circle of 50 cm radius at 98 percent of the speed of light in the doughnut of a betatron. If the flux inside the doughnut increases at a rate of 24 newton-meters/amp-sec for $\frac{1}{240}$ second, how much energy (in electron volts) will be transferred to the electron? (*Note:* Assume that the speed stays constant at approximately the speed of light.)

This problem involving induced EMF can be used to emphasize the electric field produced by a changing magnetic flux. It also contains some interesting facts about modern nuclear physics accelerators.

The energy gained by the electron will be equal to the work done on it. This work can be calculated from Fd, where F is the electric force and d is the total distance traveled. The electric force can be calculated from the induced EMF:

$$\mathcal{E} = \frac{\Delta\phi\mathcal{E}}{\Delta t} = \frac{24 \text{ nt-m}}{\text{amp-sec}} = 24 \text{ joule/coul} = 24 \text{ volts.}$$

The electric field, E, is found from the fact that this EMF corresponds to the effect of an electric field E acting through the distance of $2\pi r$. (See Section 23–6.)

$$\mathcal{E} = 2\pi rE$$

$$E = \mathcal{E}/2\pi r.$$

The distance the electron travels while it experiences this field is $d = vt$. The equivalent voltage is

$$V = Ed = (\mathcal{E}/2\pi r)(vt)$$

$$= \frac{(24 \text{ volts})(3.0 \times 10^8 \text{ m/sec})(1/240 \text{ sec})}{2(3.14)(0.50 \text{ meter})}$$

$$= 9.6 \times 10^6 \text{ volts.}$$

The electron has one elementary charge, so it gains an energy of 9.6×10^6 electron volts. In nuclear physics, one would use the abbreviation Mev for 10^6 electron volts; thus, the problem is about a 10-Mev betatron.

Alternate Solution

One can arrive at the same expression for the energy by realizing that in each revolution the electron is accelerated through a potential difference $\mathcal{E} = 24$ volts, so the total energy it gains must equal 24 electron volts multiplied by the total number of revolutions. In each revolution the electron goes a distance $2\pi r$ and altogether it goes a distance vt, so it must make $vt/2\pi r$ revolutions. Hence the energy gained is

$$\mathcal{E}(1 \text{ elem.ch.})vt/2\pi r.$$

23–25*

What determines the "equivalent current" between two parallel plates in an electric circuit? (Section 7.)

The rate at which charge is flowing onto either plate, which in turn determines the rate of change of electric flux through the region between the plates.

23–26*

Is it possible to maintain indefinitely (a) a constant magnetic circulation? (b) a constant electric circulation? (Section 7.)

(a) Yes; with a constant current.
(b) No; it is impossible to change the magnetic flux within a region at a constant rate indefinitely.

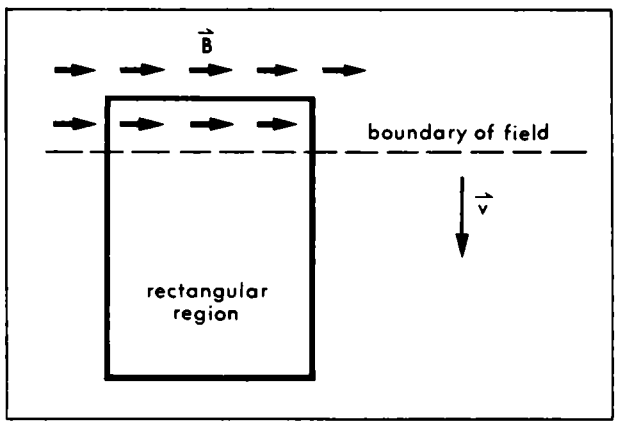
23–27*

In Fig. 23–22(c), assume that the plates started with zero charge and that the current shown has been flowing for a while. (a) What is the sign of the charges on each plate? (b) What is the direction of the electric field? (Section 7.)

(a) Negative charges on the top plate, positive charges on the bottom plate.
(b) The electric fields point upward.

23–28

Suppose that at the moment there is no electric field where you are, but a short time later there will be an electric field pointing vertically down. The boundary is moving straight toward you. When it has reached you, what will be the direction of the magnetic field at your position?



The following argument parallels the argument given in the text for the moving boundary of magnetic field. Consider a rectangular region of space which is perpendicular to the direction of the electric field and which is almost entirely in front of the boundary; one edge is just behind the boundary and parallel to it. Since the electric flux through this region is changing, there must be a magnetic circulation around the edge of the region. There is no field at all in front of the boundary. The only place there could be a magnetic field is along the edge just behind the boundary. Since the electric field points down and the area through which it passes is increasing, there is an equivalent current directed downward. Pointing our thumb down, we see that behind the boundary our fingers point to the right, so the magnetic field just behind the boundary is to our right as the boundary moves toward us.

We can now extend our region back a little further behind the boundary. The diagram shows what we would see if we looked down on the region. The electric field is down into the page. The magnetic field along the side edges is zero in front of the boundary and is perpendicular to the edges just inside the boundary. Hence, there is no contribution to the magnetic circulation from the side edges. The only place a magnetic field could contribute to the circulation is along the back edge. Again, it must point to the right. Extending this argument further, we see that everywhere behind the moving boundary there is a magnetic field to the right.

23–29*

What is the speed within a vacuum of (a) radio waves, (b) infrared waves, (c) gamma rays? (Section 9.)

(a) 3×10^8 m/sec;
(b) 3×10^8 m/sec;
(c) 3×10^8 m/sec.

23–30

When electrons moving at 3×10^7 m/sec hit the target in an X-ray tube, they are brought to rest in about an atomic diameter—in about 10^{-10} m.

(a) What is the stopping time?
(b) If you use one divided by this time to estimate the frequencies you might expect to see in the electromagnetic radiation arising from this deceleration, what frequency do you get?
(c) What wavelength does this frequency give?

This problem emphasizes the relation between acceleration and electromagnetic radiation. It also gives typical values of λ and frequency for low-energy X rays.

(a) To estimate the time, assume a constant deceleration. First, find v_{average}:

$$v_{\text{average}} = \frac{v_{\text{initial}+0}}{2} = \frac{3 \times 10^7 \text{ m/sec}}{2}$$
$$= 1.5 \times 10^7 \text{ m/sec.}$$
$$t_{\text{stop}} = \frac{d}{v_{\text{av}}} = \frac{10^{-10} \text{ m}}{1.5 \times 10^7 \text{ m/sec}}$$
$$= \underline{6.7 \times 10^{-18} \text{ sec.}}$$

(b) An estimate of the frequency, f, as $1/t_{\text{stop}}$ is:

$$f = \frac{1}{6.7 \times 10^{-18} \text{ sec}} = \underline{1.5 \times 10^{17} \text{ sec}^{-1}}.$$

(c) This frequency corresponds to a wavelength:

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8 \text{ m/sec}}{1.5 \times 10^{17} \text{ sec}}$$
$$= \underline{2.0 \times 10^{-9} \text{ meter.}}$$

Note: In the calculation above, the frequency that we estimate depends upon 1/time of stopping. Other effects may enter and determine the highest frequency seen. The highest frequency depends, according to quantum theory, on the energy of individual electrons. If that energy is low, no light of high frequency will be seen even if the stopping time is very short.

23–31

What evidence is there that sound is not electromagnetic radiation?

This easy problem can be used to review the properties of electromagnetic waves. A discussion of all the reasons for believing that sound is not electromagnetic radiation could go on interminably. A few are listed below.

(1) Sound does not travel through empty space.
(2) Sound does not travel with the speed of electromagnetic radiation.
(3) Sound is not produced by accelerating charges. It can be produced by means which are clearly nonelectric.
(4) Electromagnetic radiation of the same wavelengths as sound is well identified (ham band to radar wavelengths) and is different from sound.
(5) Sound is often primarily a longitudinal wave which cannot be polarized (while polarization is not discussed in the text, it will still have meaning to most students).

CHAPTER 24

24–1

Suppose you have a large collection of small steel balls which you can fire at different targets in turn. You cannot see the targets directly, but you can observe the paths of the balls before and after striking. In each case below, tell what you know about the target from the behavior of the balls. The balls are all fired at the target in the same direction.

(a) All the balls bounce off the target with angle of reflection equal to angle of incidence.
(b) All the balls bounce back, but with no definite relation between angle of reflection and angle of incidence.
(c) Practically all the balls go through the target, but a few are lost in it.
(d) All the balls are lost in the target.
(e) The paths of all the balls cross after reflection at a point in front of the target.

This problem is intended as the basis for a class discussion to show how "scattering" of particles from an unseen target can be used to learn some of the properties of the target.

(a) A flat plate extending over the whole target area would give specular reflection of all "bullets," provided only that the bullets collide elastically with the plate (see Fig. 4–3.) We cannot determine exactly the composition of the plate, or how thick it is. Perhaps we could learn more by firing bullets of different speeds and sizes; very fast bullets may not reflect specularly, very small bullets may perhaps go through.

Another possibility is that there are several plates in the target, all parallel, but lying in different planes. This could work for a particular direction of incidence but not for all directions; if the direction of incidence is varied, some bullets would get through, either directly or after several reflections.

(b) A rough plate extending over the whole target area would give rise to diffuse reflection (see Fig. 4–4.) The plate must be made of a material that reflects the bullets elastically, and the irregularities must be both sufficiently random and of larger size than the bullets. (We can also think of more complicated arrangements to give diffuse reflection.)

(c) If most of the bullets go through without appreciable change in direction or speed, most of the target must be more or less empty. Since some bullets do not get through, a small fraction of the target area must be occupied by something that can stop, or destroy bullets. One or more chunks of something like mud or putty would do, provided the chunks are thick enough to stop our bullets. By counting the number of bullets that get through and the number of bullets that stop we can determine the total cross-sectional area of the "mud balls" but not their number or their shape. We could find the shapes of the mud balls if we were able to aim the bullets very carefully, something which cannot be done with atomic bullets fired at atomic targets. In our example the "mud balls" must somehow be tied down, or we should be able to see them recoil out of the target area (if they are visible).

(d) A wall sufficiently thick and soft to stop the bullets apparently extends over the whole target area. The experiment does not disclose whether the wall is made of mud, snow, molasses, or what have you. Nor do we learn anything about the size and shape of the wall.

(e) A curved "mirror," concave toward the direction from which the bullets arrive, could focus the bullets. In order to bring all particles in a beam to a focus, the mirror should

be parabolic. As in part (a), the mirror must be made of a material that can reflect the bullets elastically. It should perhaps be said that this is one experiment we could not carry out with atomic bullets fired on atomic targets; in such an experiment we can only observe the changes in direction of the particles.

24–2

To help visualize the thickness of the gold foil used in the alpha-particle deflection experiment described in Section 24–1, find the number of sheets of this foil which, when stacked together, are equal to one leaf of your textbook.

Two hundred pages, or 100 leaves, of the text are 0.8 cm thick. Hence, each leaf is 8×10^{-3} cm or 8×10^{-5} m thick. Since each sheet of gold foil is 1×10^{-7} m thick, we need a stack of 800 sheets of gold foil to equal the thickness of one leaf of the textbook.

24–3

One square meter of silver foil has a mass of 5×10^{-3} kg.

(a) What is its thickness? (The density of silver is 1×10^{4} kg/m^3.)

(b) The mass of a silver atom is 1.8×10^{-25} kg. What is the length of the side of a "box" representing a single atom?

(c) What is the thickness of the foil expressed in atomic "boxes"?

(a)

$$\text{mass} = \text{volume} \times \text{density}$$
$$= \text{area} \times \text{thickness} \times \text{density},$$

and

$$\text{thickness} = \frac{\text{mass}}{\text{area} \times \text{density}}$$
$$= \frac{5 \times 10^{-3}\ \text{kg}}{(1\ \text{m}^2)(1 \times 10^{4}\ \text{kg/m}^3)}$$
$$= \underline{5 \times 10^{-7}\ \text{meter.}}$$

(b) The volume of the "box" representing the atom equals the mass of the atom divided by the density of silver. Hence, the length of one side is the cube root of this volume:

$$\text{length of side} = \left(\frac{\text{mass}}{\text{density}}\right)^{1/3}$$
$$= \left(\frac{1.8 \times 10^{-25}\ \text{kg}}{1 \times 10^{4}\ \text{kg/m}^3}\right)^{1/3}$$
$$= 3 \times 10^{-10}\ \text{meter.}$$

(c) The thickness of the foil in units of atomic "boxes" is

$$\frac{5 \times 10^{-7}\ \text{m}}{3 \times 10^{-10}\ \text{m}} = 2 \times 10^{3}.$$

24–4*

What is the evidence for our belief that alpha particles which are deflected by a large angle when passing through a thin gold foil do so in one collision? (Section 1.)

The large number of particles that come through the foil undeflected, compared with the small number that are deflected, tells us that the chance for one collision is very small; hence, the chance that one alpha particle will have two collisions is much smaller.

24–5*

What are the main features an atomic model must have in order to account for the way in which alpha particles are deflected when they pass through gold foil? (Section 1.)

An atom must be mostly empty space, with something in it which can exert a strong force on an alpha particle.

24–6*

What is a scattering angle? (Section 2.)

It is the angle between the original and final directions of motion of a scattered particle.

24–7

Suppose that a ball is rolled many times at the hill model of Fig 24–6. Each time it starts along the same path, but with a different kinetic energy. How do you expect its scattering angle θ to be related to its energy?

When a ball is rolled with increased kinetic energy, it is rolled with increased speed; it spends less time on the slope of the hill, and hence gets a smaller impulse from the hill. Consequently its change of momentum is less. In the interaction with the hill, its total energy would be conserved, so that it would leave the interaction with its kinetic energy, and its speed, unchanged. The smaller change in momentum thus means that the direction of its path changed less. We conclude that the scattering angle (for the same initial path) will decrease as the kinetic energy of the ball increases.

24–8

In a scattering experiment using gold foil, 4 alpha particles in 10,000 are scattered through an angle of

more than 5°.

(a) How many particles would be scattered through more than 5° if we doubled the thickness of the foil?

(b) If we made the foil a thousand times thicker?

This problem shows that the probability of scattering through any given angle depends only on the effective cross-sectional area of the target. In this case, since the nuclei all have the same radius, the scattering probability must depend only on the total number of nuclei per unit of cross-sectional area, hence directly on the thickness of the target.

(a) If the target thickness is doubled, the number of nuclei per unit of cross-sectional area is doubled, hence twice as many alpha particles would be scattered into the same angular interval, or 8 in 10^4.

(b) This part of the problem serves to review the reasons why very thin foils must be used in the alpha-particle scattering experiments. With the original foil, 4 alpha particles out of 10^4 were deflected more than 5°. With such a small probability of deflection the chances are overwhelming that the deflections are due to single collisions with a scattering angle of more than 5°.

If foil a thousand times thicker were used, the alpha particles would lose their kinetic energy by collisions with the electrons in the foil and would come to a stop inside. If they did get through, there would be a high chance that the deflection was caused by multiple collisions and not by a single collision.

24–9

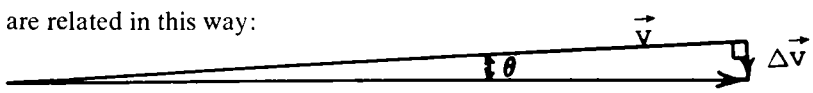
A beam of alpha particles from polonium is shot perpendicularly into the gold foil described in Section 24–1. We observe that some of the particles are scattered through an angle of about 3° or 5×10^{-2} radian. The alpha particles have a kinetic energy of 8.5×10^{-13} joule, which corresponds to a speed of 1.6×10^7 m/sec.

(a) What velocity $\Delta\vec{v}$ must be added at right angles to their original velocity to produce this change in direction?

(b) How long does it take an alpha particle to pass through a single gold atom in the foil?

(c) What is the average acceleration of one of these alpha particles if it gets its whole deflection in passing through a single gold atom?

(a) The velocities are related in this way:



Since θ is so small, its value in radians is just $\theta = \Delta v/v$, so

$$\Delta v = (1.6 \times 10 \text{ m/sec})(5 \times 10^{-2}) = \underline{8 \times 10^5 \text{ m/sec.}}$$

(b) From Section 24–1, we have a diameter of 2.5×10^{-10} meters for each gold atom. Then

$$\Delta t = \frac{2.5 \times 10^{-10} \text{ m}}{1.6 \times 10^7 \text{ m/sec}} = \underline{1.6 \times 10^{-17} \text{ sec.}}$$

(c)
$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{8 \times 10^5 \text{ m/sec}}{1.6 \times 10^{-17} \text{ sec}} = 5 \times 10^{22} \text{ m/sec}^2,$$

at right angles to the original velocity.

An acceleration of this magnitude requires a force extraordinarily large compared to forces we can exert on an alpha particle in the laboratory. In spite of the approximations made in this problem, we see that even to be scattered through small angles, the alpha particle must approach the charged nucleus very closely.

24–10*

Why is it not possible to find directly the relation between θ and b from the Rutherford scattering experiment with alpha particles? (Section 3.)

We cannot measure b directly because we do not know where each nucleus is, nor do we know the exact trajectories of the incident alpha particles.

24–11*

How is N_0, the number of particles which have scattered through all angles larger than θ, related to the aiming error b? (Section 3.)

N_θ is proportional to the square of the aiming error.

24–12*

What do the results of scattering experiments tell us about the electric field inside the gold atom? (Section 3.)

It is a Coulomb field; that is, it varies as $1/r^2$.

24–13*

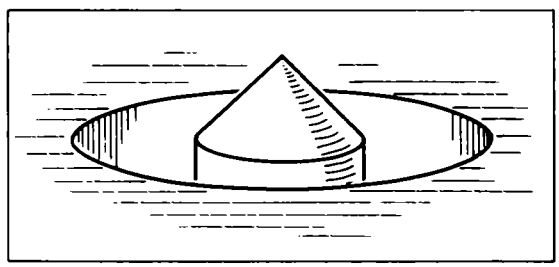
If the alpha particles approaching a nucleus with a certain aiming error b were scattered by 90°, through what range of angles were the particles scattered

which approached the nucleus with a smaller aiming error? (Section 3.)

Through angles between 90° and 180°.

24–14

Imagine a device which can drop 1000 steel balls through a circular hole 5 cm in radius and spread them evenly over the area of the circle (Fig. 24–17). In the center of the hole we fix a steel cone whose sides make an angle of 45° with its base, and whose base has a radius of 2 cm. In a group of 1000 balls, how many are scattered sideways before they reach the hole?

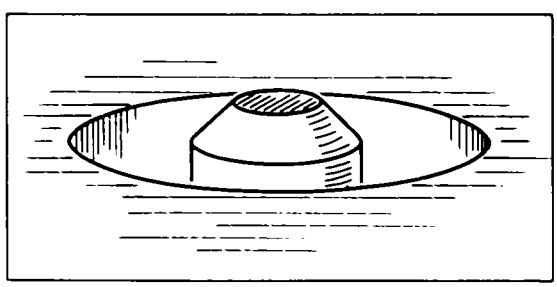


As seen from above, the cone covers a circle whose radius is 2 cm. The steel balls are spread evenly over the whole area, and since the area of a circle is proportional to the square of its radius, the number of balls which strike the cone and are scattered before they reach the hole is

$$\frac{(2\text{ cm})^2}{(5\text{ cm})^2}(1000) = \underline{160.}$$

24–15

The point of the cone in Problem 14 is sawed off to leave a smooth plane parallel to the base and 1 cm in radius (Fig. 24–18).



(a) In each group of 1000 balls, how many are turned back and how many are scattered sideways by it?

(b) Draw a graph of the scattering angle θ versus the aiming error b, measured from the center of the hole.

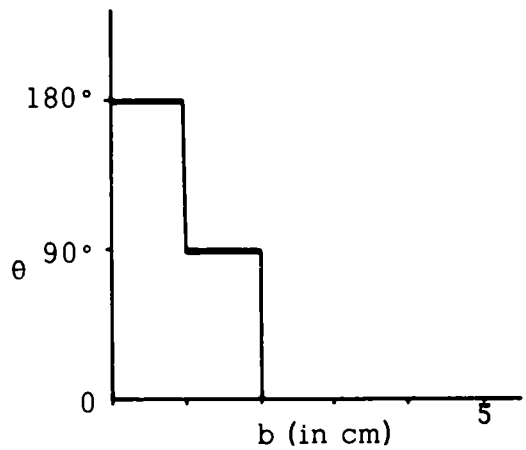
(c) Draw a graph of N_θ (the number of balls scattered through an angle greater than θ) versus θ.

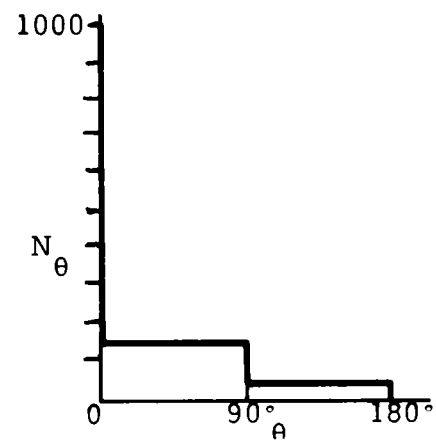
(a) The balls which strike the central plane of the cone are turned back, and those which strike the sloping sides are scattered sideways. As seen from above, these cover a circle of 1-cm radius and a ring beyond that whose outer radius is 2 cm. Then,
No. of balls turned back

$$= \frac{(1\text{ cm})^2}{(5\text{ cm})^2} \times 1000 = \underline{40.}$$

No. of balls turned sideways

$$= \frac{(2\text{ cm})^2 - (1\text{ cm})^2}{(5\text{ cm})^2} \times 1000 = \underline{120.}$$



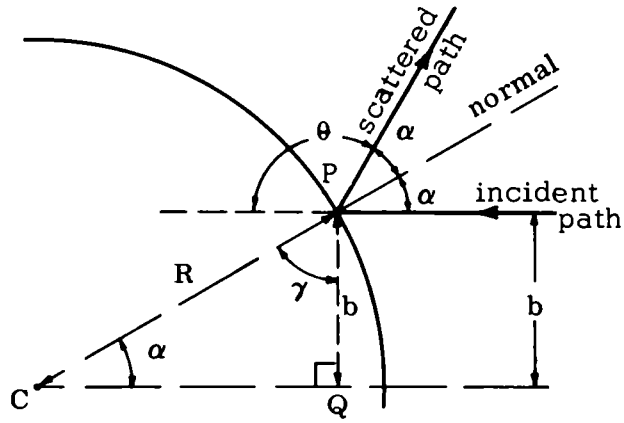


24–16

(a) To construct the curve of b vs. θ for an abrupt force, draw a fairly large circle to represent a plane cut through a rigid sphere and measure the scattering angles for various aiming errors. (Assume that the angle of reflection of a ball hitting the surface equals the angle of incidence. Why?)

(b) From your curve of b vs. θ construct a graph of N_θ vs. θ and compare it with Fig. 24–14.

(c) Explain why this relationship is independent of the energy of the particles fired at the sphere.



(a) As the particle reaches the surface of the sphere it experiences a sudden change in momentum. We assume two things:

(1) The collision is elastic because the force is not only abrupt (i.e., acts almost instantaneously), but also is the same when the particle comes in and when it goes out;

(2) The mass of the particle is very small compared to the mass of the target sphere.

Assumption 2 implies that the kinetic energy transferred to the target sphere in the collision is negligible. Assumption 1 then requires that the kinetic energy of the particle before and after collision be the same (nearly). Thus the *magnitude* of the momentum of the particle is the same before and after collision: $p_1 = p_2$.

Assumption 1 also requires that the force on the particle depend only on its position relative to the sphere. In particular, the *direction* of the force must depend only on the position, not on its velocity (no friction). By symmetry the force must be normal to the surface of the target sphere. It follows that there is no change in the tangential component of momentum of the particle during the collision: $\vec{p}_{t1} = \vec{p}_{t2}$.

We now draw a triangle to represent the momentum of the particle, and its normal and tangential components, before collision. We also draw the corresponding triangle after collision.

Since $p_1 = p_2$ and $p_{t1} = p_{t2}$, the two right triangles are equal (one is the mirror image of the other). Therefore $\alpha = \beta$.

We now draw a cross section through the sphere on the plane defined by the center of the sphere and the straight-line path of the incident particle:

C = center of sphere

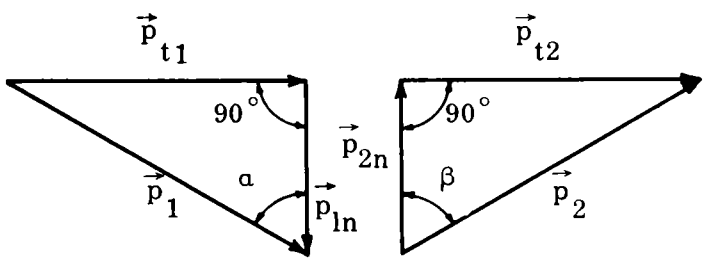
b = aiming error

θ = angle of scattering = $180° - 2\alpha$

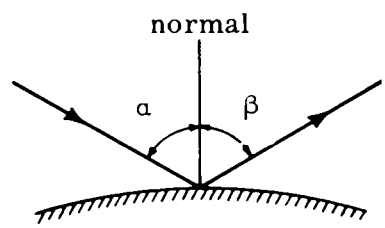
$\theta/2 = 90° - \alpha = \gamma$.

We can draw a number of paths for different values of the angle α and measure the corresponding aiming error. Alternatively we can look up b/R in a table of cosines. From the triangle CPQ we see that:

$$b/R = \cos\gamma = \cos(\theta/2).$$

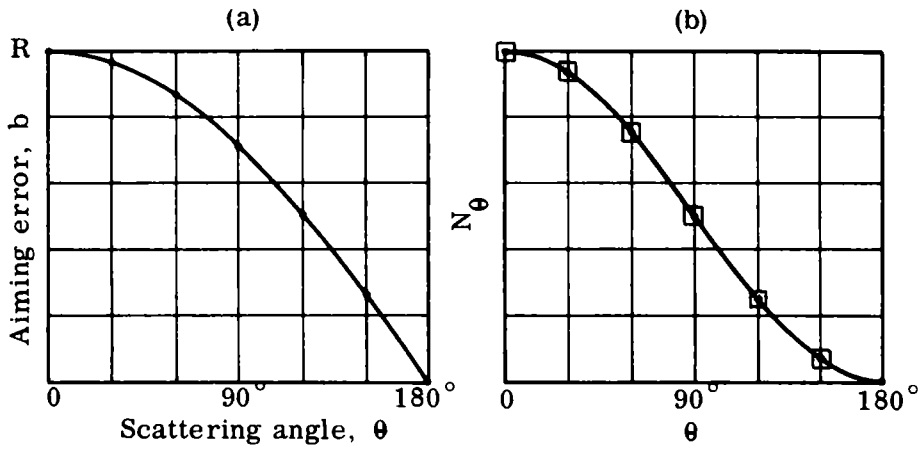


Before collision | After collision



Path of particle

θ	(a) $\frac{b}{R}$	(b) $(b/R)^2$
0	1.000	1.000
30°	0.966	0.933
60°	0.866	0.750
90°	0.707	0.500
120°	0.500	0.250
150°	0.259	0.067
180°	0.000	0.000



By either method we find the results shown in column (a) of the table above and plotted on the graph (a).

(b) With n targets per unit area in the beam, the probability of scattering through an angle greater than θ is given by $n\pi b^2$. The number of scatters, N_θ, through an angle greater than θ is therefore proportional to b^2 and hence to $(b/R)^2$. The calculated values of $(b/R)^2$ are shown in the preceding table, column (b), and were used to draw the graph (b) of N_θ vs. θ. Points read from Fig. 24–14 are shown on the same graph as well as in the table. They agree when multiplied by some constant so as to make them agree at one point, say $\theta = 0$.

(c) The result (a) was obtained without mention of the energy. The scattering is independent of energy as long as the assumptions are valid. The original result is a very good approximation as long as the assumptions are valid.

24–17

From the graph in Fig. 24–13, determine the ratio of the number of particles scattered through an angle greater than 15° to those scattered through an angle greater than 22.5°.

Because the graph is small, only an approximate answer is possible. The ratio of

$$\frac{N_{15°}}{N_{22.5°}} \approx \frac{1.2}{0.6} \approx 2.$$

24–18*

How many times as close can an alpha particle of given energy get to the center of the nucleus of an aluminum atom as it can to the center of a gold nucleus? (Section 4.)

About 6 times as close.

24–19*

How close can polonium alpha particles get to the center of an aluminum nucleus? (Section 4.)

$r_0 = 7 \times 10^{-15}$ meter.

24–20*

Which conservation law is used in calculating the distance of closest approach? (Section 4.)

The law of conservation of energy.

24–21

In nuclear fission, the nucleus of an atom of uranium 238 sometimes breaks into just two fragments, owing to the Coulomb force of repulsion between its protons. Imagine that the split has just happened, and the two fragments are still touching each other (roughly 10^{-14} m apart, center-to-center) and at rest. Find their total kinetic energy when they are very far apart, if one has $Z_1 = 42$ elementary charges and the other has $Z_2 = 50$ elementary charges. Give your answer in joules and in millions of electron volts (Mev).

This problem and the following call for the application of conservation of energy in cases involving electrical potential energy. This problem presents a simplified picture of nuclear fission which is

useful only for an order-of-magnitude calculation.

Since the only forces acting on the two fragments are the electrical forces of repulsion exerted by each fragment on the other, the electrical potential energy of interaction of the two fragments plus the sum of their energies remains constant. As the fragments fly apart, the potential energy decreases with a corresponding increase in the sum of their kinetic energies. The initial potential energy is

$$U_{coul} = \frac{kZ_1Z_2}{r}$$

$$= \frac{2.3 \times 10^{-28}\ \text{nt-m}^2/(\text{elem.ch.})^2 \times 42\ \text{elem.chgs.} \times 50\ \text{elem.chgs.}}{10^{-14}\ \text{m}}$$

$$\approx 5 \times 10^{-11}\ \text{joule} = 300\ \text{Mev.}$$

This will all be converted into kinetic energy of the fragments if we take the simple model proposed. Actually, some of it goes into excitation of the fragments and a very small fraction into extracting a few free neutrons, which also are liberated in most fission reactions.

24–22

(a) Two protons are released from rest 10^{-10} m apart. What will their kinetic energy be at large separation?

(b) Suppose one of the protons is nailed down. What will the kinetic energy be at large separation?

(a) The initial potential energy of the protons due to the Coulomb force is

$$k\frac{q_1q_2}{r} = 2.3 \times 10^{-28} \times \frac{(\text{newton-meter})^2(1\ \text{elem.ch.})^2}{(\text{elem.ch.})^2(10^{-10}\ \text{meter})}$$

$$= 2.3 \times 10^{-18}\ \text{joule},$$

where the zero of potential energy has been taken at large (infinite) separation. The final potential energy is zero. Since the initial kinetic energy is zero, the final kinetic energy is 2.3×10^{-18} joule $= 14.4$ ev.

Each proton has one half of the total kinetic energy (by conservation of momentum) so that the kinetic energy of each proton is 1.2×10^{-18} joule.

(b) Because no energy can be stored or dissipated, the movable proton must carry all the kinetic energy, 2.3×10^{-18} joule. The fixed proton does not move, and has zero kinetic energy.

24–23

Three experiments were done with the same alpha particle beam, the same counters at the same positions, and different foil targets. Five counters were arranged with respect to each target and the beam to count particles scattered by the angles given in the table. The time interval of counting was the same for all three experiments. The foils were designed so that the beam passed through the same number of atoms per square centimeter regardless of which target was used. The results presented in the table below give the numbers of particles scattered into the same range of scattering angles. For example, the count at 30° includes all particles scattered in any direction in the range between 29° and 31°; the count at 60°, all those scattered in any direction in the range between 59° and 61°, etc.

SCATTERING ANGLE	NUMBER COUNTED		
	Target 1	Target 2	Target 3
30°	2790	35,920	102,810
60°	346	4451	12,760
90°	100	1288	3685
120°	39	496	1423
150°	14	187	532

(a) Plot the number scattered versus the scattering angle for each target. (Plot all three graphs on the same axes.)

(b) Target 1 was aluminum. What were the other targets?

This problem shows the connection between the angular distribution of scattered particles and the atomic number of the scattering nucleus. Note that the experimental conditions presented in the problem (identical counters, identical geometry, identical numbers of atoms in the beam path, etc.) ensure that the observed differences can result only from differences in the nuclei in the target material.

(a) A plot of the data (which we omit) shows, qualitatively, the same angular dependence for each target. For a quantitative evaluation it is simpler to compare the counting rates numerically.

(b) The probability of scattering into a given range of angles should be proportional to the square of the nuclear charge (see Section 24–4). We calculate the ratio of the number of counts for targets 2 and 3 to the number of counts for target 1. The results are:

θ	Target 2/Target 1	Target 3/Target 1
30°	12.9	36.8
60°	12.9	36.9
90°	12.9	36.8
120°	12.7	36.5
150°	13.4	38.0
Average:	13.0	37.0

We note that the ratios are the same at all angles, as we would expect from theory. Since the ratios of counts should be equal to the square of the ratio of nuclear charges, we have:

$$\left(\frac{q_2}{q_1}\right)^2 = 13.0, \quad \frac{q_2}{q_1} = 3.60;$$

$$\left(\frac{q_3}{q_1}\right)^2 = 37.0, \quad \frac{q_3}{q_1} = 6.08.$$

Assuming the nuclear charge of aluminum to be 13 elementary charges, we have

$$q_2 = q_1(3.60) = (13)(3.60) = 46.8 \text{ elem. ch.}$$
$$q_3 = q_1(6.08) = (13)(6.08) = 79.0 \text{ elem. ch.}$$

Target 2 was probably silver ($Z = 47$) and target 3 was probably gold ($Z = 79$). The consistency of the data given is not quite good enough to guarantee that we are right, but we are certainly within one unit of atomic number. In fact, the data given are the results of experiments using aluminum, silver, and gold.

24–24

(a) Explain why the gravitational force between an alpha particle and a gold nucleus has little effect on the scattering of alpha particles by gold.

(b) What is the ratio of the electric force on an alpha particle to the gravitational force when the separation of an alpha particle and a gold nucleus is 10^{-14} m? When it is 10^{-13} m?

This problem illustrates the enormous magnitude of electrical forces relative to gravitational forces.

(a) The gravitational force between an alpha particle and a nucleus is much smaller than the coulomb force at any separation distance. For a quantitative comparison, see part (b).

(b) The electrical force between two charged objects separated by a distance r, from Coulomb's law, is

$$F_e = \frac{kq_1q_2}{r^2},$$

while the gravitational force, from Newton's law (Chapter 13), is

$$F_g = \frac{Gm_1m_2}{r^2}.$$

The ratio F_e/F_g at *any* given distance is then

$$\frac{F_e}{F_g} = \frac{kq_1q_2}{Gm_1m_2}.$$

Substituting the correct values for gold and helium, we get

$$\frac{F_e}{F_g} = [(2.3 \times 10^{-28} \text{ nt-m}^2/(\text{elem.ch.})^2) \times (79 \text{ elem.ch.})(2 \text{ elem.ch.})]/[(6.7 \times 10^{-11} \text{ nt-m}^2/\text{kg}^2) \times (3.3 \times 10^{-25} \text{ kg})(6.6 \times 10^{-2} \text{ kg})] = \underline{2.5 \times 10^{35}} \text{ for any distance.}$$

24–25

An alpha particle of kinetic energy 1.0×10^{-12} joule is scattered by a gold nucleus.

(a) What is the minimum possible distance of closest approach?

(b) What must be the aiming error if the alpha particle is to get this close?

(c) How close could a proton of the same energy approach? A neutron?

In solving this problem, we follow the argument used in the text in Section 24–4.

(a) and (b) The potential energy at the minimum separation distance, r_o, is

$$U = \frac{kq_1q_2}{r_o}.$$

If the collision is head-on (the aiming error is zero), the alpha particles will come to rest at a minimum distance r_o such that $U(r_o)$ is equal to the initial kinetic energy of the system. Solving,

$$r_o = \frac{kq_1q_2}{E_K} = \frac{(2.3 \times 10^{-28} \text{ nt-m}^2/(\text{elem.ch.})^2) \times (79 \text{ elem.ch.})(2 \text{ elem.ch.})}{1.0 \times 10^{-12} \text{ joule}} = \underline{3.6 \times 10^{-14} \text{ meter.}}$$

(c) For the proton, $q_2 = 1$ elem. ch.; hence r_o is half as large as in (a), or $\underline{1.8 \times 10^{-14} \text{ m.}}$

Since the neutron has no charge, it is not repelled by the Coulomb barrier and can get as close as it pleases to the nucleus for any energy. Actually, there are important short-range nuclear forces which come into play for distances of the order of the nuclear radius. These forces also act on charged particles in the nucleus and those fired at the

nucleus with enough energy to bring them through the Coulomb field into the region of the nucleus.

24–26

In one scattering experiment a proton beam strikes a gold target. In a second experiment a beam of deuterons (atomic mass = 2, atomic number = 1) is used. The kinetic energy of the particles is the same in both beams. How will the minimum distance of closest approach to gold nuclei of the two kinds of beams compare?

This simple problem illustrates that the minimum distance of closest approach (for Coulomb scattering) depends only on the kinetic energy and *charge* of the bombarding particles and scattering nucleus. The minimum distance of closest approach, r_o, is determined by the point at which the bombarding particle is brought to rest in a head-on collision. At that point the particle's potential energy is equal to its initial kinetic energy,

$$\frac{kq_1q_2}{r_o} = \text{original } E_K.$$

Since the two particles, proton and deuteron, have the same charge and kinetic energy, the minimum distance of closest approach must be the same in both cases.

As an extension of this problem, students might be asked to discuss the case in which the two particles have the same speed, rather than the same kinetic energy. In this case, the kinetic energy, $\frac{1}{2}mv^2$, of the deuteron (mass twice that of the proton) is twice the proton kinetic energy, and the minimum distance of closest approach for the deuteron will be half that for the proton.

24–27

(a) What fraction of its energy is lost by an alpha particle in a head-on collision with an electron? (The mass of an alpha particle is about 7200 times that of an electron.)

(b) Approximately how many such collisions must occur before the energy of the alpha particle is reduced by 1 percent?

(a) We can find the speed, v'_e, of the electron and the speed, v'_α, of the alpha particle after the collision by applying conservation of energy and momentum. The formulas are given in Section 15–7; they are:

$$v'_e = \frac{2\,m_\alpha}{m_\alpha + m_e} v_\alpha;\; v'_\alpha = \frac{m_\alpha - m_e}{m_\alpha + m_e} v_\alpha.$$

Since the mass, m_α, of the alpha particle is much greater than the mass, m_e, of the electron, the results show that the speed of the alpha particle is only slightly smaller after collision than before, as we would have expected. The speed acquired by the electron is given with sufficient accuracy by:

$$v'_e = 2\,v_\alpha.$$

To find the kinetic energy lost by the alpha particle, we calculate the kinetic energy gained by the electron:

$$E'_e = \tfrac{1}{2}\,m_e(v'_e)^2 = \tfrac{1}{2}(m_e/m_\alpha)m_\alpha(2\,v_\alpha)^2$$
$$= \frac{4\,m_e}{m_\alpha}\left(\frac{1}{2}\,m_\alpha {v_\alpha}^2\right)$$
$$= \frac{4\,m_e}{m}E_\alpha = \frac{4}{7200}E_\alpha = \underline{E_\alpha/1800.}$$

The energy lost by the alpha particle is equal to this by energy conservation.

(b) Eighteen head-on collisions would be required to make the alpha particle lose 1/100 of its energy. It may be interesting to note that 1 percent is approximately the energy lost by an alpha particle from a polonium source in traversing a gold foil 10^{-7} meter, or 400 atoms, thick. The way in which the alpha particle loses energy is by collisions with atomic electrons. Not all collisions are head-on, so that more collisions are involved than we have actually calculated here.

24–28†

(See the Appendix in the text.)

24–29

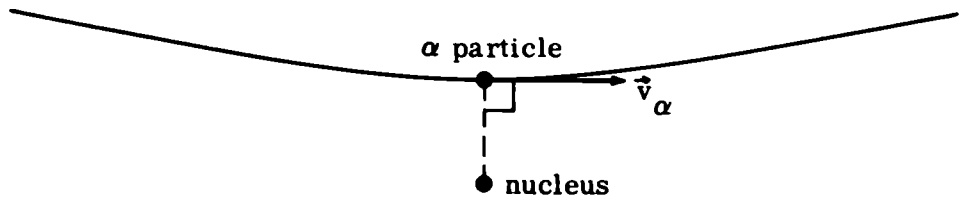
(a) If the distance of closest approach of the alpha particle in Problem 28 was 1.65×10^{-13} m, what was its speed at this point?

(b) At what speed was it moving toward the nucleus at that point?

(a) Since energy is conserved the sum of the kinetic and potential energies remains constant:

$$\underset{\text{kinetic energy}}{\tfrac{1}{2}\,mv^2} + \underset{\text{potential energy}}{\frac{kq_\alpha q_N}{r}} = E, \text{ the total energy.}$$

When the alpha particle is a long way from the nucleus and moving with speed v_o, r is so large that the potential energy is negligible, and the total energy is



$$E = \tfrac{1}{2}mv_0^2$$
$$= \tfrac{1}{2} \times 6.62 \times 10^{-27} \times (1.00 \times 10^7)$$
$$= 3.31 \times 10^{-13} \text{ joule.}$$

When $r = 1.65 \times 10^{-13}$ meter the potential energy is

$$k\frac{q_\alpha q_N}{r} = 2.31 \times 10^{-28} \text{ joule-m/(elem.ch.)}^2 \times \frac{(2 \text{ elem.ch.})(79 \text{ elem.ch.})}{1.65 = 10^{-13} \text{ m}}$$
$$= 2.21 \times 10^{-13} \text{ joule.}$$

Hence, the kinetic energy at the distance of closest approach is

$$E_K = \tfrac{1}{2}mv^2 = (3.31 - 2.21) \times 10^{-13} \text{ joule}$$
$$= 1.10 \times 10^{-13} \text{ joule.}$$

Then

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2 \times 1.10 \times 10^{-13}}{6.62 \times 10^{-27}}}$$
$$= \underline{5.80 \times 10^6 \text{ meters/sec.}}$$

(b) At the distance of closest approach the component of velocity along the line to the nucleus is zero.

24–30

In the hydrogen atom, the electron is on the average 5×10^{-13} m from the positively charged proton.

(a) Estimate the average force between the electron and proton.

(b) If the atom were placed between the plates used in the Millikan experiment of Section 19–8, how many of the 90-volt batteries would we need to put in series with the plates to tear the atom apart?

(a) $$F = k\frac{q_1 q_2}{r^2} = \frac{2.3 \times 10^{-28}(1 \times 1)}{(5 \times 10^{-11})^2}$$
$$= 9 \times 10^{-8} \approx 10^{-7} \text{ newton.}$$

(b) The plates of the Millikan experiment described in Section 19–8 exerted a force of $\approx 10^{-14}$ newton on each elementary charge when three 90-volt batteries were connected in series with them. In order to increase the force to 10^{-7} newton, we would need 10^7 times as many 90-volt batteries.

Obviously, this is not the mechanism of ionization in a gas, because ionization occurs long before such large voltages are applied. A more effective process uses collisions with charged particles, where the necessary electric field is produced by a single elementary charge close by ($\sim 10^{-10}$ m) instead of an enormous number of charges at much larger distances ($\sim 10^{-3}$ m).

CHAPTER 25

25–1*

How would the two graphs in Fig. 25–2 look if the electrons were ejected according to the wave model of light? (Section 1.)

The time at which a first jump occurred would always be later for weak light.

25–2

In a certain camera, at dusk, the light that gets through the lens is so dim that in 1/100 second one photon on the average lands in 10 mm² of the film. If you want to take a finely detailed, sharp picture, what should you do?

To take a finely detailed picture we need more photons. To get them we must use a long exposure time.

25–3

With a motion-picture camera you could determine the time at which the charge on each speck was changed for a large number of specks in the apparatus of Fig. 25–1. What relationship would you expect between the number of specks on which the charge changed and the time after the light was turned on? Does the data given in Fig. 25–2 support your prediction?

With constant intensity of illumination, one would expect that the number of specks on which charge

changed during a short time interval would be proportional to the number of specks on which charge had not changed up to the start of that interval. One would observe fewer and fewer specks changing charge in each succeeding equal time interval. This is what is observed in Fig. 25–2.

25–4*

A surface ejects electrons when hit by green light, but none when hit by yellow light. Do you expect that electrons will be ejected when the surface is hit (a) by red light? (b) by blue light? (Section 2.)

(a) No.
(b) Yes.

25–5

Some photographic materials can be handled safely in red light but are spoiled instantly when white light is turned on. How do you account for this?

Light of shorter wavelength is more effective in spoiling photographic materials, just as it is in ejecting electrons from metals. White light contains a mixture of colors from red to violet. Hence, it contains shorter wavelengths than red light alone.

25–6

Suppose that a photograph similar to Fig. 25–4 had been made with X rays of wavelength about 0.1 angstrom. In what respects would you expect the new photograph to resemble Fig. 25–4? In what respects would it be different?

The fog droplets will be larger, but there will be a weakening of the beam as before.

25–7

Assume that the entire negative plate in the cell illustrated in Fig. 25–5 is always illuminated by a constant source of light capable of producing a photoelectric current. What would be the effect on this current of
(a) doubling the area of the plates?
(b) halving the distance between the plates?

(a) The current will be doubled.
(b) The current will be unchanged.

25–8*

In Fig. 25–7, which points on the curve were obtained by using violet light? (Section 3.)

The points corresponding to frequencies of 6.8 and 7.4×10^{14} cycles/sec.

25–9*

What is the minimum energy, in electron volts, needed for photons to eject electrons from the photoelectric surface used in Fig. 25–7? (Section 3.)

1.4 ev.

25–10

A good mirror reflects about 80 percent of the light which hits it. How could you determine whether 20 percent of the photons were not reflected, or whether all of the photons were reflected but each with 20 percent less energy?

If each photon lost 20% of its energy, the color of the reflected light would be shifted to that of a longer wavelength than the incident light. Since the reflected light has the same color as the incident light, we conclude that 20% of the photons are absorbed completely.

25–11

Red light has a wavelength of about 6500 angstroms.
(a) Estimate its frequency.
(b) Estimate the energy in joules of one photon of red light.
(c) Estimate the energy in electron volts of one such photon.
(d) Find out approximately the longest and shortest wavelengths you can see, and compute the energy of the corresponding photons.

(a) $$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/sec}}{6.5 \times 10^{-7} \text{ m}} = \underline{4.6 \times 10^{14} \text{ cycles/sec.}}$$

(b) $$E = h\nu = 6.6 \times 10^{-34} \text{ joule-sec} \times 4.6 \times 10^{14} \text{ cycles/sec} = \underline{3.0 \times 10^{-19} \text{ joule.}}$$

(c) $$E = 3.0 \times 10^{-19} \text{ joule} \times \frac{1 \text{ ev}}{1.6 \times 10^{-19} \text{ joule}} = \underline{1.9 \text{ ev.}}$$

(d) You can see light of all wavelengths from 4000 A (violet) to 7000 A (red). Using the fact that $E = h\nu = hc/\lambda$, we can set up a proportion to find the energies of the 4000-A and 7000-A photons from the energy of the 6500-A photons computed above.

$$\frac{E_{7000}}{E_{6500}} = \frac{6500}{7000};$$

$$E_{7000} = 1.9 \times \frac{6500}{7000} = \underline{1.8 \text{ ev.}}$$

$$\frac{E_{4000}}{E_{6500}} = \frac{6500}{4000};$$

$$E_{4000} = 1.9 \times \frac{6500}{4000} = \underline{3.1 \text{ ev.}}$$

25–12

Light of 5000-angstrom wavelength illuminates a surface. What voltage is needed to stop all the electrons emitted from the surface if their binding energy to the surface is 2.0 electron volts?

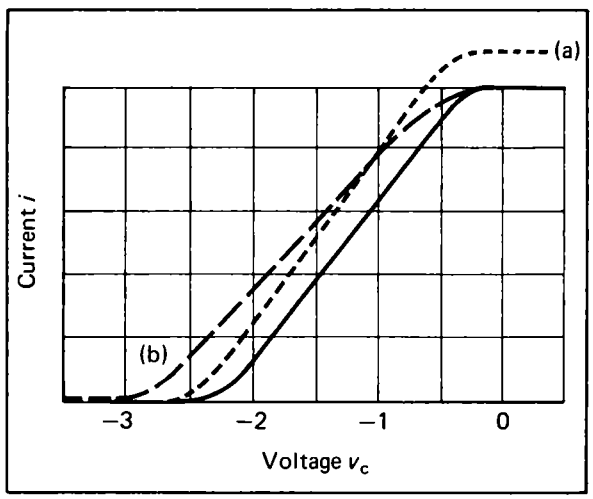
The energy of the 5000-A photons is $E = 2.5$ ev. Part of this energy, 2.0 ev, is used up getting the electron out of the metal, while the remainder, 0.5 ev, remains with the fastest electrons as kinetic energy. It therefore will take a retarding potential of 0.5 volt to stop these electrons. (Most of the electrons which emerge lose some of this 0.5 ev of energy in collisions inside the solid.)

25–13

Sketch the graph of Fig. 25–6, then upon it sketch graphs that will show the effect of

(a) increasing the intensity of the light source.

(b) decreasing the wavelength of the light.



Note that increasing the intensity of illumination increases the current, but does not change the cut-off potential; decreasing the wavelength of the light makes the cut-off potential more negative, but does not change the current.

25–14

(a) What is the ratio of the energy of a photon of wavelength 4500 A (blue light) to the energy of a photon of wavelength 6300 A (red light)?

(b) What is the ratio of the maximum energy of electrons emitted from a surface with a binding energy of 1.8 electron volts using the same red and blue lights?

(a) $$\frac{E_{\text{blue}}}{E_{\text{red}}} = \frac{hc/\lambda_b}{hc/\lambda_r} = \frac{\lambda_r}{\lambda_b} = \frac{6300\text{A}}{4500\text{A}} = \underline{1.4.}$$

(b) $$E_{\text{red}} = \frac{1.24 \times 10^4 \text{ ev} - \text{A}}{6300\text{A}} = 1.97 \text{ ev}$$

$$E_{\text{blue}} = 1.4\, E_{\text{red}} = 2.76 \text{ ev}$$

$$E_{\text{electron}} = E_{\text{photon}} - E_{\text{binding}}$$

for red light:

$$E_{\text{electron}} = 1.97 \text{ ev} - 1.8 \text{ ev} = .17 \text{ ev}$$

for blue light:

$$E_{\text{electron}} = 2.76 \text{ ev} - 1.8 \text{ ev} = .96 \text{ ev}$$

$$\frac{E_{\text{electron}} \text{ using blue light}}{E_{\text{electron}} \text{ using red light}} = \frac{.96 \text{ ev}}{.17 \text{ ev}} = \underline{5.6.}$$

25–15†

(See the Appendix in the text.)

25–16

(a) Take a pack of ordinary playing cards (from which the jokers are removed), thoroughly shuffle them, and draw a card. After noting which card it is, return it to the pack, shuffle, and draw again. After ten draws, note how many times each of the following were drawn:

(I) A face card.

(II) A card on which a 3 is printed.

(III) A card on which an even number is printed.

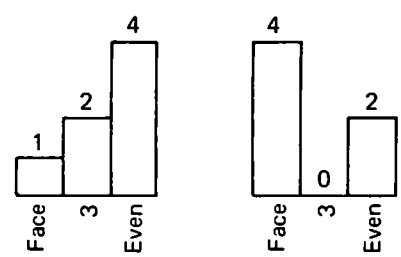
(b) Make a bar graph, after the fashion of Fig. 25–9, to display the results of ten draws.

(c) Make a bar graph to display the results you would expect to obtain as the result of about 5200 draws.

(d) In what respects are your graphs like those of Fig. 25–9?

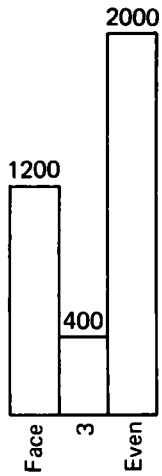
(a) Because of the small number of draws, the numbers of cards of each class obtained will vary widely, as indicated by the typical bar graphs given in answer to (b).

(b)



After ten draws

(c)



After 5200 draws

(d) One observes the same effects as are noticed in Fig. 25–9(c); the pattern built up by random chance is highly erratic for a small number of cases, but builds up to a characteristic distribution after a large number of cases, that changes very little with additional cases.

25–17

In the three-month experiment described in Section 25–3, G. I. Taylor estimated the energy of the light reaching his photographic plate. He found it was 5×10^{-13} joule per second. (He obtained that value by comparing the average blackening of the plate with that produced in 10 seconds by a candle 2 yards away, without any absorbing screens of smoked glass.) From that estimate, calculate the average distance between photons, as follows:

(a) Assume the wavelength of the useful light was about 5000 angstrom units. What energy did each photon carry?

(b) Calculate from the given flow of energy the average time that elapsed between the arrival of one photon and the next.

(c) From the average time, calculate the average distance between one photon and the next in the beam of light.

(d) If you were asked, "How many photons are in the box at any chosen instant," you would have to answer "None" most of the time, but on looking again and again and again, you should expect to "see" one photon in the box about once in so many trials. How many? Assume the box was 1.2 meters long.

(a) $$E = h(c/\lambda) = \frac{6.6 \times 10^{-34}\,\text{joule-sec} \times 3.0 \times 10^{8}\,\text{m/sec}}{5000\text{A} \times 10^{-10}\text{m/A}}$$

$$E = \underline{4.0 \times 10^{-19}\,\text{joule.}}$$

(b) The average time between the arrival of each photon is

$$\frac{4.0 \times 10^{-19}\,\text{joule}}{5 \times 10^{-13}\,\text{joule/sec}} = \underline{8 \times 10^{-7}\,\text{sec.}}$$

(c) Since photons in air travel at 3×10^{8} m/sec and they arrive, on the average, 8.0×10^{-7} sec apart, if the beam were in space, the photons would be separated by 8.0×10^{-7} sec $\times$ 3×10^{8} m/sec = $\underline{240\ \text{meters.}}$

(d) To answer this question, it is helpful to picture a beam in space with an average spacing between photons of 240 meters, but with any individual photon likely to be anywhere along the beam at any instant of time because of the random nature of the emission process. Then if you make the further assumption that the longitudinal extent of a photon is very short, you can make the following argument. A box 1.2 meters long placed in the beam extends for 1/200 of the average distance between photons. Therefore, when you look in the box at any instant of time you will see a photon only once in every $\underline{200\ \text{times}}$ that you look.

25–18

Use the language of photons to describe how the pattern of Fig. 8–5 is formed with very dim light.

The pattern of Fig. 8–5 was produced by monochromatic red light from a small source passing through two narrow slits placed close together. The path of any single photon from the source passing through one of the slits is a matter of chance, and no prediction can be made as to where it will go. However, the intensity of the various regions of the interference pattern is related to the probability of a photon arriving at a particular point of the pattern. Where the maxima of the pattern occur, there is a high probability of photons arriving there; conversely, there is a low probability of photons arriving at the minima. The interference pattern can be observed only after many photons have passed through the slits.

25–19

An ordinary flashlight has a bulb which draws 0.25 amp at 3.0 volts, and concentrates the emitted light into a beam about 10 cm^2 in area. Assume that 1.0 percent of the energy consumed by the bulb is turned into visible light of wavelength 5.8×10^3 angstrom units.

(a) How many photons are there in each meter along the axis of the beam?

(b) How many photons per second arrive on each square centimeter of a surface illuminated by this flashlight?

(a) $$\frac{\text{Photons}}{\text{Meter}} = \frac{\text{Photons/sec}}{\text{Speed (m/sec)}}$$

$$= \frac{(E_{\text{emitted}}/\text{sec})/(E/\text{photon})}{\text{Speed (m/sec)}}$$

$$= \frac{Vi \times .01/(hc/\lambda)}{c} = \frac{10^{-2} Vi\lambda}{hc^2}$$

$$= \frac{10^{-2}(3.0 \text{ volts})(2.5 \times 10^{-1} \text{ amp}) \times (5.8 \times 10^{-7} \text{ m})}{(6.62 \times 10^{-34} \text{ joule-sec}) \times (3.0 \times 10^8 \text{ m/sec})^2}$$

$$\frac{\text{Photons}}{\text{Meter}} = 7.3 \times 10^7.$$

(b) $$\frac{\text{Photons/sec}}{\text{cm}^2}$$

$$= \frac{10^{-2} Vi\lambda}{(hc)(10 \text{ cm}^2)}$$

$$= \frac{(10^{-2})(3.0 \text{ volts}) \times (2.5 \times 10^{-1} \text{ amp})(5.8 \times 10^{-7} \text{ m})}{(6.62 \times 10^{-34} \text{ joule-sec}) \times (3.0 \times 10^8 \text{ m/sec})(10 \text{ cm}^2)}$$

$$= 2.2 \times 10^{15} \text{ photons/cm}^2\text{-sec}.$$

25–20

A typical FM radio station broadcasts with a wavelength of 3 meters. Estimate:

(a) the energy in one photon of this radiation.

(b) the number of photons broadcast per second if the radiated power is 10 kilowatts.

(c) roughly the number of photons received by a receiving set during one vibration of the letter "s" in the audio pattern, if the receiver is 100 km from the station. (Make the following simplifying assumptions:

(i) the radiation spreads uniformly in all directions along radii from the source so that an inverse-square law holds for energy flow. (This is not true for any ordinary antenna.)

(ii) the receiving antenna collects radiation from an area of 1 $meter^2$ perpendicular to the line of travel of waves.

(iii) the sound of the letter "s" involves air oscillations of frequency about 4000 cycles per second.)

(d) Will you ordinarily notice the individual photons in receiving the letter "s"?

(a) $E = h\nu = h(c/\lambda)$

$$= \frac{6.6 \times 10^{-34} \text{ j-sec} \times 3 \times 10^8 \text{ m/sec}}{3}$$

$$= \underline{6.6 \times 10^{-26} \text{ joule.}}$$

(b) If each photon carries about 10^{-25} joule, the number of photons required to produce 10^4 joules is

$$\frac{10^4 \text{ joules}}{10^{-25} \text{ joule}} = 10^{29}.$$

Consequently, $\underline{10^{29} \text{ photons/sec}}$ are required to produce 10^4 joules/sec (or 10 kilowatts).

(c) When the radio waves get out to a radius of 100 km, their total energy is spread over an area A equal to the surface of a sphere of 100-km radius:

$$A = 4\pi R^2 = 4\pi \times 10^{10} \text{ m}^2.$$

A portion of this surface with an area of 1 m^2 will capture a fraction equal to $1 \text{ m}^2/(4\pi \times 10^{10} \text{ m}^2)$ of the total energy. The receiver gets this fraction of the 10^{29} photons/sec emitted by the transmitter, or $1/(4\pi \times 10^{10}) \times 10^{29} \approx 10^{18}$ photons/sec.

One vibration of the sound "s" lasts 1/4000 sec; so 1/400 sec $\times 10^{18}$ photons/sec $\approx \underline{10^{14} \text{ photons}}$ arrive at the receiver during one vibration of the letter "s."

(d) You would *not* notice one photon in 10^{14}!

25–21

In a particular X-ray tube electrons hit a target after they have been accelerated by a potential difference of 20,000 volts. As they decelerate to a stop in the target, a few of them emit X-ray photons.

(a) Why is there a definite minimum wavelength of photons observed from this tube? (This limit is known as the Duane-Hunt limit.)

(b) Calculate that minimum wavelength.

(a) Since each X ray is generated by a single electron, the maximum energy that a single X ray can have is equal to the kinetic energy that the electron gives up as it is decelerated to a stop in the target. The wavelength corresponding to this maximum energy is

$$\lambda = \frac{c}{\nu} = \frac{hc}{E_{max}}.$$

Any smaller wavelength would give a larger photon energy than is available.

(b) $\lambda = \dfrac{1.24 \times 10^4 \text{ ev-A}}{2.00 \times 10_4 \text{ ev}} = \underline{0.620 \text{ angstrom.}}$

CHAPTER 26

26–1*

What are the discrepancies between the predictions of the Rutherford model about the emission of light by atoms and the observed emission of light? (Section 2.)

These discrepancies are discussed in Section 26–1. An electron orbiting the nucleus undergoes accelerated motion and must radiate electromagnetic energy. As it loses energy, the diameter of its orbit should decrease, and its period should decrease also. Hence, the frequency of the light it emits should increase. Any group of many atoms should be at various stages of this process, and together should produce a continuous spectrum of light. Actually, the spectrum of a gas shows only a set of discrete frequencies.

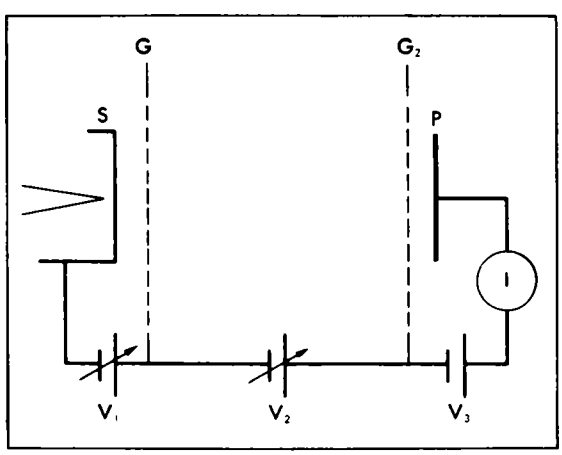
26–2*

An electron whose kinetic energy is 4.2 ev collides with a mercury atom. What will be the kinetic energy of the electron after the collision? (Section 2.)

4.2 ev.

26–3

Figure 26–9 is a schematic sketch of a vacuum tube in which electrons emitted by S can be accelerated by either of the potential differences V_1 or V_2. Suppose the tube contains some mercury vapor and is used to perform the Franck-Hertz experiment.



(a) If $V_1 = 0$ volts, $V_2 = 5.0$ volts, and the field between G_1 and G_2 is uniform, in what region of the space between G_1 and G_2 can electrons give up 4.9 ev in inelastic collisions?

(b) If $V_1 = 0$ volts and $V_2 = 10.0$ volts, in what region of the space between G_1 and G_2 can electrons give up 4.9 ev in inelastic collisions? In what region 6.7 ev?

(c) If $V_1 = 10$ volts and $V_2 = 0$ volts, how far do the electrons have to go through the gas before they can lose 4.9 electron volts in inelastic collisions? How far before they can lose 6.7 ev? What electron energies should it be possible to measure to the right of G_2?

(d) The 6.7-ev transfer of energy is observed when the apparatus is operated as in (c), but it is not observed when the apparatus is operated as in (b). Explain this result in terms of your answers to (b) and (c).

(a) The inelastic collisions can occur only after electrons have been accelerated through a potential difference of 4.9 volts. This would happen only <u>very close to G_2.</u>

(b) The 4.9-ev energy is reached just before the electron is half the distance from G_1 to G_2. The mercury vapor can take another 4.9 ev of energy from electrons in the <u>last half</u> of the distance G_1 to G_2. Electrons would give up 6.7 ev in the <u>last third</u> of the distance G_1 to G_2.

(c) Electrons will now have 10 ev of energy when they enter the gas through G_1. Each electron can immediately lose either 4.9 or 6.7 ev. Those that lose 4.9 ev can again lose 4.9 ev. The electron energies which it would be possible to measure to the right of G_2 would be:

$$10.0 \text{ ev} - 0.0 \text{ ev} = 10.0 \text{ ev}$$
$$10.0 \text{ ev} - 4.9 \text{ ev} = 5.1 \text{ ev}$$
$$10.0 \text{ ev} - 6.7 \text{ ev} = 3.3 \text{ ev}$$
$$10.0 \text{ ev} - 2 \times 4.9 \text{ ev} = 0.2 \text{ ev}.$$

(d) In (c), an electron has sufficient energy to cause either transition when it reaches the gas. However, in (b) each electron reaches 4.9 ev at half the distance G_1 to G_2 and can give up this amount of energy in a collision after this point. If the 6.7 ev is not observed, this means that most electrons have given up the 4.9 ev of energy before they have reached

the position where they can gain 6.7 ev, which is about two thirds of the distance.

26–4

(a) What is the significance of the constant B that appears in the equation for the photoelectric effect in Section 25–3?

(b) How do the values of B for the four metals exhibited in Fig. 25–7 compare with their ionization energies? (Ionization energies of Li, Mg, and Cu are 5.36 ev, 7.61 ev, and 7.68 ev, respectively.)

(c) What conditions seem to influence the amount of work required to remove an electron from a metal?

(a) B is the energy required to remove an electron from the metal.

(b) All values of B are less than the ionization energies.

(c) The amount of work seems to be influenced by whether the atom is isolated or bound to other atoms.

26–5

In the experiment of Franck and Hertz, schematically represented in Fig. 26–2, electrons are used to bombard the mercury atoms. What advantages do the electrons possess for this purpose over protons?

It is much easier to produce an electron beam.

26–6*

Mercury vapor is bombarded by electrons whose kinetic energy is 6.67 ev. What are the possible energies of the emitted photons? (Section 3.)

6.67 ev, 4.86 ev, 1.81 ev.

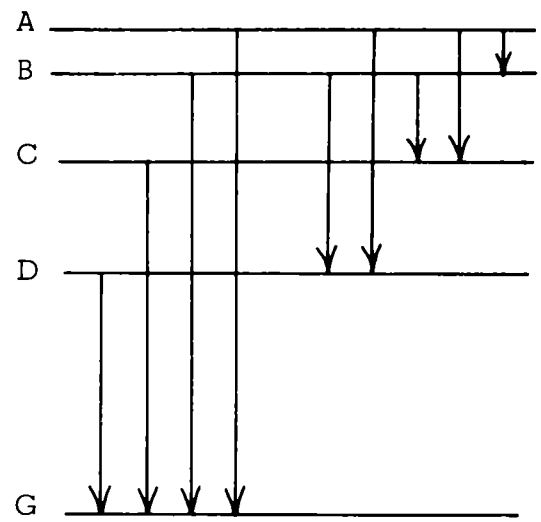
26–7*

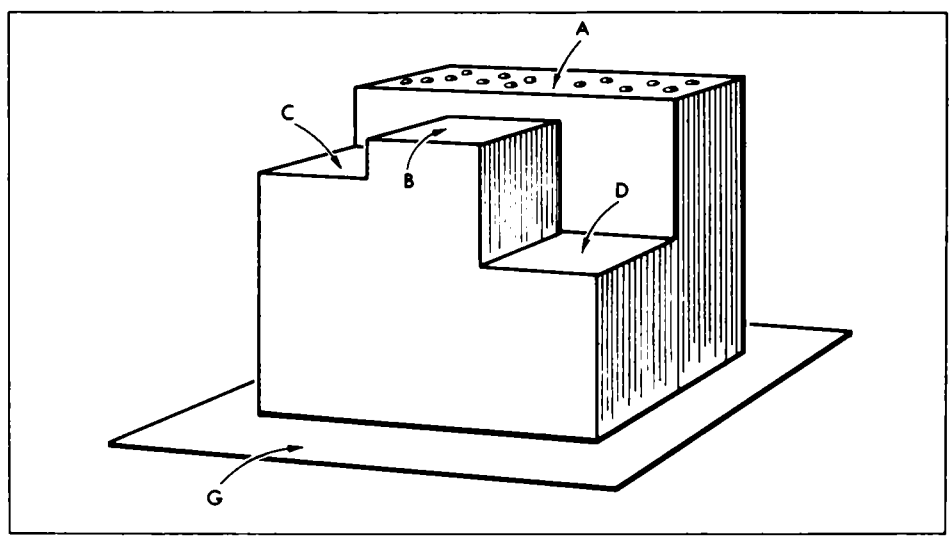
Suppose a certain atom emits a photon with a wavelength of 6838 A. How much energy does the atom lose? (Section 3.)

1.81 ev.

26–8

Figure 26–10 shows a mechanical model of energy levels with platforms at four different levels above the ground. A number of uniform putty balls are located on top of A. When balls are pushed over the side from A and land somewhere below, their temperature will rise in direct proportion to their





loss in potential energy. Balls which land on *B*, *C*, and *D* are again pushed until they reach the ground. How many different changes in temperature are possible? Draw the "energy levels" and mark the possible jumps.

As shown in the diagram, there are nine different temperature changes possible. Notice that the transition from *C* to *D* is impossible. This is an Illustration of what happens also in atoms. Not all energy levels in an atom are connected through photon emission, although the emission is possible as far as energy is concerned. These missing lines are referred to as "forbidden lines."

26–9

Suppose you are going to bombard a vapor of cesium atoms with 4.00-electron-volt electrons. Use Fig. 26–2 to predict some of the things that you would expect to happen.

This problem in energy conservation, for electron collisions, can also illustrate emission spectra.

Electron Collisions. Since the electrons have greater kinetic energy than that needed to strip away one atomic electron, a number of things can happen. The electron may excite one of the discrete states, in the process losing 1.38 ev or 2.30 ev. It may scatter elastically, losing no kinetic energy. If it ionizes the atom, the atomic electron and the bombarding electron share the remaining kinetic energy. 3.87 ev is required to tear away the atomic electron, so they will have 4.00 − 3.87 ev = 0.13 ev between them.

Emission Spectra. Since some of the atoms are ionized, all possible emission lines should occur. An ionized atom will encounter some slowly moving electron, moving with a very small kinetic energy of about 3/2 kT, where T is the temperature of the cesium gas (see Section 17–3). (For ordinary temperatures, this will be less than 0.1 ev.) The electron is attracted by the net charge on the ion. It can be "captured" into any of the discrete states by emitting a photon to conserve energy. The diagram shows some possible photons which can be emitted as the atom gradually relaxes to its ground state.

Note: Figure 26–2 shows only a few levels. There are many other levels which lie close together below the ionization energy. Therefore the incoming electron can lose many other possible amounts of energy and also many other spectral lines can be emitted.

26–10

If ordinary table salt is introduced into a gas flame or a soft glass rod is heated in it, the flame is colored with the characteristic light from sodium atoms, whose wavelength is 5890 angstroms. This light is emitted by electrons returning to the ground state from the first excited energy level.

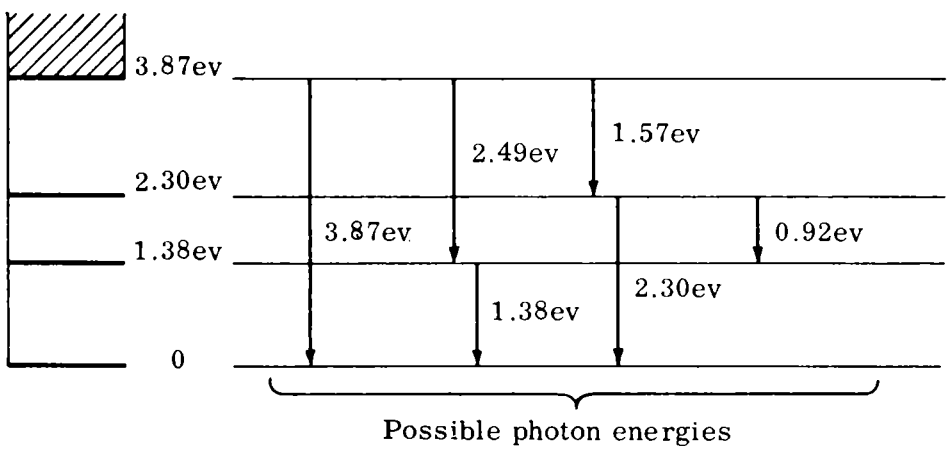
(a) What is the energy of this level above the ground state, in electron volts?

(b) What would be the temperature of a gas whose molecules had an average kinetic energy of this amount?

(c) The average temperature of the molecules in a gas flame is about 2100°K. How is it possible for a gas flame to raise sodium atoms to the first excited energy level?

(a) $E = \dfrac{1.24 \times 10^4 \text{ ev-A}}{5890 \text{ A}} = \underline{2.1 \text{ ev.}}$

(b) $E = 3/2(kT)$.



Hence,

$$T = 2/3(E/k)$$
$$= \frac{2(2.1 \text{ ev})(1.6 \times 10^{-19} \text{ joule/ev})}{3(1.4 \times 10^{-23} \text{ joule/ K})}$$
$$= \underline{1.6 \times 10^4 \text{ °K.}}$$

(c) Although the average kinetic energy of the molecules in a gas flame corresponds to a temperature of 2100°K and amounts to 0.27 ev, there are many molecules with greater or less energy, and enough with ten times the average energy to excite the sodium atoms by collisions.

26–11

If we heat helium gas to a high enough temperature, the average kinetic energy of an atom in its thermal motion will be so high that an inelastic collision between two helium atoms can easily excite one of them to its first excited state at 19.8 electron volts above the ground state. Estimate this temperature, obtaining any data you need from Chapter 18.

This problem illustrates why atoms are usually found in their lowest energy state.

The electron will be excited only when the kinetic energy of a single gas molecule, $3/2kT$ see Section 17–3), becomes comparable to 19.8 ev. Then, on collision of two atoms, their translational kinetic energy can be used up in exciting one of them.

At this energy the temperature of the gas is

$$T = 2/3(E_K/k)$$
$$= \frac{2(19.8 \text{ ev})(1.6 \times 10^{-19} \text{ joule/ev})}{3(1.38 \times 10^{-23} \text{ joule/°K})}$$
$$\approx \underline{10^5 \text{ °K.}}$$

The temperature at the surface of the sun is only 6000°K, so even there, helium atoms would be predominantly in their ground state.

26–12*

Compare the amount of energy absorbed when mercury vapor is bombarded with a 6-ev electron and when it is bombarded by a 6-ev light photon. (Section 4.)

The atom absorbs 4.86 ev from the electron. It absorbs no energy from the photon.

26–13

White light, having passed through sodium vapor, is examined with a spectroscope, and the dark lines characteristic of the absorption spectrum of sodium are observed.

(a) What happens eventually to the energy absorbed by the sodium atoms?

(b) Why, then, are the absorption lines dark?

(a) It is re-emitted.

(b) The re-emission takes place in all directions, leaving the transmitted beam weaker and hence dark by contrast with light of adjacent frequencies that has not been absorbed.

26–14

If you point a good spectroscope at an arc lamp, you will see a continuous spectrum. If you point it out the window on a bright day, you will again see a continuous spectrum; but this time, with a narrow enough slit, you will see a number of dark lines crossing the spectrum parallel to the slit. (If you have access to a good spectroscope and have never done this, it is worth doing.) What is the origin of these lines? What can be deduced from a study of them?

These lines are produced by absorption of light coming from the photosphere of the sun as it passes through gases in the atmospheres of the sun and the earth. Elements present in these atmospheres can be identified from the absorption spectra.

26–15

Figure 26–11 is a schematic diagram of another vacuum tube in which electrons can be accelerated by V_1 and the energies of the electrons measured by adjusting V_2 until no current is shown on the meter. Suppose this tube contains mercury vapor.

As in the Franck-Hertz experiment, the energy of electrons arriving at P shows an abrupt decrease when V_1 is 4.9 volts. They also show an abrupt decrease when V_1 is 9.8 volts. Does this mean that mercury atoms have an energy level at 9.8 volts? Explain.

No. The electrons have made two collisions, each involving a transfer of 4.9 ev to the mercury atoms.

CHAPTER 27

27–1*

What is the wavelength of an electron whose speed is 1.0×10^7 m/sec? (Section 1.)

$\lambda = 0.73$ angstrom.

27–2*

As the angle θ in Fig. 27–2 is increased, the beam will again reflect very strongly out of the crystal. What must $2d \sin \theta$ be equal to at this angle? (Section 1.)

$2d \sin \theta = 2\lambda$.

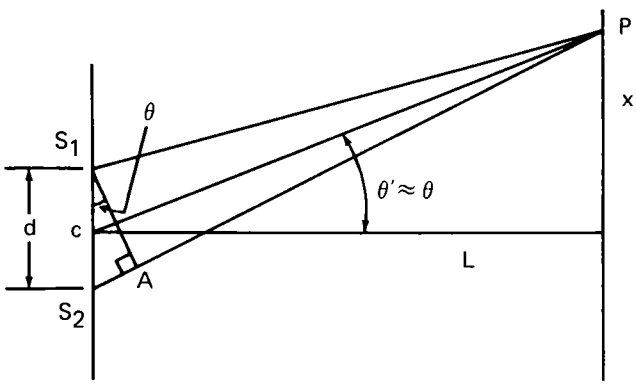
27–3

The layers of atoms in crystals are known to be a few angstroms apart, say 3×10^{-10} m. X rays from ordinary commercial X-ray tubes are found to be diffracted at quite large angles, such as 10°, by such crystals acting as interference gratings. Make a rough estimate of the energy of photons in such X rays.

From Figure 27–2 of the text we see that $\lambda = 2d \sin \theta$. Since $\sin 10° \approx 0.17$, then $\lambda = 2(3 \times 10^{-10}\text{ m})(0.17) = 1 \times 10^{-10}\text{ m} \approx 1$ angstrom unit. The photon energy is given by $E = h(c/\lambda) = 1.24 \times 10^4$ ev-A/1 A $\approx \underline{10^{-4}\text{ ev}}$.

27–4

Electrons passing through a double slit (fantastic but possible, as discussed in the next question) make an interference pattern on a remote screen. If the gun voltage used to provide the original stream of electrons with kinetic energy is changed from 50 volts to 5000, what does this do to the spacing of the fringes?



The analysis of two-slit interference has been given in Section 7–4. Referring to the figure, bright fringes occur at P whenever the length of AS_2 is equal to a whole number of de Broglie wavelengths of the electrons, i.e., $AS_2 = n\lambda$, where $n = 0, 1, 2, 3$, etc. So, from Section 7–4 we have $x = n\lambda L/d$, and the distance of separation on the screen between fringes is $\Delta x = \lambda L/d$. For the electrons,

$$\lambda = h/p = h/\sqrt{2mE}.$$

Hence,

$$\Delta x = \frac{hL}{d\sqrt{2mE}},$$

so the spacing Δx is inversely proportional to $\sqrt{E}$. Changing the energy of a beam of electrons from 50 to 5000 electron volts increases E by a factor of 100, $\sqrt{E}$ by a factor of 10, and therefore reduces Δx tenfold.

27–5

A narrow stream of 100-ev electrons is fired at two parallel slits very close together. The distance between the slits is estimated to be 10 angstrom units. The electrons passing through the slits reach a screen 3 meters away, and form a pattern of interference fringes.

[It seems almost impossible to make such a pair of slits in any real sheet of matter, made of atoms which are themselves 1 angstrom or more in diameter, and even more impossible to measure their separation, since light has a wavelength of thousands of angstroms. Yet the equivalent of this pair of slits can be made—has been made and used. It is the "biprism" of Fig. 27–5(b).]

(a) Estimate the distance between one bright fringe and the next.

(b) The bright fringes would appear as bright marks on a fluorescent screen sufficiently sensitive to glow when bombarded with 100-ev electrons (or as dark marks on a photographic film, or as large pointer readings on some meter attached to an electron collector). What is the essential difference between the bright-fringe electrons and those that reach the screen in a dark fringe? Are they bigger? More massive? Of better quality? Of greater charge, or what?

(a) In the discussion of problem 4, we showed that

$$\Delta x = \frac{hL}{d\sqrt{2mE}}.$$

Here,

$$\Delta x = \frac{(6.6 \times 10^{-34}\text{ joule-sec})(3\text{ m})}{(10 \times 10^{-10}\text{ m})\sqrt{2(9.1 \times 10^{-10}\text{ kg})(100\text{ ev})(1.6 \times 10^{-19}\text{ joule/ev})}}$$

$$= \underline{0.36\text{ m}}.$$

(b) There is no difference whatever between the bright fringe electrons and those that reach the screen in the dark fringe. It is only that more electrons arrive at the bright fringes (this is what makes the fringes bright) than

arrive at the dark fringes. The fact that the dimensions of the diffracting slits are comparable to the de Broglie wavelength of the electron means that the behavior of the electrons is dominated by their wavelike nature. This means that the position at which the electrons will arrive is governed by a complicated probability pattern. The particular destination, within this pattern, of any individual electron is unpredictable; its mass, charge, energy, and size, are, however, identical with those of all of the rest of the 100-ev electrons.

27–6

In a certain type of vacuum tube, electrons escaping from the hot cathode pass through a grid maintained at a potential of −10 volts at a distance of 0.5 mm from the cathode, then hit a plate maintained at a potential of +110 volts at a distance of 2.0 mm from the cathode.

Must the wave behavior of the electrons be taken into account in describing their passage from the cathode to the plate? Explain.

No. One can see from Table 1 that the wavelength of 100-ev electrons is 1.2×10^{-10} m, far less than the distances between the electrodes.

Students may wish to verify the wavelength given in Table 1. This may be done as follows:

$$E_K = qV$$
$$\lambda = \frac{h}{p}.$$
$$p = \sqrt{2mE_K}$$
$$= \sqrt{2mqV}$$
$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For 100-volt electrons,

$$\lambda = \frac{6.6 \times 10^{-24}}{\sqrt{2 \times 0.91 \times 10^{-30} \times 1.6 \times 10^{-19} \times 10^{2}}}$$
$$= \frac{6.6 \times 10^{-34}}{\sqrt{29 \times 10^{-48}}}$$
$$= \frac{6.6 \times 10^{-24}}{5.4 \times 10^{-24}}$$
$$= 1.2 \times 10^{-10}\ \text{m}.$$

27–7

(a) What is the wavelength of X rays whose photons each carry 40,000 electron volts of energy?

(b) At about what energy do electrons have a de Broglie wavelength equal to that of 40,000-volt X rays? (Give your answer in electron volts.)

(c) What energy baseballs?

(d) What is the wavelength of a baseball moving at 10 meters per second?

(a) Using the information from Section 25–3 that $\lambda = 1.24 \times 10^4/E$, we find that 40,000-ev rays have wavelengths of $(1.24 \times 10^4)/40{,}000 = 0.31$ A.

(b) Using the de Broglie relationship, the momentum p of *any* particle with this wavelength is given by

$$p = h/\lambda = 6.62 \times 10^{-34}\ \text{joule-sec}/0.31 \times 10^{-10}\ m = 2.13 \times 10^{-23}\ \text{kg-m/sec}.$$

For an electron, $m_e = 9.11 \times 10^{-31}$ kg,

$$E_e = p^2/2m_e = \frac{(2.13 \times 10^{-23})^2}{2 \times 9.11 \times 10^{-31}}$$
$$= 2.49 \times 10^{-16}\ \text{joule}$$
$$= 1.56 \times 10^{3}\ \text{ev}.$$

Thus electrons with this wavelength must have an energy of about 1600 ev.

(c) For a baseball, the momentum, p, is the same. The mass of a standard baseball, m_b, is five ounces or 0.14 kg. Since, with constant p, $E \propto 1/m$, we can obtain the baseball energy, E_b, by proportion:

$$\frac{E_b}{E_e} = \frac{m_e}{m_b};$$
$$E_b = 2.5 \times 10^{-16}\ \text{joule} \times \frac{9.1 \times 10^{-31}}{0.14}$$
$$= 1.6 \times 10^{-45}\ \text{joule, or } 1.0 \times 10^{-26}\ \text{ev}.$$

The velocity of this baseball can be obtained from p:

$$v_b = p/m_b = \frac{2.13 \times 10^{-23}}{0.14}\ \text{m/sec}$$
$$= 1.5 \times 10^{-22}\ \text{m/sec}.$$

(d) If $v_b = 10$ m/sec, then $p = mv_b = 1.4$ kg-m/sec. Then,

$$\lambda = \frac{h}{p} = \frac{6.60 \times 10^{-34}}{1.4} = 4.7 \times 10^{-34}\ \text{m}$$
$$= 4.7 \times 10^{-24}\ \text{A}.$$

The moral is clear: we never have to worry about the wave-nature of baseballs.

27–8

The sizes of atomic nuclei are hard to measure with alpha particles, because we have to look for small

differences from Coulomb scattering. Neutrons are often used.

(a) What are the de Broglie wavelengths of neutrons with kinetic energy 10^4 electron volts? 10^6 electron volts? 10^8 electron volts?

(b) Compare these wavelengths with the diameter of a gold nucleus (15×10^{-15} m).

(c) Look back at Fig. 27–6 to see how waves behave when they meet an obstacle. Suppose we fire a stream of neutrons at a small target of gold. For which of the neutron waves in (a) do you expect to find

(i) most of the neutrons pass by the nuclei undisturbed while some are absorbed and some bounce off, all like balls in Newtonian mechanics?

(ii) most of the neutrons pass the nucleus apparently undisturbed and the rest come out of the gold target equally in all directions, regardless of the shape of the nucleus?

(d) With neutrons of which energy would you prefer to try measuring the nuclear size?

This problem introduces some ideas and orders of magnitude of nuclear physics.

(a) The mass of a neutron is approximately equal to the mass of the proton, 1.7×10^{-27} kg. Consider the case $E = 10^4$ electron volts = 1.6×10^{-15} joule. Then,

$$p = \sqrt{2m_n E}$$
$$= \sqrt{2 \times 1.7 \times 10^{-27} \times 1.6 \times 10^{-15}}\ \text{kg-m/sec}$$
$$= 2.3 \times 10^{-21}\ \text{kg-m/sec}.$$

The de Broglie relationship gives us that

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}\ \text{joule-sec}}{2.3 \times 10^{-21}\ \text{kg-m/sec}}$$
$$= \underline{2.9 \times 10^{-13}\ \text{m}}.$$

The other values of E increase by factors of 10^2 and 10^4, and since $\lambda \propto p^{-1}$, i.e. $\lambda \propto E^{-1/2}$, λ will be decreased by factors of 10 and 100, respectively.

(b) In tabular form, the results are as follows:

E	λ	λ/diameter of gold nucleus
10^4 ev	2.85×10^{-13} m	19
10^6 ev	2.85×10^{-14} m	1.9
10^8 ev	2.85×10^{-15} m	0.19

(c) (i) When the wavelength of a "particle" is small compared with the objects it encounters, it tends to behave like a classical particle. In this case, the 10^8-ev neutrons would behave in this way. Since the nucleus is not another billiard ball, but a complicated congregation of nucleons, the incident neutron may sometimes be absorbed into the nuclear motion, instead of scattering off the surface like a billiard ball.

(ii) It is at the other extreme, where λ is much greater than the size of the object, that the incident particle will not "see" the detailed structure of the scatterer, but for all cases comes out as a spherical wave. Here the 10^4-ev neutrons would do this.

In both cases most of the neutrons will not be affected by the scatterer, but for different reasons: in (i), the geometric area of the nuclei is such a tiny part of the target area. For (ii), the overall influence of the scattering object on the wave vanishes as its size becomes small—there just isn't much surface area for the waves to slap against.

(d) Since the 10^8-ev neutrons behave somewhat like billiard balls in colliding with gold nuclei, they would be better for measuring nuclear size than the other two groups. By counting the number of neutrons per m^2 in the neutron beam and the number absorbed or deflected from the beam by a thin sheet of gold nuclei, we could calculate the fraction of the square meter covered by the gold nuclei. Then if we computed the number of nuclei by weighing the sheet of gold, we could get the area per nucleus.

27–9

The relation between photon energy in electron volts and the wavelength of light in angstroms is shown in Section 25–3.

(a) Work out a similar relation between the kinetic energy of a particle and its de Broglie wavelength.

(b) If the particle is an electron and its kinetic energy is generated by accelerating it through a potential difference, relate the potential difference in volts to the de Broglie wavelength of the electron in angstroms.

(c) What is the energy of electrons whose wavelength is the same as that of yellow light?

(a) The relation between the kinetic energy E_K of a particle and its momentum p is

$$p = (2mE_K)^{1/2}.$$

Since $\lambda = h/p$, the desired relation is

$$\lambda = \frac{h}{(2mE_K)^{1/2}}.$$

(b) The kinetic energy of an electron which is accelerated through a potential difference V is $E_K = qV$, where q is the charge on the electron. Hence, if V is in volts, the wavelength in meters is

$$\lambda = \frac{h}{\sqrt{2mqV}}$$
$$= \frac{6.6 \times 10^{-34} \text{ joule-sec}}{\sqrt{2(9.1 \times 10^{-31} \text{ kg})(1.6 \times 10^{-19} \text{ coul})V}}$$
$$= \frac{1.2 \times 10^{-9}}{\sqrt{V}} \text{ meter},$$

and λ in angstroms is thus

$$\lambda = \frac{12}{\sqrt{V}} \text{ angstroms.}$$

This formula will give a good approximation of the wavelengths for electrons of energies up to 2500 electron volts, where the electron has then a velocity close to 1/10 the velocity of light; above this energy, relativity becomes important.

(c) From part (b), if λ is in angstroms then

$$V = \frac{(12)^2}{\lambda^2} = \frac{144}{(5900)^2} = 4 \times 10^{-6} \text{ volt.}$$

Note: This extremely small voltage explains why we cannot produce electron waves of wavelengths comparable to those of visible light. Any beam of electrons from which we may start has a spread in energy large compared to 4×10^{-6} ev; electrons "at rest" at room temperature have a mean kinetic energy of 2.5×10^{-2} ev.

27–10

Borrow a cymbal from the school band (or use your own). Hold the cymbal with the handle down and strike the rim with your finger, or a small rubber hammer. With a little practice, you should be able to form standing waves along the rim. Describe them.

There is no substitute for trying this. Nodal lines are formed radially outward from the center; as one section of the rim moves up, adjacent sections move down. Nodal lines may also form as concentric circles around the center of the cymbal; in this case, if the whole rim is moving up, the next inner section will be moving down. Frequently there is a complex combination of nodal lines of both types.

27–11

Waves travel at 2 meters/sec in a spring fastened firmly at both ends to rigid walls 8 m apart.

(a) What frequencies of vibration could be found in standing waves formed on this spring?

(b) Could you produce standing waves corresponding to all these frequencies by shaking the spring at its midpoint?

(a) 0.125 sec^{-1}, 0.25 sec^{-1}, 0.375 sec^{-1}, ...

(b) No; those with a node at the midpoint cannot be generated.

27–12*

When a length d is chosen for the "box" discussed in Section 27–5, does this restrict the particle in the "box" to only one fixed energy? (Section 5.)

No; since the only restriction is that 2d equal a whole number of wavelengths, there are an infinite number of energy values which the particle could have.

27–13*

What is the order of magnitude of the kinetic energy of an electron (mass 10^{-30} kg) bouncing back and forth in a "box" of length 1 A. (Section 5.)

10^{-17} joule.

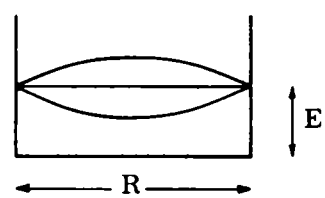
27–14

(a) Estimate the kinetic energy of a neutron in a nucleus of radius

$$R = 6 \times 10^{-15} \text{m}.$$

(This is a big nucleus.) Assuming the nucleus is a box of length R will be good enough.

(b) If the nucleus is smaller, should the kinetic energy be smaller?



This problem will help students appreciate the different orders of magnitude of nuclear energies compared with atomic energies.

(a) $\lambda = 2R$; $p = h/\lambda = h/2R$.

$$E = p^2/2m = h^2/8mR^2$$

$$= \frac{(6.6 \times 10^{-34})^2}{8 \times 1.7 \times 10^{-27} \times (6 \times 10^{-15})^2} \text{ joule}$$

$$= \underline{9 \times 10^{-13} \text{ joule}}$$

$$= \frac{9 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ ev} \approx 5.5 \times 10^6 \text{ ev}.$$

This is about a million times as big as atomic energies. This is the characteristic difference between nuclear and atomic processes, and it explains, for example, why fusion and fission nuclear bombs are so much more powerful than ordinary chemical explosives.

(b) The energy depends on $1/R^2$. As R decreases, $\underline{E \text{ must increase}}$, and rapidly. The enormous energies associated with nuclear events are due to the small nuclear size, in just this way. We obtained a factor 10^6, even though a neutron is 2000 times heavier than an electron, in itself an effect which would tend to decrease energies by that factor.

27–15

Suppose that an electron is confined within a small cubic crystal about 1 micron (10^{-6} m) on a side. Would quantum effects be detectable if you observed changes in the energy of the electron? (Hint: Calculate the lowest possible energy, considering the electron as a standing wave within the crystal.)

From Section 27–6, the expression for the energy is

$$E_o = 3h^2/8\,md^2$$

$$E_o = \frac{3 \times (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-6})^2}$$

$$E_o = 1.8 \times 10^{-25} \text{ joule} = 1.1 \times 10^{-6} \text{ ev}.$$

The largest *changes* in energy will be comparable to the lowest energy—the order of 10^{-6} ev. Such changes would be undetectable at ordinary temperatures, which correspond to average kinetic energies of the order of 10^{-2} ev; even at 10°K the average kinetic energy of an atom is about 10^{-3} ev.

27–16

Suppose the particle between fixed walls (Section 27–5) is charged and can, therefore, radiate. What is the frequency of its radiation as it goes from the first excited state to the ground state? Express your answer in terms of h, m, and d.

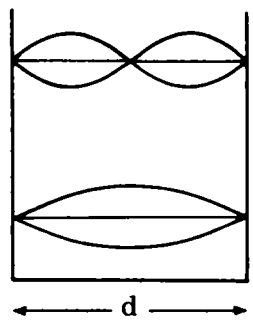
For the two states in question,

$$\lambda_2 = d, \quad p_2 = \frac{h}{\lambda_2} = \frac{h}{d};$$

$$\lambda_1 = 2d, \; p_1 = \frac{h}{\lambda_1} = \frac{h}{2d}.$$

The two energies are:

$$E_1 = \frac{{p_1}^2}{2m} = \frac{h^2}{8md^2}; \quad E_2 = \frac{{p_2}^2}{2m} = \frac{4h^2}{8md^2}.$$



The radiated photon will have energy equal to $E_{\text{photon}} = E_2 - E_1 = 3h^2/8md^2$. Since $E_{\text{photon}} = h\nu$, then $\underline{\nu = 3h/8md^2}$.

The "frequency" of the charged particle (the classical frequency with which it shoots back and forth across the box) is just $v_1/2d$, where v_1 is the classical velocity, i.e., $v_1 = p_1/m$. Thus, the classical particle frequency for the ground state is

$$\frac{p_1}{2m_1 d} = \frac{2h}{8m_1 d^2}.$$

For the first excited state it will be twice this, since $p_2 = 2p_1$. The average of these two frequencies is just the photon frequency. It is not accidental that the two kinds of frequency are comparable in magnitude, but the fact that the wave mechanical frequency is the arithmetic mean of the two lowest "classical" frequencies is a coincidence.

27–17

Suppose an electron moves so as to remain always in one plane within a square of side length d.

(a) How would its minimum energy compare with that of an electron confined to a cubical box of edge length d?

(b) Compare the wavelengths of the radiation

emitted when each of these electrons makes a transition from its first excited state to its ground state.

(a) $E_K = \frac{p_x^2 + p_y^2}{2m}$.

$\frac{n_x\lambda_x}{2} = d;\ \lambda_x = \frac{2d}{n_x}$.

$\frac{n_y\lambda_y}{2} = d;\ \lambda_y = \frac{2d}{n_y}$.

$p_x = \frac{h}{\lambda_x};\ p_y = \frac{h}{\lambda_y}$.

$p_x = \frac{n_x h}{2d};\ p_y = \frac{n_y h}{2d}$.

$E_K = \frac{(n_x^2 + n_y^2)h^2}{8md^2}$.

When $n = 1$, $E_o = \frac{2h^2}{8md^2}$.

For the cubical box, $E_o = \frac{3h^2}{8md^2}$.

Hence the minimum energy is $\frac{2}{3}$ that of the electron confined in the cubical box.

(b) For the electron in a box:

$$E_2 = \frac{12h^2}{8md^2};\ E_1 = \frac{3h^2}{8md^2}$$

$$E_{photon} = E_2 - E_1 = \frac{9h^2}{8md^2} = h^2 = \frac{hc}{\lambda}.$$

$$\lambda = \frac{8md^2c}{9h}.$$

For the electron in a square:

$$E_2 = \frac{8h^2}{8md^2};\ E_1 = \frac{2h^2}{8md^2}.$$

$$E_{photon} = \frac{6h^2}{8md^2} = \frac{hc}{\lambda}.$$

$$\lambda = \frac{8md^2c}{6h}.$$

Hence the wavelength of the radiation emitted by the electron in the square is $\frac{3}{2}$ that emitted by the electron in the box.

27–18

Suppose the particle described in Section 27–6 were confined in a rectangular box whose dimensions were d, d, and $2d$. How would the minimum energy of the system compare with that when it is confined to a cube of side d?

As before, $p_x = \frac{n_x h}{2d}$ and $p_y = \frac{n_y h}{2d}$.

But in this case, $\frac{n_z\lambda_z}{2} = 2d$,

$$\lambda_z = \frac{4d}{n_z}, \quad \text{and} \quad p_z = \frac{n_z h}{4d}.$$

Then $E_x = \frac{p_x^2 + p_y^2 + p_z^2}{2m}$

$$= \frac{h^2}{2md^2}\left(\frac{n_x^2}{4} + \frac{n_y^2}{4} + \frac{n_z^2}{16}\right)$$

$$= \frac{h^2}{32md^2}(4n_x^2 + 4n_y^2 + n_z^2).$$

For the ground state, $E_o = \frac{9h^2}{32md^2}$.

The energy of the ground state is $\frac{3}{4}$ of the energy of the ground state of the particle confined in the cube.

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